Engaging older adults with mathematics

NANAMIC Annual Conference

26th June 2025

David Martin

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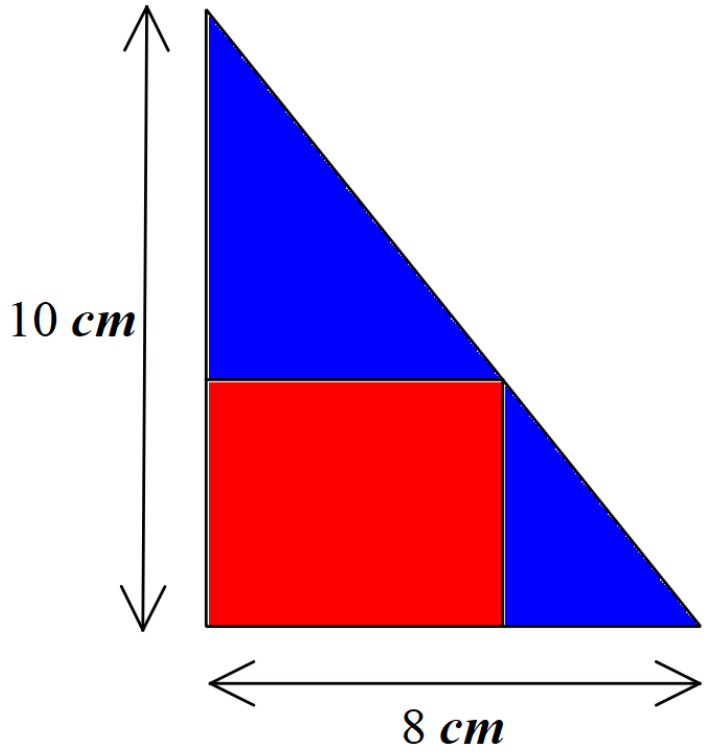
Title: Engaging older adults with mathematics
Session Leader: David Martin

Brief description of session: What might learning mathematics look like for the older adult learner? David will be sharing from his many years of experience as the u3a National Subject Advisor for Mathematics and Statistics. The session will be a blend of presentation and engagement in sample activities.

Profile: David has been a mathematics researcher; mathematics education consultant; FE teacher, teacher trainer and external examiner. As the National Subject Adviser for Mathematics and Statistics for the u3a he works nationally to engage and reengage the retired with mathematics.

Some problems offer rich opportunities for sharing our different approaches

What is the maximum area of a rectangle that can fit within a right-angled triangle, as shown in the diagram?



1. How many different methods could you use to solve this?
2. What would be your method of solution?
3. What are the advantages and disadvantages of your method?

We might derive an equation for the area

The triangles ABC and ADF are similar so AB/BC = AD/DF = 10/8.

Let the width (BC) of the rectangle be w and the height (BD) be h.

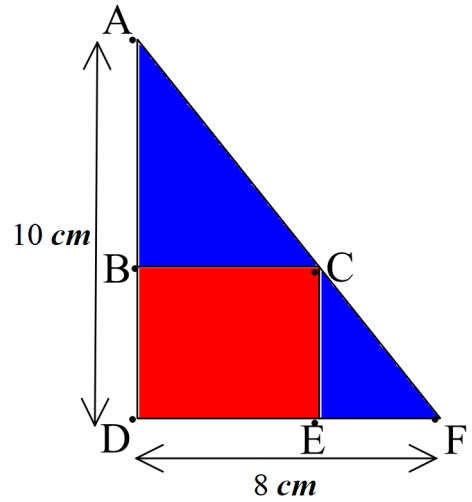
Then AB = 10 - h.

We then have (10 – h)/w = 10/8

Hence, 10w = 80 – 8h

w = 8 – 4h/5.

The area of the rectangle is



$hw = h \times (8 - 4h/5) = 8h - 4h^2/5$ and we want to find the maximum value.

This can be done in several ways.

This equation would not be unique

An alternative method for generating a formula for the area:

Sliding the triangle to position its right angle at the origin, the equation of the hypotenuse is $y = -5/4\ x + 10$

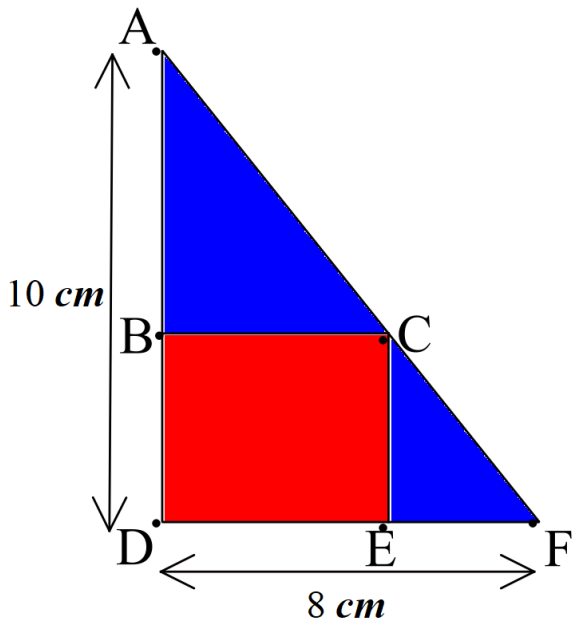
So, with rectangle height h and width w

$h = -5/4\ w + 10$ and area of triangle is

$hw = w(-5/4\ w + 10) = -5/4\ w^2 + 10w$

We seek the maximum of

$-5/4\ w^2 + 10w$



And then draw up a table of areas

Maximum Area of $8h - 4h^2/5$ [or of $10w – 5w^2/4$]

