Fun with Platonic Solids
NANAMIC

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Part One – Introduction to the Platonic Solids

Part Two – Introducing ten activities

Part Three – Sharing approaches and sample solutions

Why worth getting to know the Platonic solids?

Dodecahedron - ‘One of the greatest most mysterious archaeological objects’

Icosahedron – Coronavirus

Octahedron - Natural diamond crystals

Cube - Common six-sided dice as are sugar cubes.

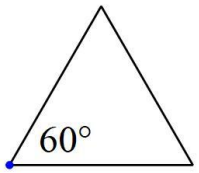
Tetrahedron - Methane has a structure with a carbon atom at its centre and a hydrogen atom at each of its four corners. These corners are called vertices.

Plato in 360 BC associated Tetrahedron with fire ; Cube with earth ; Octahedron with air ; Icosahedron with water ; Dodecahedron with the cosmos or universe

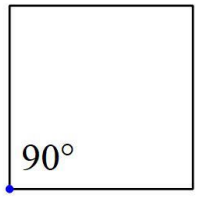
Euclid’s Elements 300 BC builds to the properties of the Platonic solids in Book XIII

In addition, to their great mathematical interest these five platonic solids are worth getting to know better and today through ten activities we will seek to do that.

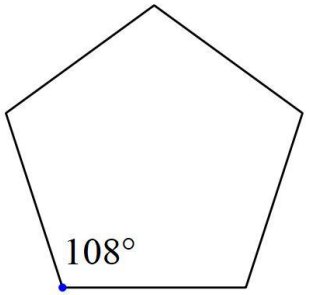
Platonic Solids – what are they?



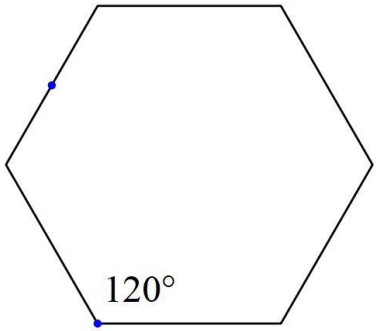
Equilateral Triangle



Square



Pentagon



Hexagon

Some regular polygons – and their internal angles.

In a Platonic solid

1. Each face is an identical (congruent) regular polygon.
2. An equal number of faces meet at each vertex.

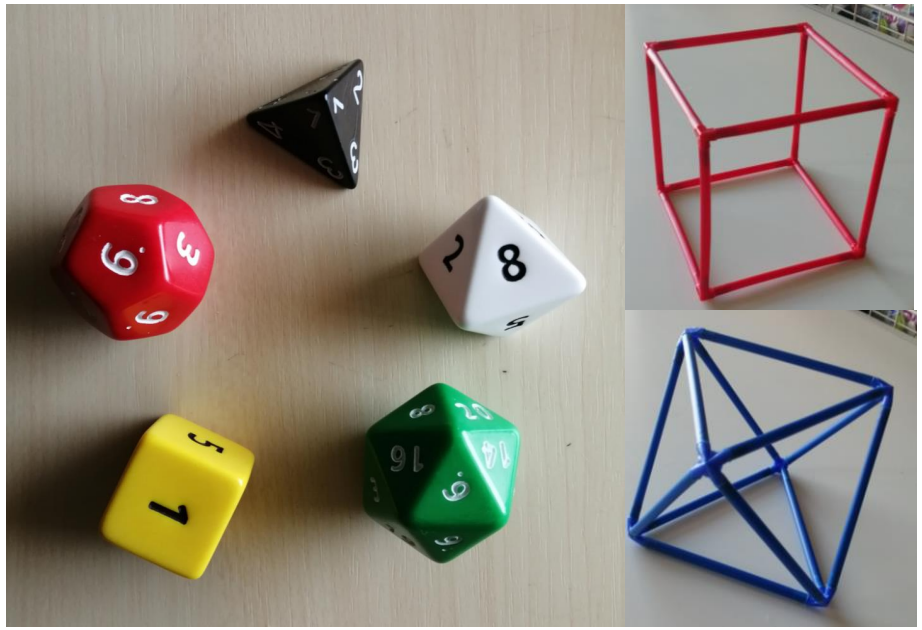
For example, in a cube

1. Each face is an identical square
2. Three faces meet at each vertex

For example, in a tetrahedron

1. Each face is an identical equilateral triangle
2. Three faces meet at each vertex

Examining and building Platonic solid models develops awareness
Some Platonic solid models – using nets and ‘straws’



Dodecahedron

Icosahedron

Hexahedron

Tetrahedron

Octahedron

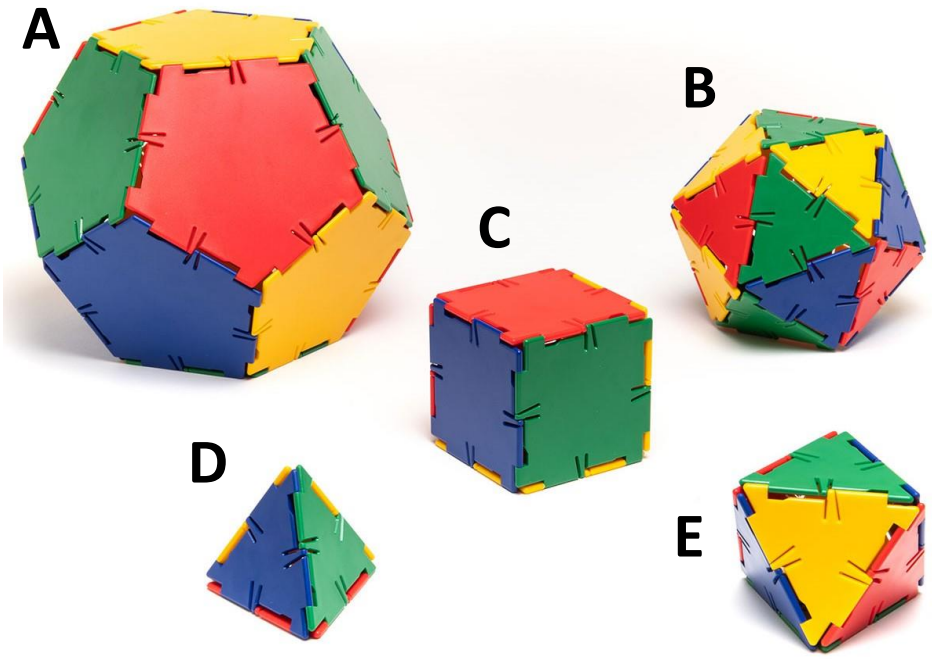
With thanks to Polydron
(https://www.polydron.co.uk)

Part Two – Introducing ten activities

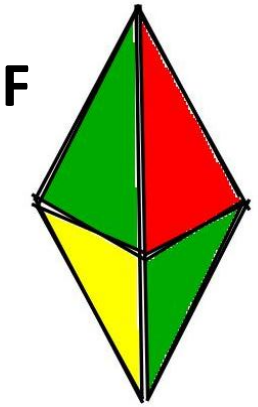
Activities 1 to 5
1. What is a Platonic solid?
2. What is a valid cube net?
3. Number of cubic dice?
4. Ways to colour a tetrahedron
5. Examine the cube’s geometry

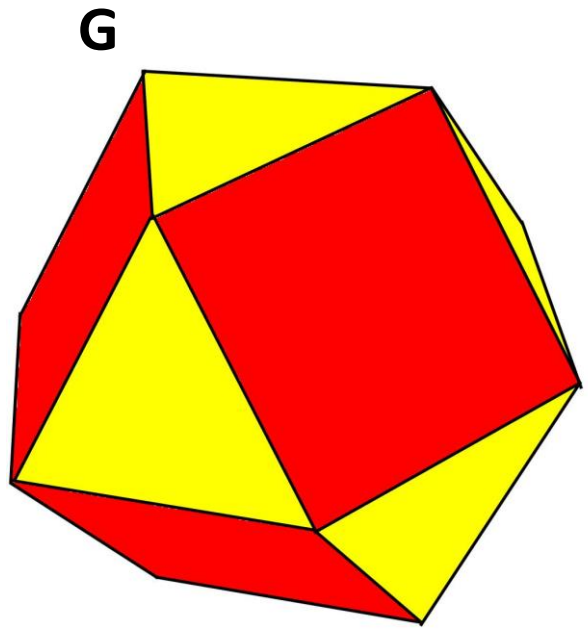
Activities 6 to 10
6. Find and apply a formula
7. How many Rubik’s Cubes?
8. Classifying Platonic Solids
9. Properties of Platonic Dice
10. Why only five Platonic Solids?

Q1. Platonic Solids – identical regular polygon faces and identical vertices



Platonic Solids
Match the Platonic Solids A to E
1. Tetrahedron
2. Cube - Hexahedron
3. Octahedron
4. Dodecahedron
5. Icosahedron

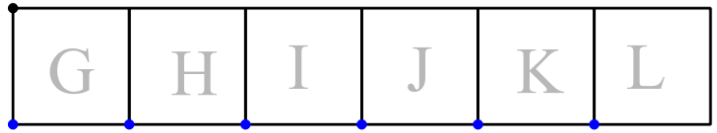




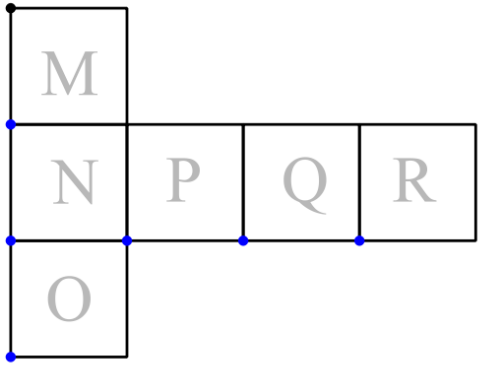
Non examples of Platonic Solids
F (Triangular Bipyramid with its six faces)
G (Cuboctahedron) are not Platonic Solids
Why are these not Platonic Solids?

Q2. Cube nets

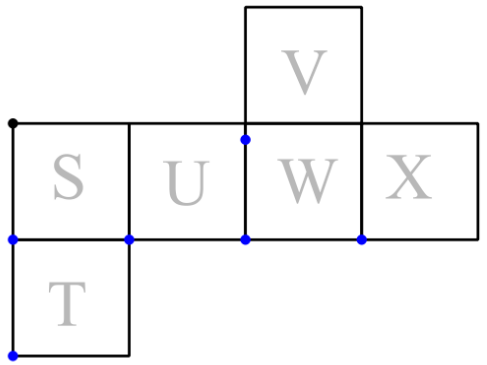
A



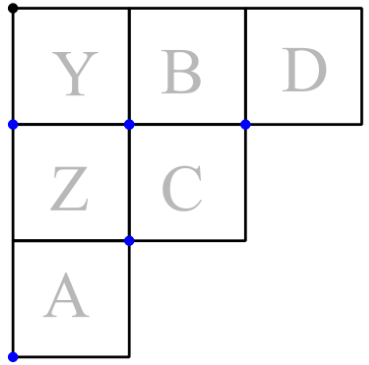
B



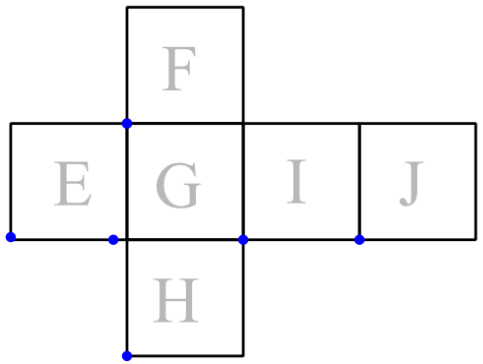
C



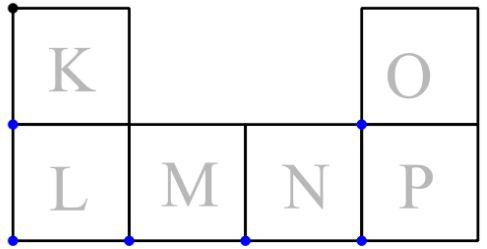
D



E

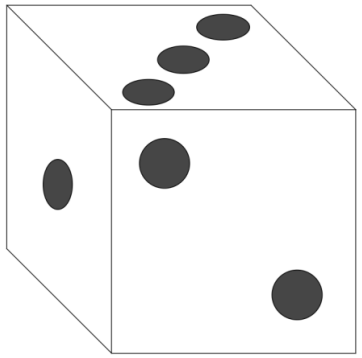


F

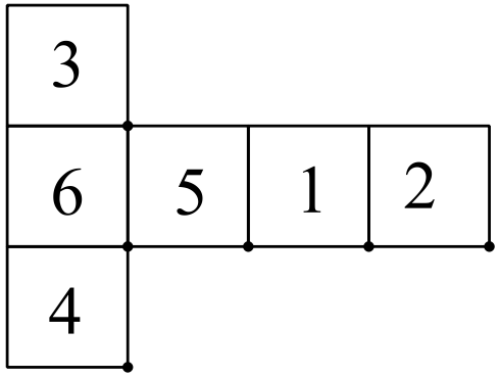


Which of these nets do not form cubes?