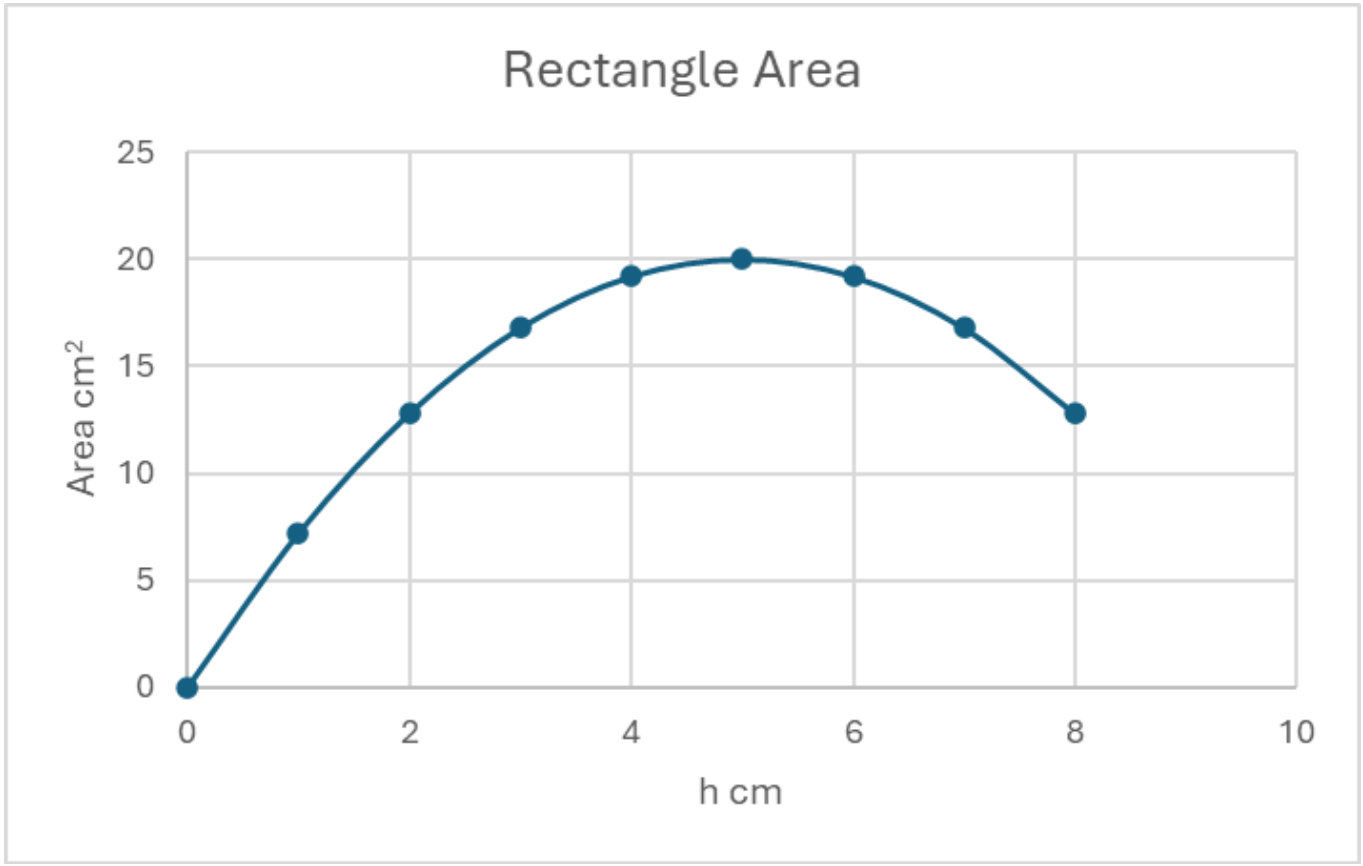
We could complete a table of values of Area for various values of h.

h	Area
3	16.8
4	19.2
5	20.0
6	19.2
4.9	19.992
5.1	19.992

This suggests a maximum area of around $20cm^2$

Or draw a graph of areas

We could draw a graph of Area = $8h - 4h^2/5$



The graph has a maximum of around 20 cm^2

Or we might rearrange the formula

We could rearrange the formula for the Area = $8h - 4h^2/5$ to more easily identify where it is a maximum through completing the square.

Area = $8h - 4h^2/5 = 4/5(10h – h^2) = 4/5(25 - (5 – h)^2)$

which has a maximum of 4/5 x 25 = 20 when h = 5

Or use differentiation

We could use differentiation to find the stationary point shown on the graph.

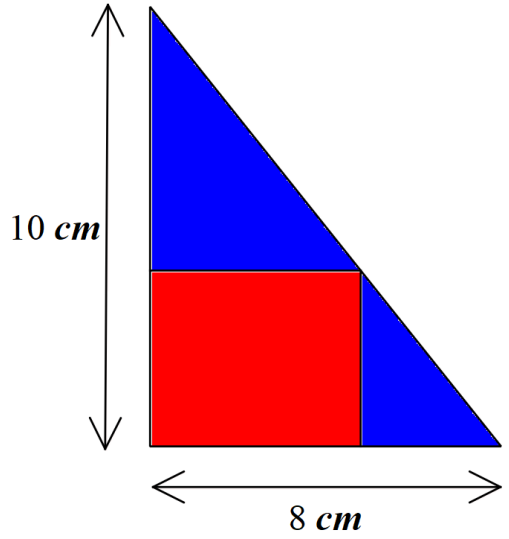
The gradient $8 – 8h/5 = 0$ when $h = 5$ and area = $20cm^2$.