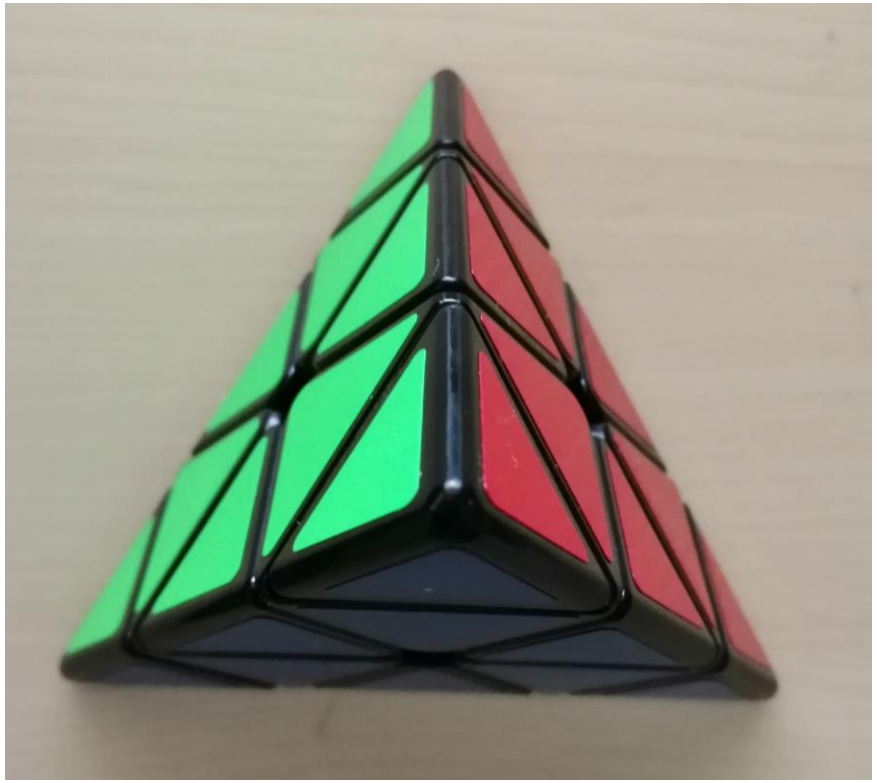
Q3. How many different dice?



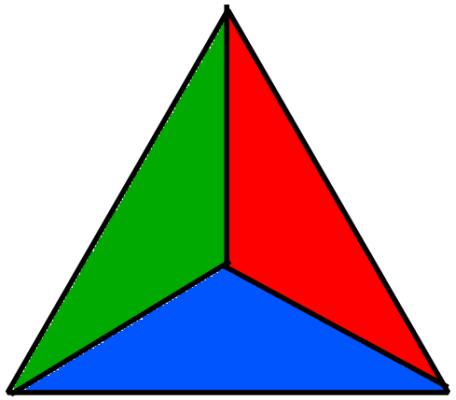
Opposite sides of a six-sided die sum to 7 i.e., 6 is opposite 1, 5 is opposite 2, and 4 is opposite 3. How many possible dice are there?



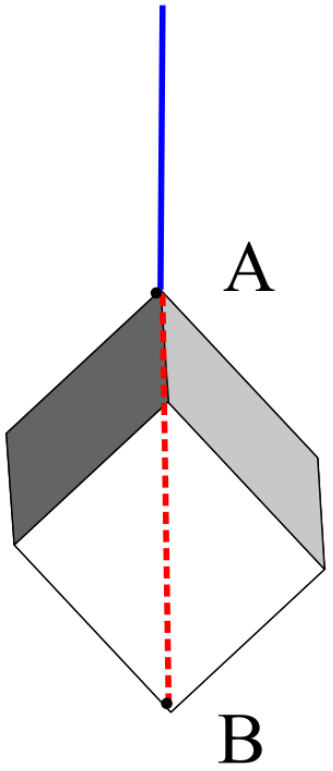
Q4. How many ways to paint a tetrahedron?



A tetrahedron is to be coloured
In Red, Green, Blue and Yellow,
with a different colour on each face.
How many ways could this be done?



Q5. Shortest distance over a cube



A cube with 5cm edges is suspended by one of its vertices at A.
An insect starting at A wishes to crawl to B
It takes the route shown in red
Could the insect have taken a shorter route?

Q6. What are the numbers of Vertices, Edges and Faces and how are these numbers connected?

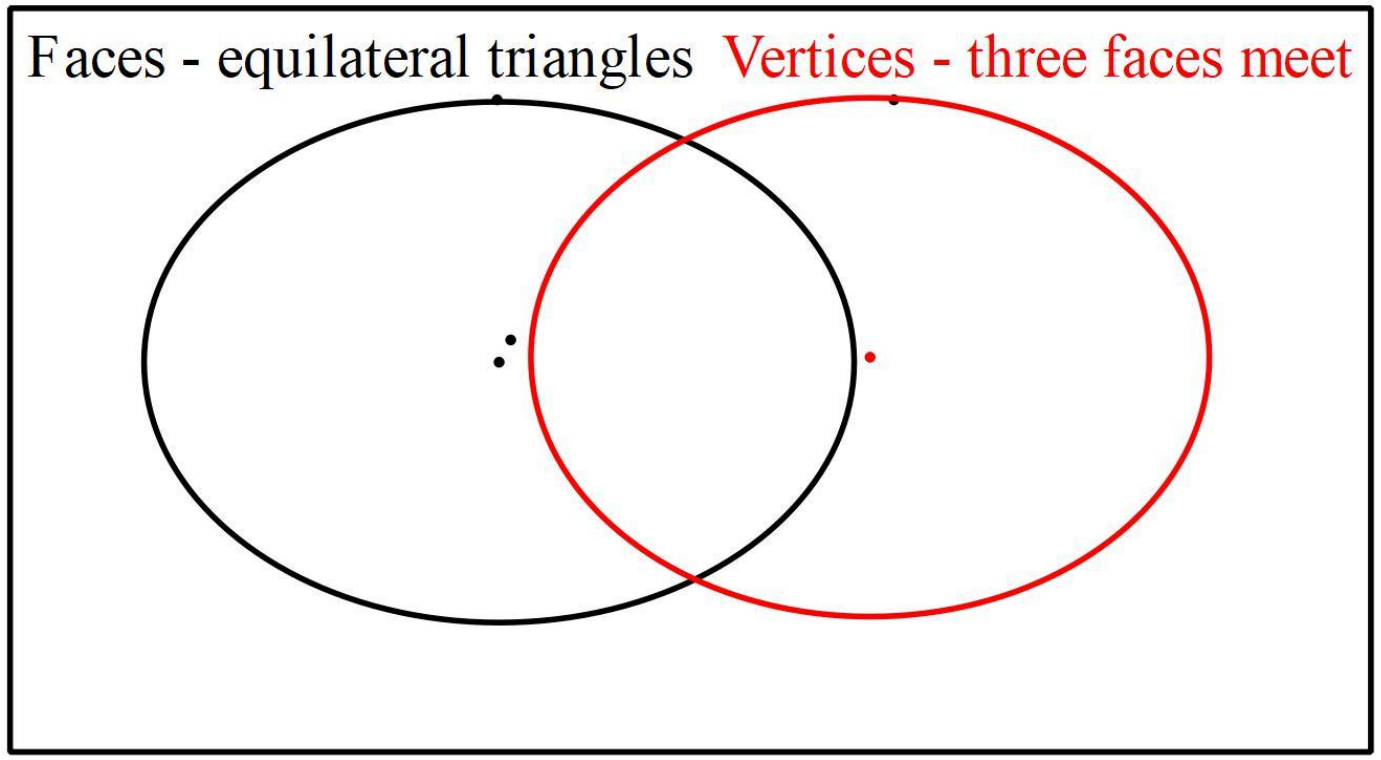
Platonic Solid	**Vertices (V)**	**Edges (E)**	**Faces (F)**
Tetrahedron			
Cube (Hexahedron)			6
Octahedron	6		8
Dodecahedron	20	30	12
Icosahedron	12	30	20

Find the formula connecting V, E and F and use it to check and complete the table

Q7. How many Rubik's Cubes?

Opposite sides of a Western six-sided Rubik's cube are White and Yellow (W + Y); Blue and Green (B + G); Red and Orange (R + O). Note that adding Yellow to the first colour gives the opposite colour. How many different Rubik's cubes are possible colouring in this way?

Q8. Classifying Platonic Solids – similar and different



The Platonic Solids

Tetrahedron
Cube (Hexahedron)
Octahedron
Dodecahedron
Icosahedron

Complete the Venn diagram with the names of the five Platonic solids

Q9. Platonic dice

The dice on the left are made from Platonic solids. The cubic die can be used to randomly select between six alternatives, in this case between 1, 2, 3, 4, 5 and 6.

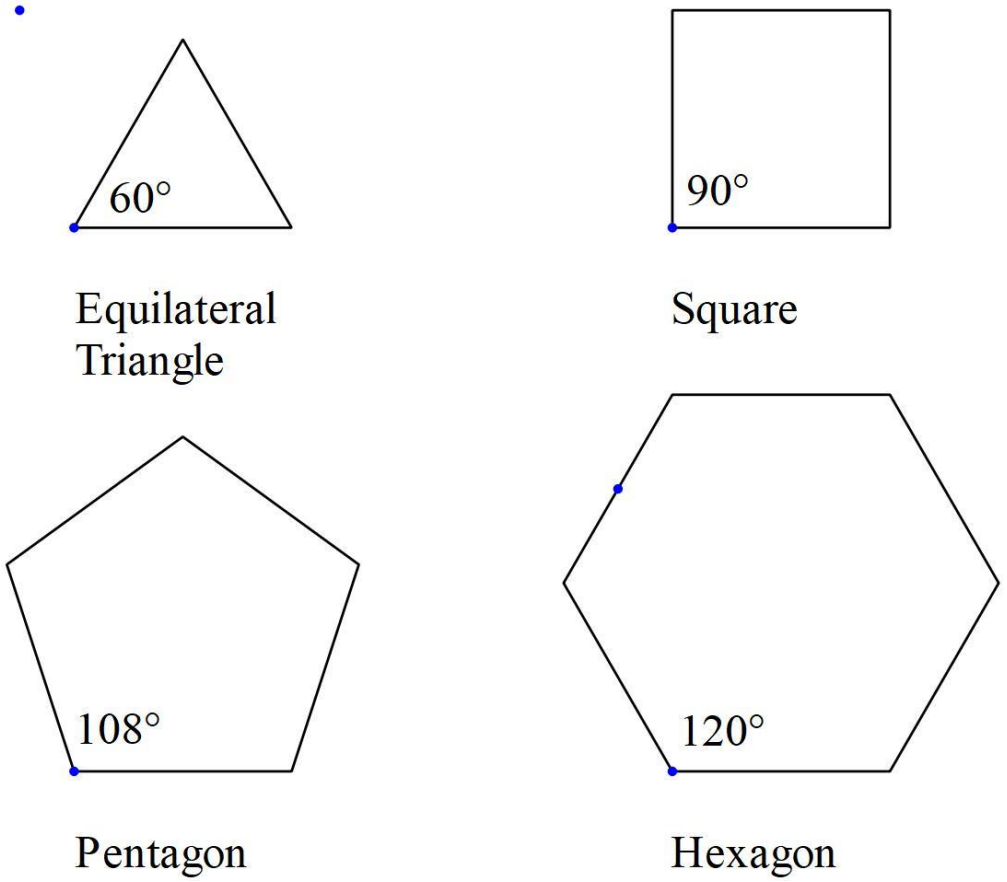
1. How many alternatives for each die?

2. Why is the way you read the score on the tetrahedral die different from the others?

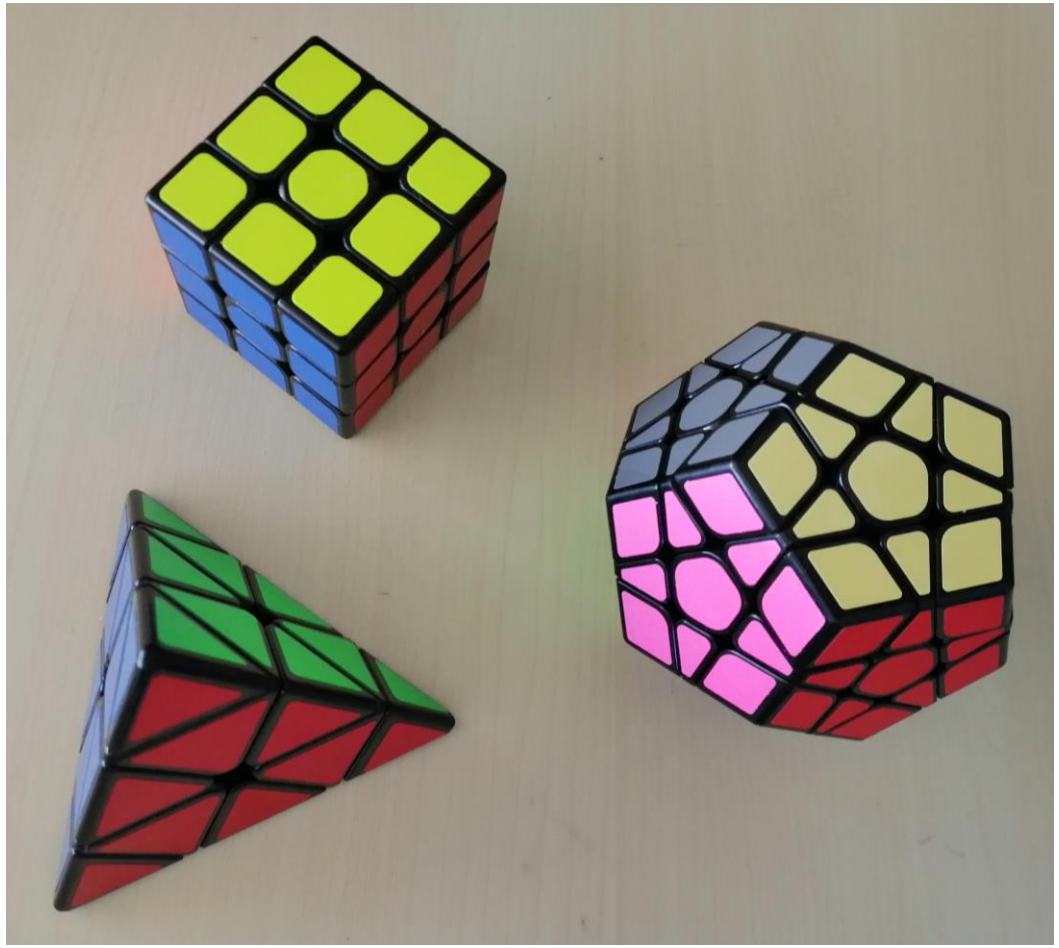
3. What do you notice about the two dice below?