This gradient changes from being positive before $h = 5$ to being negative after $h = 5$, as expected for a maximum.

This is shown by differentiating $8 – 8h/5$ again to give the negative $-8/5$.

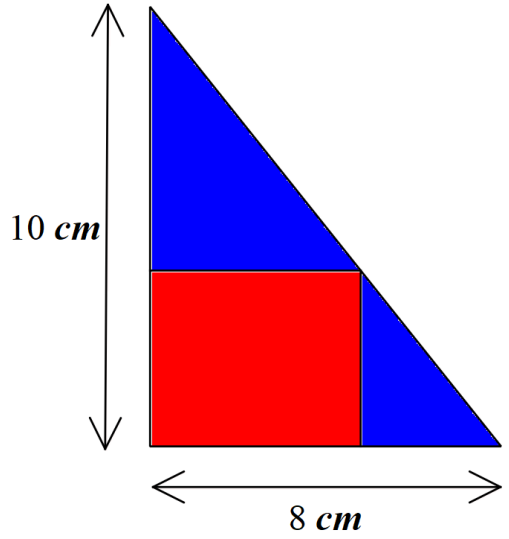
Or compare different possible rectangles

Draw in rectangles of different widths and use the measurements of corresponding heights to determine an estimate for the maximum area.



Or construct different physical models

Construct models of the problem, and fill each with a layer of ball bearings to determine, physically, which contains the greatest number, hence covers the greatest area.



Or work from the general to the specific

For a general triangle of height H and width W

$(H - h)/w = H/W$

i.e., $w = W(H - h)/H$

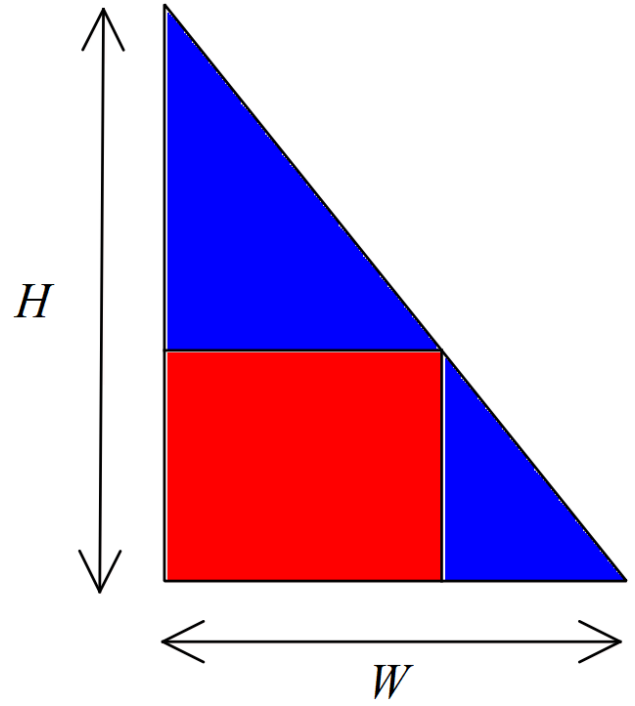
Hence, rectangle Area = $wh = W(H - h)h/H$

$= Wh - Wh^2/H$

This has a maximum when $W - 2Wh/H = 0$

i.e., when $h = H/2$ and $w = W/2$

So, for the given triangle, $h = 10/2 = 5$ cm and $w = 8/2 = 4$ cm

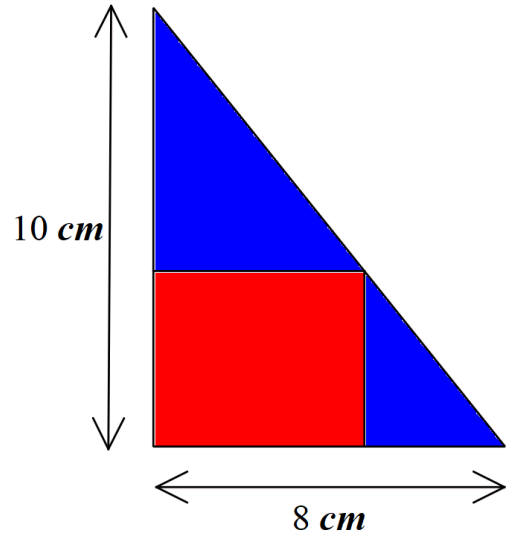


Or try an intuitive method

Intuitively, if the triangle was isosceles with both sides 8cm, then by symmetry we might be led to believe that the maximum area is that of a 4cm by 4cm square of area $16cm^2$.

Stretching the y axis by a factor 10/8 all areas, including the maximum area, stretches by this factor to $16x10/8 = 20cm^2$

How confident are we that this method is valid?



Problems that promote discussion

Such problems enable different approaches to be shared and discussed.

Solutions using different levels of expertise are possible.

This problem can be set in a useful context e.g., a rectangular building is to be built on a right-angled triangular plot as shown in the 1:100 scale drawing. What is the greatest floor area that the building could occupy?