Q10. Platonic Solids have identical (regular polygon) faces and identical vertices – Why only five Platonic solids?

Some regular polygons and their internal angles



Three of the Platonic solids



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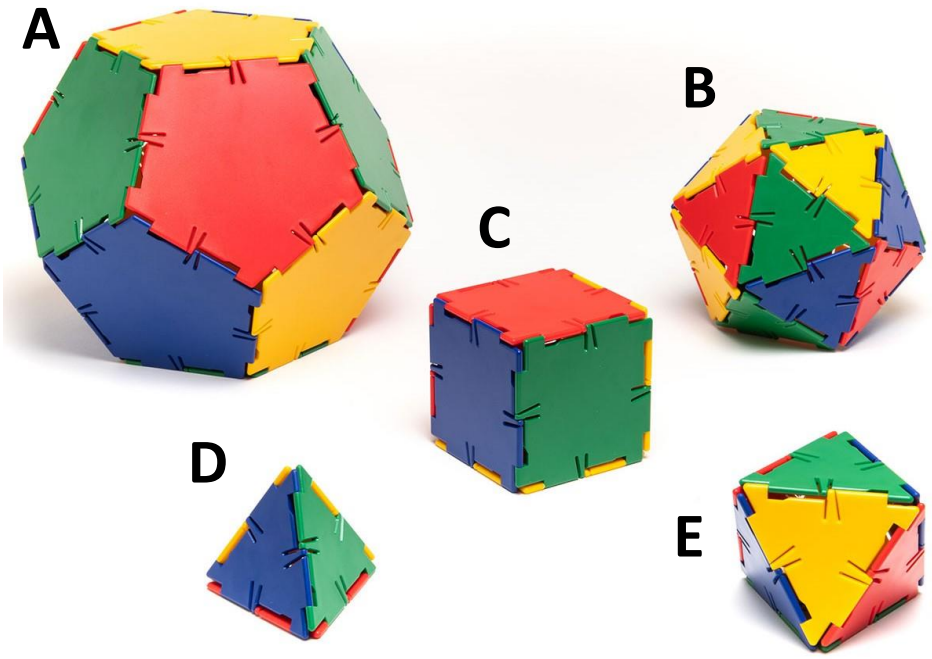

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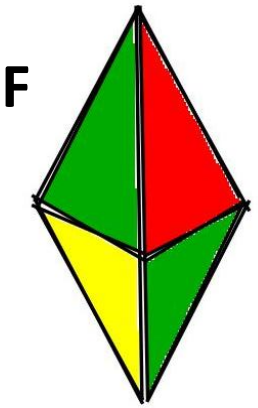

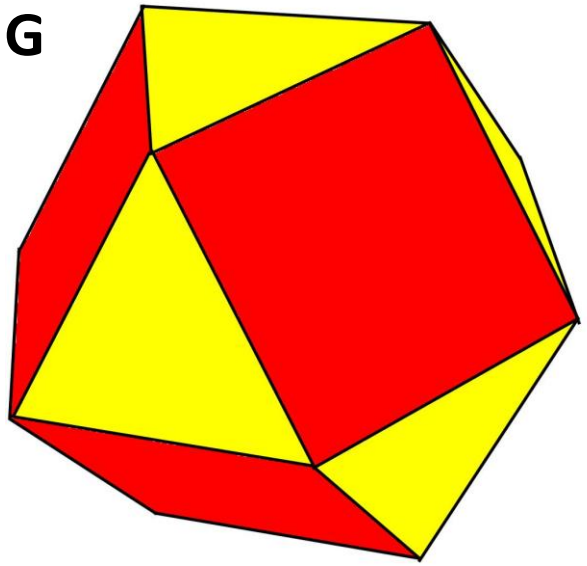

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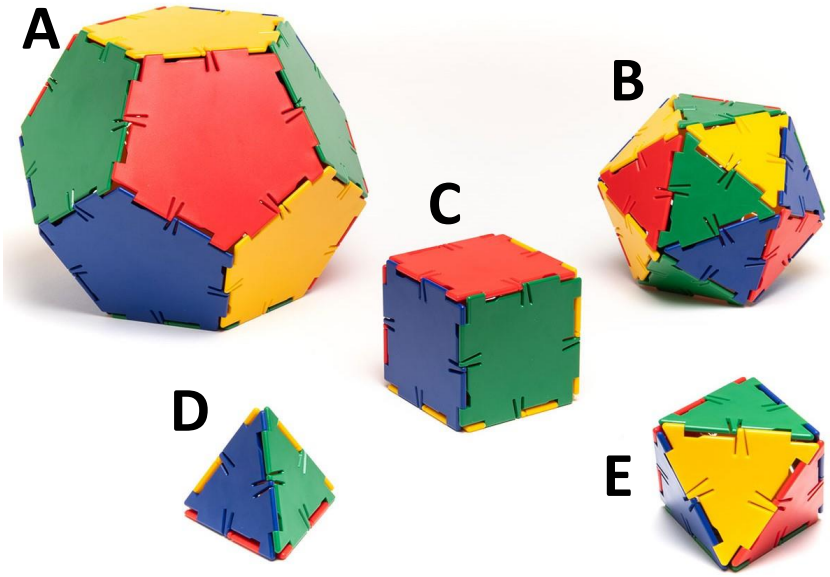


Platonic Solids

Match the Platonic Solids A to E

1. Tetrahedron *(D)*
2. Cube – Hexahedron *(C)*
3. Octahedron *(E)*
4. Dodecahedron *(A)*
5. Icosahedron *(B)*

Non examples of Platonic Solids

F (Triangular Bipyramid with its six faces)

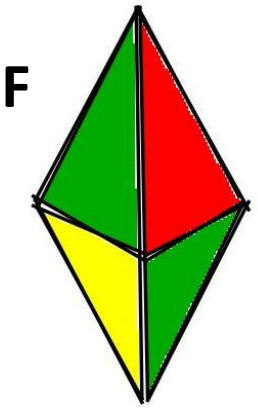
G (Cuboctahedron) are not Platonic Solids

Why are these not Platonic Solids?

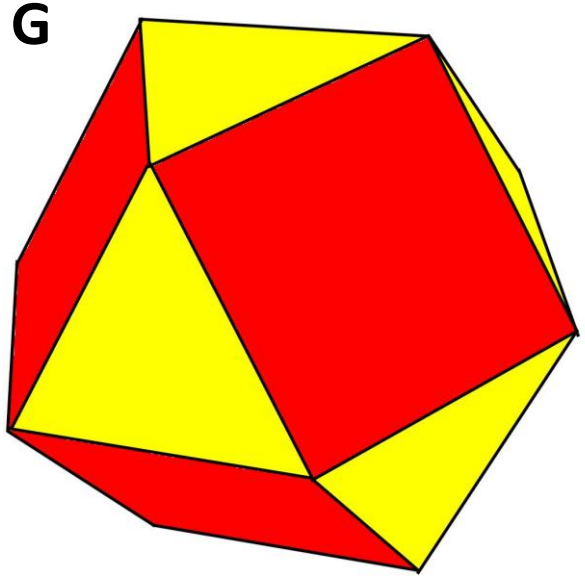
F – mix of 3 and 4 triangular faces meet at vertices.