The challenge has a moderate threshold [problems such as the MaxBox problem of maximising the volume of an open box created from cutting out the corners of a sheet of metal and then folding up the four sides has a lower threshold as results (in this case volumes) can be more easily worked out for a range of side lengths.]

Our session is about Engaging older adults with mathematics - the so called Third Age

Defining the Third Age

1st Child: childhood and school education.
2nd Younger adult: work and bringing up children.
3rd Older adult: ceased full time work and children have flown the nest.
[perhaps progressing to the 4th Age of frailty and dependence]

The Third age does not have a lower age limit but simply refers to:
Adults who have ceased full time work and children have flown the nest.

The term Elderly refers to those 65 and over

When hearing the word Elderly, what comes to mind?

This road sign?

Failing senses

Reduced mobility

Fragility

Slowing down

Reduced cognition

Isolation

Loneliness

Or of active engagement in e.g., archaeology, canoeing, circus skills, practical science and walking football (some u3a activities)

Removed pictures of u3a members taking part in the above activities.

Session outline

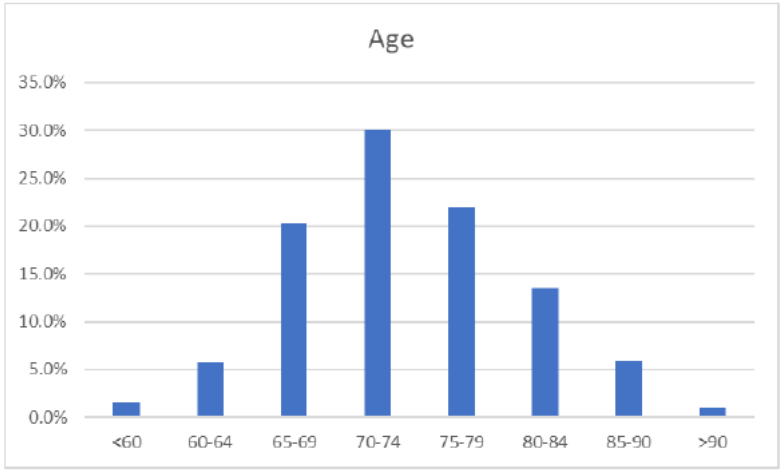
- What is the Third Age?
- Lifelong learning opportunities in mathematics
- Engaging the retired in mathematics e.g., through the u3a
- Introducing 6 of the 1000 weekly maths challenges

u3a has no lower age limit: 85% are 65 to 84

Q1 Gender

Male	29.1%
Female	70.0%
Neither	0.2%
PNS*	0.6%
	100.0%

*Prefer not to say



	Total	Men	Women
<60	1.5%	1.2%	1.7%
60-64	5.8%	4.2%	6.4%
65-69	20.4%	19.3%	20.8%
70-74	30.2%	31.3%	29.7%
75-79	21.8%	21.4%	22.0%
80-84	13.5%	14.9%	12.9%
85-90	5.8%	6.6%	5.5%
>90	1.0%	1.1%	0.9%
	100.0%	100.0%	100.0%

Average age 73.6 years

70% are female

73.6 average age

Many adults participate in mathematics

Using everyday mathematics e.g., which is the better deal – three for the price of two or buy one and get the second half price?; income tax; % discount; litres of paint required

Completing mathematical puzzles in newspapers and puzzle books

Popular maths books e..g., Maths 100 Ideas in 100 words by Sam Hartburn et al.

Watching tv programs e.g., with Marcus du Sautoy and Hannah Fry

Listening to radio programs e.g., ‘More or Less’ with Tim Harford

Some of the impacts of Third Age

Potentially more time to fill.

- As time at work and with children diminishes.
- With the death of a partner or parent.
- Moving to be close to children, leaving behind old commitments.

A danger of increased isolation and loneliness, with an opportunity to: expand on old and engage in new activities

What might motivate the retired to engage with others to learn mathematics?

Given not seeking further qualifications to gain entry to further study or to progress at work.

Maybe it is about keeping our brains active, a lifelong interest in learning things new and a wish for social engagement.

A 2019 u3a membership survey of 1% of the 400,000 members from a random selection of the 1000 u3as sought to answer questions of motivation.

General motivation – keeping brain active, continuing to learn and engaging with others

Q29 With which of the following statements do you agree? Mark all which apply.
U3A membership has:

Improved my social life	82.2%
Broadened or enhanced my interests	72.3%
Increased my knowledge and understanding	69.5%
Given me new interests	67.8%
Improved my physical or mental health	42.9%
Given me new or enhanced skills	40.2%

What might be different about learning for **older** adults?

- set the curriculum themselves
- learn through activities of personal relevance
- are more likely to have reduced motor, sensory and cognitive function
- have collaborative peer learning as a common setting
- utilise a rich variety of past experiences
- can be both a learner and a teacher in different contexts

Definition: The theory and practice of teaching and learning children (Pedagogy), adults (Androgogy) and older adults (Geragogy - Eldergogy)

u3a, a little bit of background – part one.

1973 in Toulouse, France - local University led ‘Université troisième âge’.

1982 in Cambridge, UK - peer learning led u3a (Third Age Trust).

Currently over 400,000 members; 40,000 interest groups; 1000 local u3as (1 virtual) affiliated to a central Third Age Trust providing guidelines, organisational templates, insurance, website and on-line events.