G – mix of 4 triangular and square faces at vertices

F

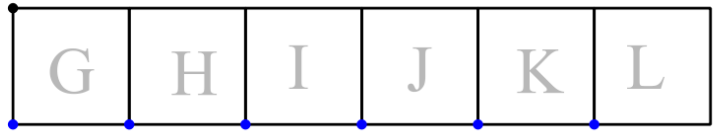


G

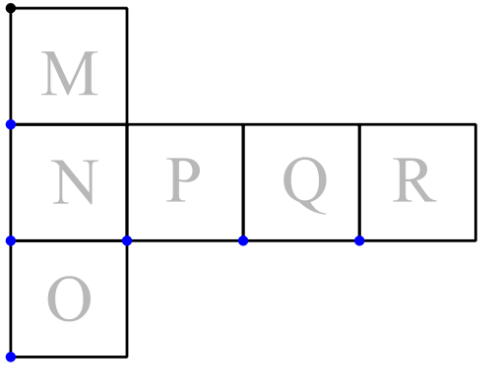


Q2. Cube nets

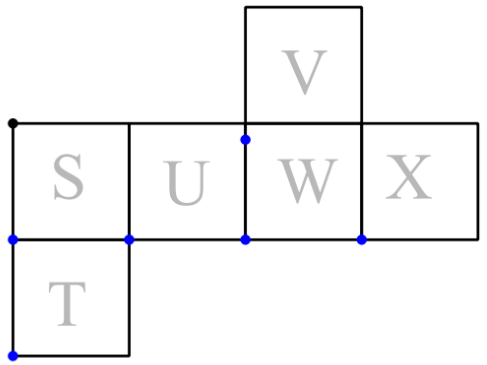
A



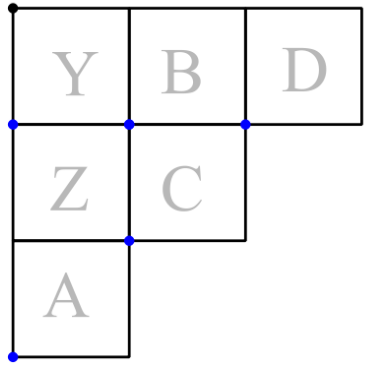
B



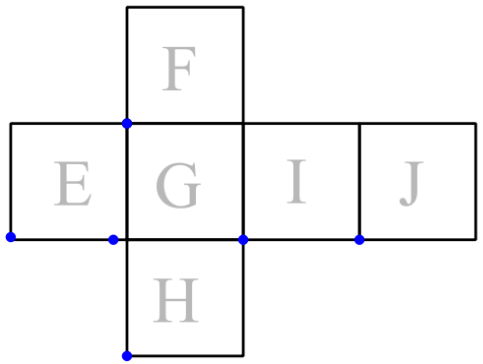
C



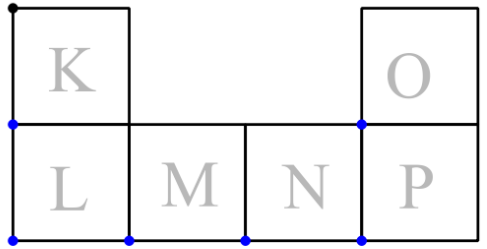
D



E



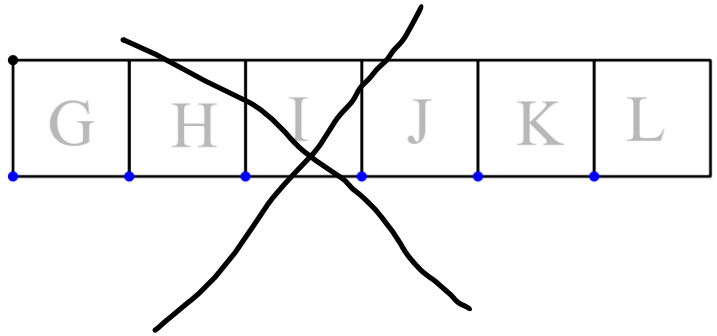
F



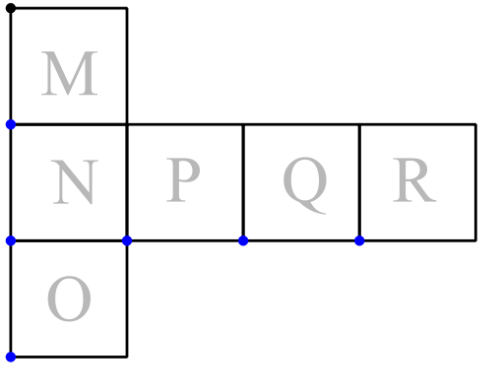
Which of these nets do not form cubes?

S2. Cube nets

A



B



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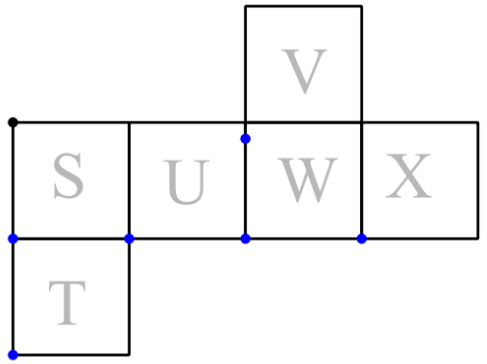
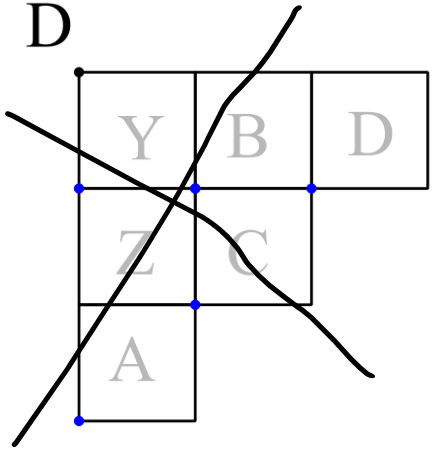
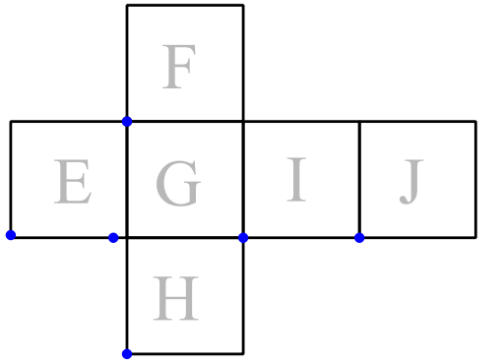