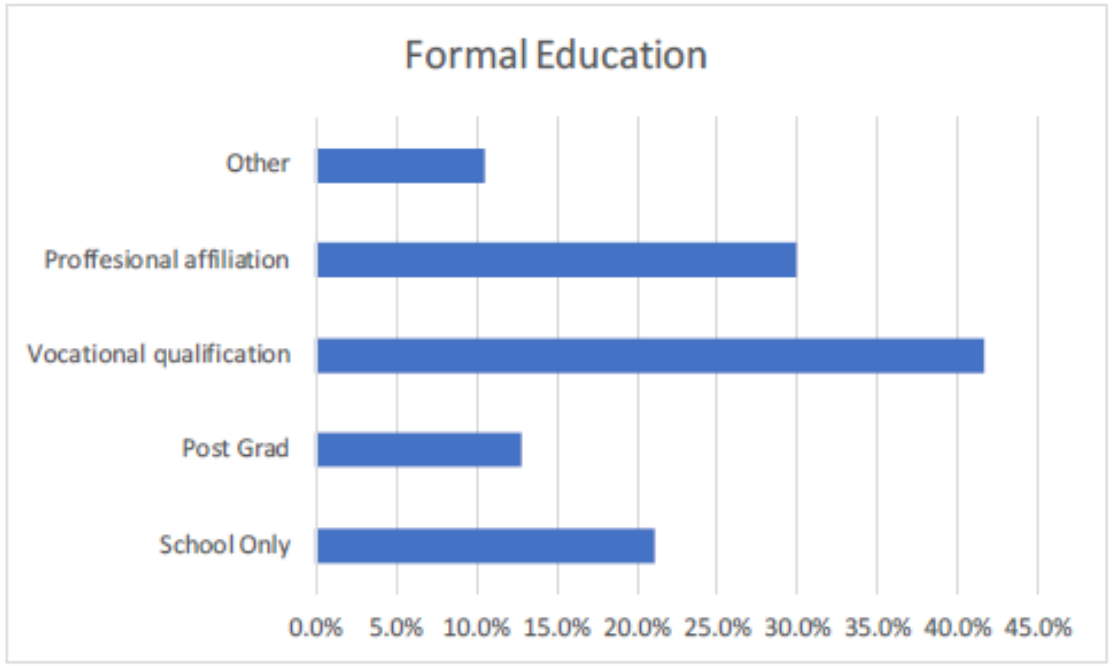
u3a, a little bit of background – part two.

- Typically, a local u3a provides a monthly main meeting with an external speaker and an average of forty different interest groups.
- Nationwide maybe 10% Mathematics and Statistics groups; one virtual
- Levels of mathematics studied from maths for fun to degree level
- The National Subject Advisor for Mathematics and Statistics supports members wishing to engage with mathematics including setting up and maintaining interest groups.
- In addition, the advisor provides weekly problems, organises online events and produces a termly newsletter.
- Most members have experienced education and training beyond school

Formal education – 20% only school education

Q13 At which levels have you experienced formal education? (Mark all that apply)

School (Only)	21.0%
Degree	33.2%
Post Grad	12.8%
Vocational qualification	41.6%
Professional affiliation	29.9%
Other	10.4%



Notes. It is assumed that the majority of people attend school. The percentage shown is for those respondents with no other qualifications beyond school.

Case Studies - part one

Case study one – about to start a group

We are just about to start a maths group - called Maths for Fun. We had our first session and discovered that although some of our students had O level maths they were scared of maths.

So, we are hoping that you will help us with a strategy for tackling their problems and getting them to love maths.

Case study two - just started a group

I have just started a Maths for Grandparents group. The reason for starting the group was the need to speak the same maths language as my grandsons and have a chance of not confusing them with their homework.

If you have any suggestions, we would be pleased to receive them.

Case Studies - part two

Case study three – started a group a year ago

Having seen an article in the Third Age about Maths for Fun I was persuaded to start such a group. This has been running successfully now for over a year. However, I am beginning to run out of ideas for where to take the group.

I would appreciate any assistance you can offer.

Case Study four – started a group two years ago

Our Group started 2 years ago. Initially, we were a small group of people, mostly of O level and a little better standard, and the leader took us through the current course requirements with some A level syllabus. We are a little too small to split into an advanced and a medium group, but we worry about the content of our studies to maintain the interests of such a wide compass of abilities.

Do you have any helpful suggestions?

Puzzling request

Dear David: David is 30, Nigel is 60, and John (me) is 90 this year. I would be delighted if you would construct one of your puzzles involving 30, 60 & 90 for our family enjoyment

Puzzling request

Dear David: David is 30, Nigel is 60, and John (me) is 90 this year. I would be delighted if you would construct one of your puzzles involving 30, 60 & 90 for our family enjoyment

David, Nigel and John are celebrating the oldest, John's birthday. Their combined ages matches that of the highest score achievable with three darts on a dartboard. David's age is the difference between John's age and Nigel's age. Ten years ago John was four times as old as David. How old are David, Nigel and John?

What curriculum do mathematics groups study?

Overall, there is little overlap between two interest group's footprint of level, topics and sequencing.

The content and sequencing decisions reflect members interests and expertise.

With an intent to meet the social, thinking and learning needs of members

In self selecting, roll on roll off, peer learning interest groups

Its impact is measured, in part, by interest groups feedback and sustainability

What's on offer is developed by each interest group, merging members interests and expertise

It is for the retired; no entry requirements and not leading to any exit qualifications

Usually delivered weekly or monthly **in a morning or afternoon session**.

An example of an interest group, my maths for fun group

- Membership of about 16
- Expertise from rusty O’level to doctorate in mathematics
- Membership includes retired FE mathematics teachers, an estate agent, mental health nurse, school support staff,…
- Meets for free in a Fire station community room with display and kitchen facilities
- No set curriculum covers history of mathematics, sharing problems and solutions
- Develops mathematical skills in context – e.g. percentages as income tax, discount
- Key activity is sharing of approaches to problem solving, building on past experiences.

u3a opportunities for members to engage with mathematics

- Membership of a local mathematics interest group or an online group
- Online learning events with mathematics communicators e.g., Rob Eastaway, Ron Knott,…
- Weekly problems uploaded each week in batches of four
- Termly newsletter with contributions from e.g., Colin Foster, Ems Lord, David Martin, Cheryl Periton, Anne Watson,… on a variety of topics.
- Termly problem sheet

u3a Maths Challenge

u3a Maths Challenge

Four problems are uploaded weekly.

Sample solutions are uploaded the following week.

The first two are intended to be easier than the second two.

Used by a number of interest groups as focus of activity.

1000 problems and solutions have been uploaded in the past five years.

https://www.u3a.org.uk/learning/learning-activities/maths-challenge

Illustrative u3a Weekly Problems

Four problems are uploaded weekly.

Sample solutions are uploaded the following week.

The first two are intended to be easier than the second two.

Question 1.

50 people attended a conference, and each of them shook hands (this was a pre-Covid conference) with all the other delegates once and only once. How many handshakes were there altogether?

Question 2.

Two trains, A and B, are travelling towards each other on parallel tracks. Train A is travelling at a speed of 110 km/h, and Train B is travelling at a speed of 90 km/h. The fronts of the two trains are initially 350 kilometres apart. Calculate the time it will take for the fronts of the two trains to pass each other.

Question 3.

Simplify $\sqrt{\frac{1}{2}+1}$ x $\sqrt{\frac{1}{3}+1}$ x $\sqrt{\frac{1}{4}+1}$ x $\sqrt{\frac{1}{5}+1}$ x $\sqrt{\frac{1}{6}+1}$ x $\sqrt{\frac{1}{7}+1}$

Which of the following three questions is
The easiest and why?
The hardest and why?

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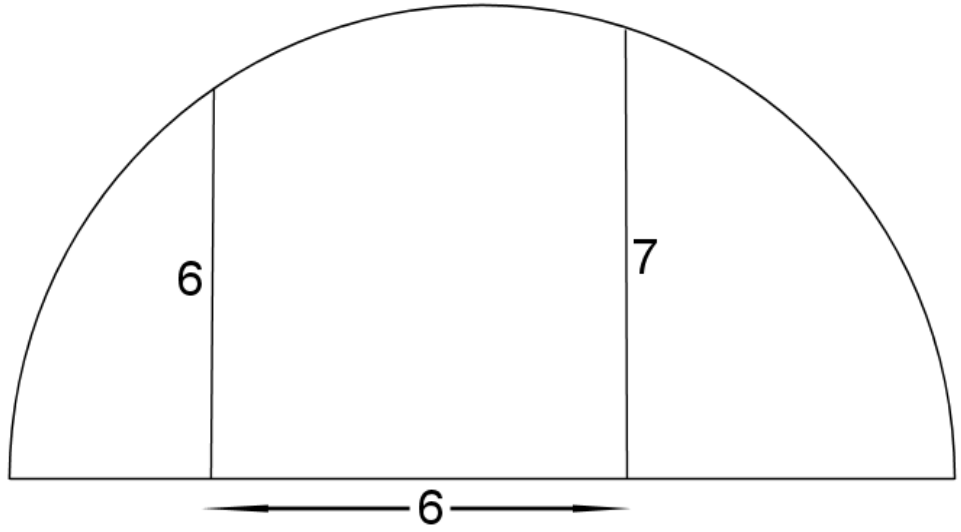
Question 4

Two vertical lines within a semicircle have heights of 6 and 7 units respectively and are 6 units apart:

What is the radius of the semicircle?



Question 5.

Is it possible to express 2027 as the sum of the squares of two whole numbers?

Question 6.

Each point in the plane is coloured either red or blue. Show that there exists at least one pair of points a metre apart which have the same colour.

What makes these problems a greater challenge?

Question 4.

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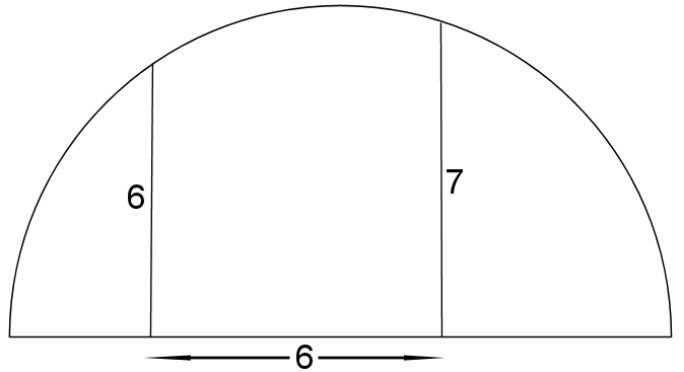