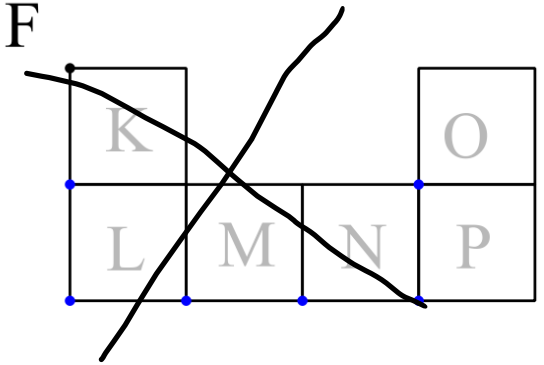
D



E

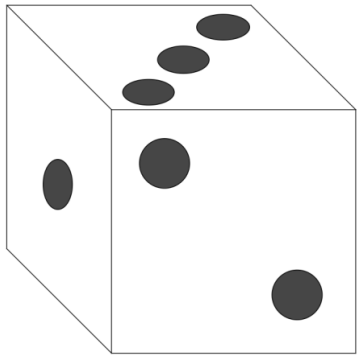


F

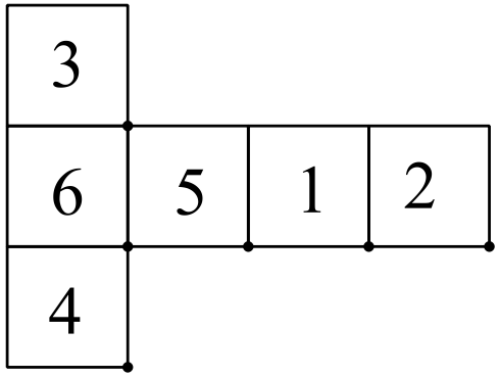


Which of these nets do not form cubes?

Q3. How many different dice?

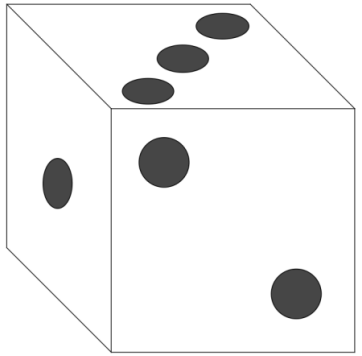


Opposite sides of a six-sided die sum to 7 i.e., 6 is opposite 1, 5 is opposite 2, and 4 is opposite 3. How many possible dice are there?

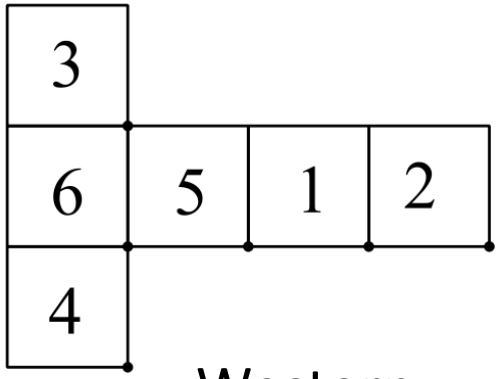


Spotlight

S3. How many different dice?



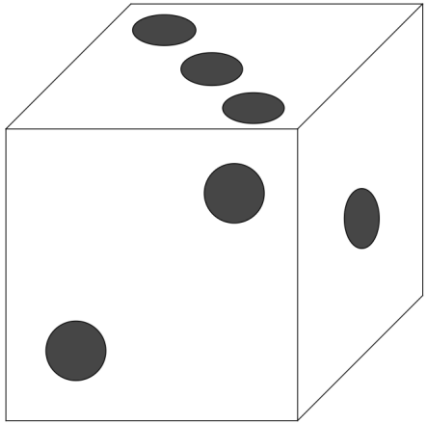
1,2,3 Anticlockwise



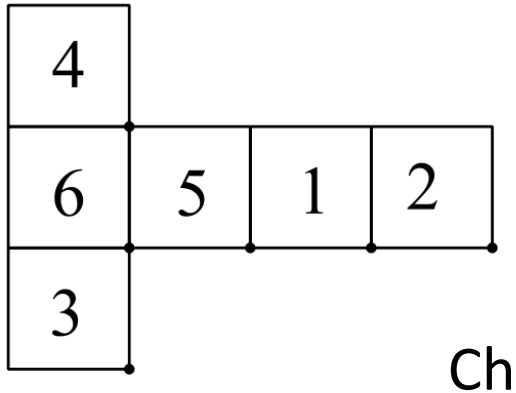
Western – right-handed

Opposite sides of a six-sided die sum to 7 i.e., 6 is opposite 1, 5 is opposite 2, and 4 is opposite 3. How many possible dice are there?

Placing the die with 3 uppermost and 1 facing, the 2 could either be to the right or left.