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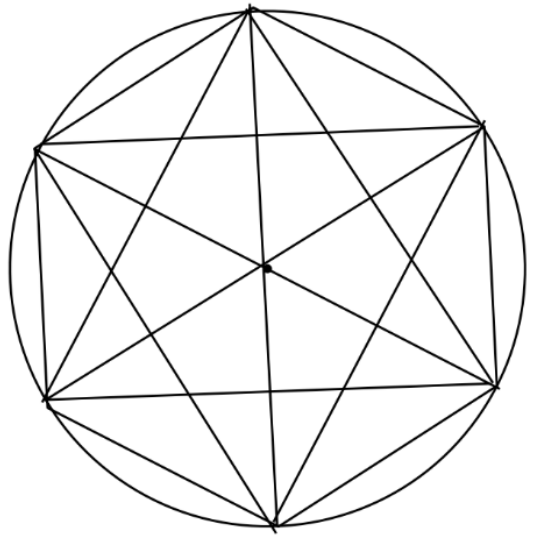
The questions with sample solutions

Question 1.

50 people attended a conference, and each of them shook hands (this was a pre-Covid conference) with all the other delegates once and only once. How many handshakes were there altogether?

Solution 1.

Each person shook hands with 49 other people, but each handshake involved two people. So, the total number of handshakes was 50 x 49 / 2 = 1225.
We can see this diagrammatically by marking fifty points around a circle and joining pairs of points
(For clarity we show this with just six points. Each point represents a person and each of the 6 x 5 / 2 = 15 lines represents a handshake)
Combinatorically we are counting the number of ways of selecting a set of two from fifty. i.e.,



$$\binom{50}{2} = \frac{50!}{(50-2)!\,2!} = \frac{50!}{48!\,2!} = \frac{50 \times 49}{2} = 1225$$

Question 2.

Two trains, A and B, are travelling towards each other on parallel tracks. Train A is travelling at a speed of 110 km/h, and Train B is travelling at a speed of 90 km/h. The fronts of the two trains are initially 350 kilometres apart. Calculate the time it will take for the fronts of the two trains to pass each other.

Solution 2.

Two trains, A and B, are travelling towards each other on parallel tracks. Train A is travelling at a speed of 110 km/h, and Train B is travelling at a speed of 90 km/h. The fronts of the two trains are initially 350 kilometres apart. Calculate the time it will take for the fronts of the two trains to pass each other.

Solution
Since the trains are moving towards each other, their relative speed is the sum of their individual speeds.
Relative speed = 110 km/h + 90 km/h = 200 km/h
Time = Distance ÷ Relative speed = 350 km ÷ 200 km/h = 1.75 hours
Therefore, it will take 1 hour 45 minutes for the two trains to reach their passing point.

Question 3.

Simplify $\sqrt{\frac{1}{2}+1}$ x $\sqrt{\frac{1}{3}+1}$ x $\sqrt{\frac{1}{4}+1}$ x $\sqrt{\frac{1}{5}+1}$ x $\sqrt{\frac{1}{6}+1}$ x $\sqrt{\frac{1}{7}+1}$

Solution 3.

Simplify $\sqrt{\frac{1}{2}+1}$ x $\sqrt{\frac{1}{3}+1}$ x $\sqrt{\frac{1}{4}+1}$ x $\sqrt{\frac{1}{5}+1}$ x $\sqrt{\frac{1}{6}+1}$ x $\sqrt{\frac{1}{7}+1}$

Solution

The expression can be written as

$\sqrt{\frac{3}{2}} \times \sqrt{\frac{4}{3}} \times \sqrt{\frac{5}{4}} \times \sqrt{\frac{6}{5}} \times \sqrt{\frac{7}{6}} \times \sqrt{\frac{8}{7}}$ or $\sqrt{\frac{3 \times 4 \times 5 \times 6 \times 7 \times 8}{2 \times 3 \times 4 \times 5 \times 6 \times 7}}$

cancel $\sqrt{\frac{(3 \times 4 \times 5 \times 6 \times 7) \times 8}{2 \times (3 \times 4 \times 5 \times 6 \times 7)}}$ leaving $\sqrt{\frac{8}{2}} = \sqrt{4} = 2$

Question 4

Two vertical lines within a semicircle have heights of 6 and 7 units respectively and are 6 units apart:

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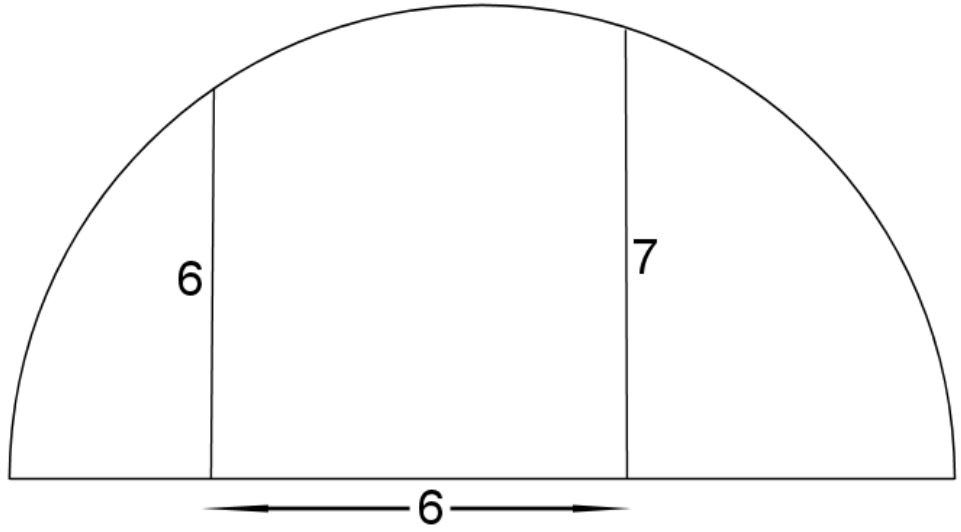

Solution 4

Two vertical lines within a semicircle have heights of 6 and 7 units respectively and are 6 units apart:

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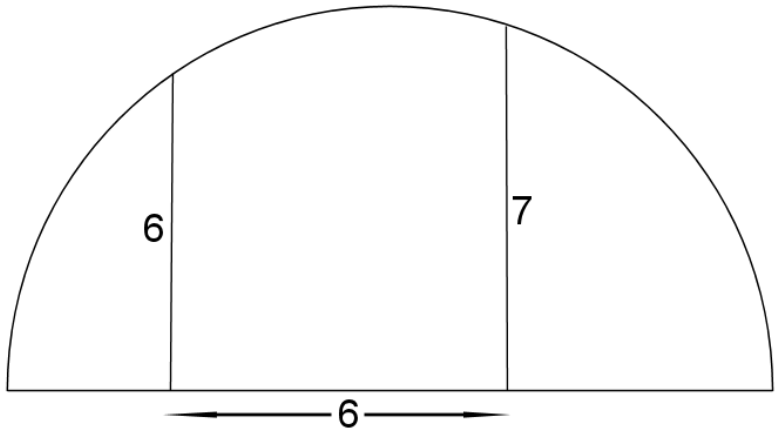
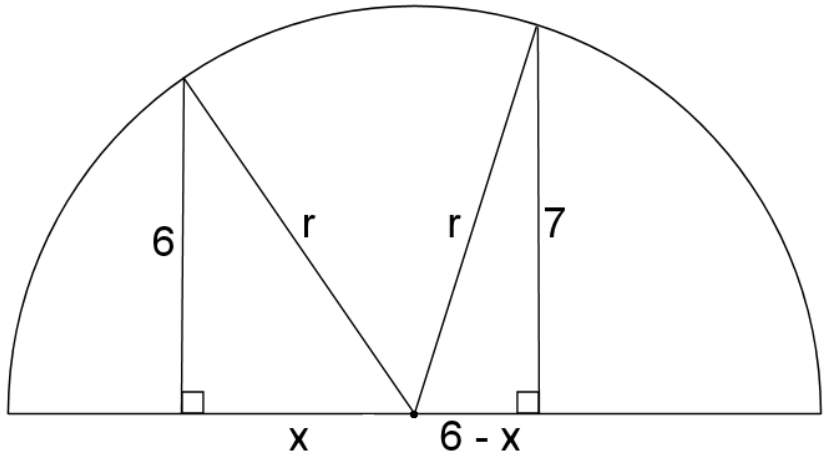


Solution



$x^2 + 6^2 = r^2 = (6 - x)^2 + 7^2$

So, $x = 49/12$

The radius r of the semicircle is therefore 7.26 to 2 decimal places.

Question 5.

Is it possible to express 2027 as the sum of the squares of two whole numbers?

Solution 5.

Is it possible to express 2027 as the sum of the squares of two whole numbers?

Solution

The answer is no.

We prove the result by contradiction.

Suppose m and n are two whole numbers such that $m^2 + n^2 = 2027$.

One of m and n (say n) must be odd and the other one must be even:

Hence $2027 = m^2 + n^2 = (2a)^2 + (2b + 1)^2$ where a and b are whole numbers.

So, $2027 = 4(a^2 + b^2 + b) + 1$

But, 2027 cannot be written as a multiple of 4 plus 1.

Question 6.

Each point in the plane is coloured either red or blue. Show that there exists at least one pair of points a metre apart which have the same colour.

Solution 6.

Each point in the plane is coloured either red or blue. Show that there exists at least one pair of points a metre apart which have the same colour.

Solution

Draw any equilateral triangle of side one metre. At least 2 of the vertices must have the same colour.

u3a resources

u3a Maths Challenge (1000 problems and solutions)
https://www.u3a.org.uk/learning/learning-activities/maths-challenge

u3a Maths and Stats (subject advice including newsletters & resources)
https://www.u3a.org.uk/learning/subjects/maths-and-stats

u3a Online learning events (this part is for u3a members only)
https://www.u3a.org.uk/events/educational-events

u3a publications

1. Learning not Lonely

https://www.u3a.org.uk/our-impact/u3a-reports/learning-not-lonely

u3a report based on a literature review focussed on the impact of lifelong learning on confidence, mental health, building friendships and communities and learning skills, and on member responses to questionnaire (801) and focus groups.

2. Learn to Live

https://www.u3a.org.uk/our-impact/u3a-reports/learn-to-live

u3a report on the value of a low cost, non-formal learning model for promoting health and wellbeing in later life

3. The U3A Story

Beckett, F. (2014). The U3A Story, 2nd edn, Bromley, Kent: The Third Age Trust.

A short history of the u3a

Engaging older adults with mathematics

NANAMIC Annual Conference

26th June 2025

Thank you

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u3a Maths Challenge (1000 problems and solutions)

https://www.u3a.org.uk/learning/learning-activities/maths-challenge

u3a Maths and Stats (subject advice including newsletters & resources)

https://www.u3a.org.uk/learning/subjects/maths-and-stats