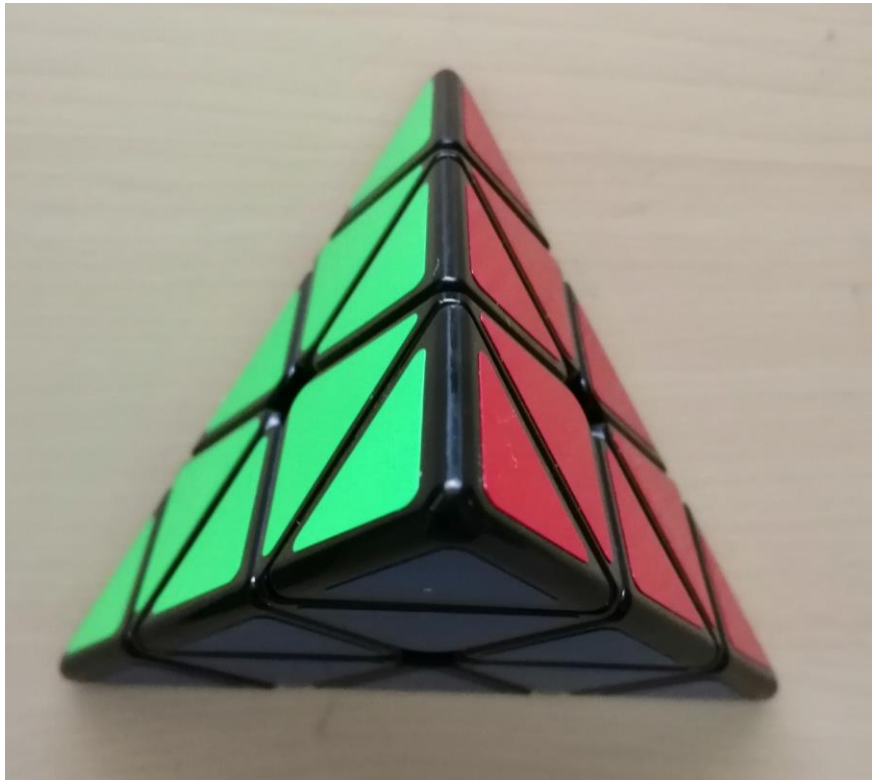


1,2,3 Clockwise

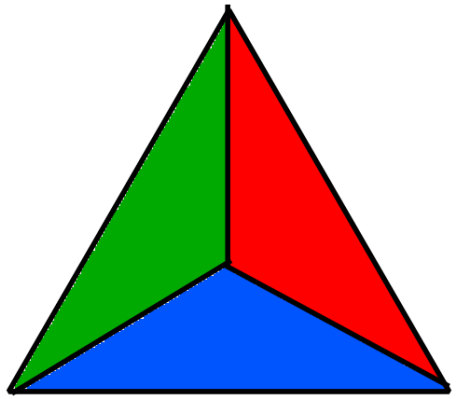


Chinese – left-handed

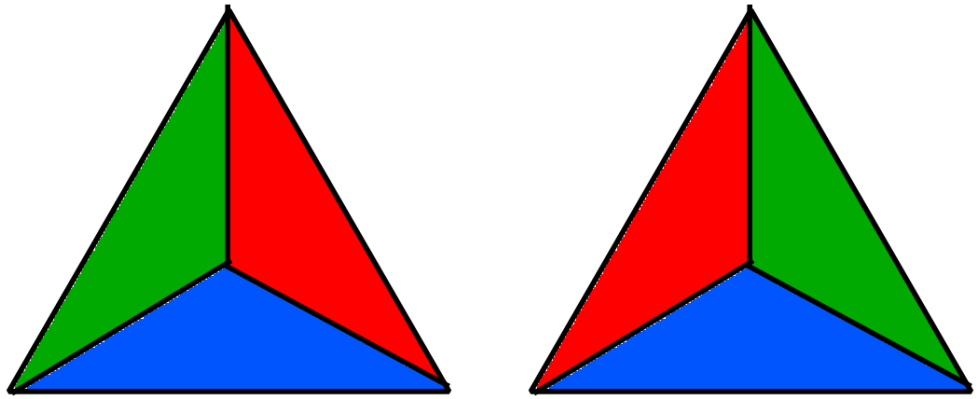
Q4. How many ways to paint a tetrahedron?



A tetrahedron is to be coloured
In Red, Green, Blue and Yellow,
with a different colour on each face.
How many ways could this be done?

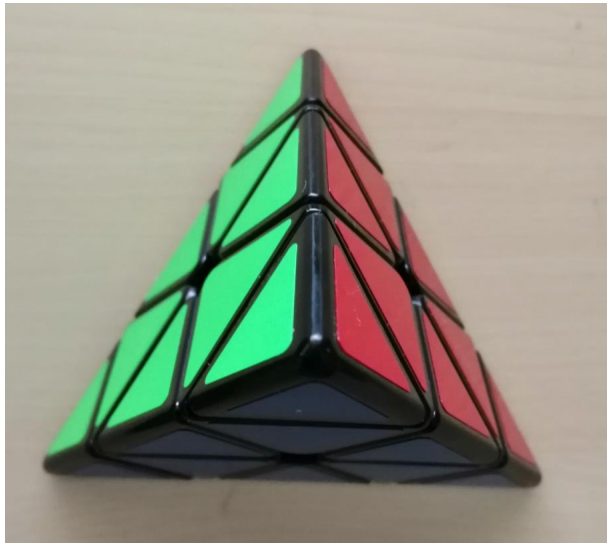


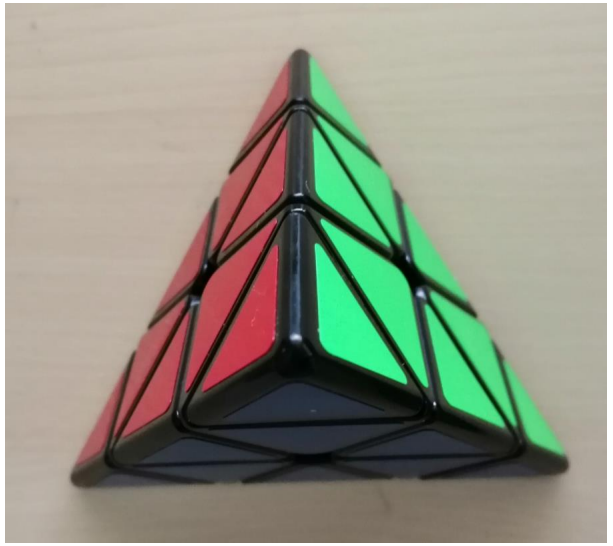
S4. How many ways to paint a tetrahedron?



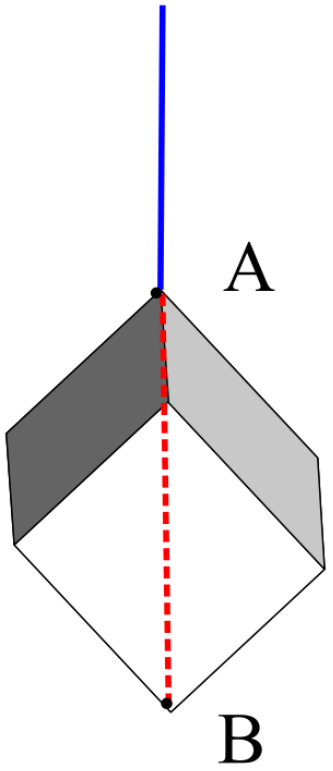
A tetrahedron is to be coloured
In Red, Green, Blue and Yellow,
with a different colour on each face.
How many ways could this be done?

Placing the tetrahedron on its Yellow base with the Blue face facing, the Red face could either be on the right or the left. There are, therefore, two ways to colour.



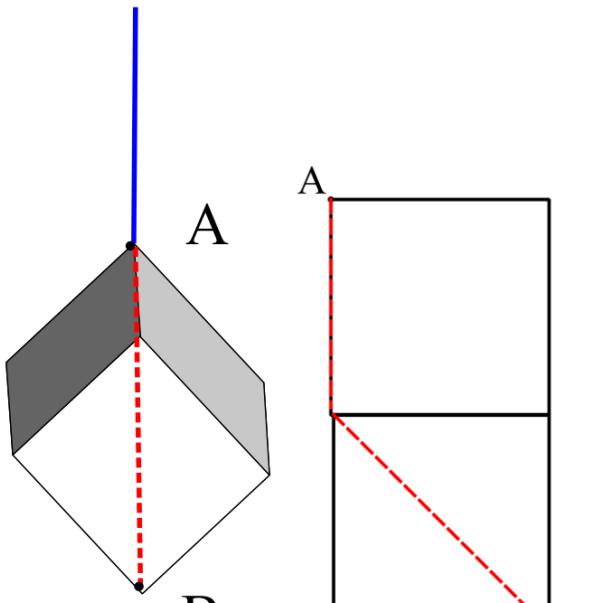


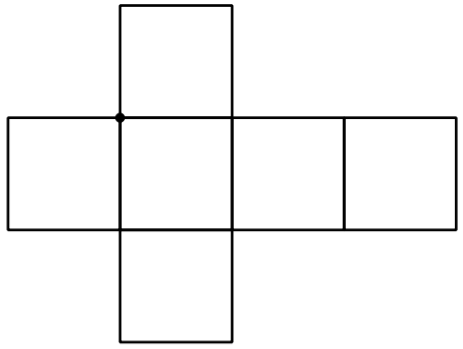
Q5. Shortest distance over a cube

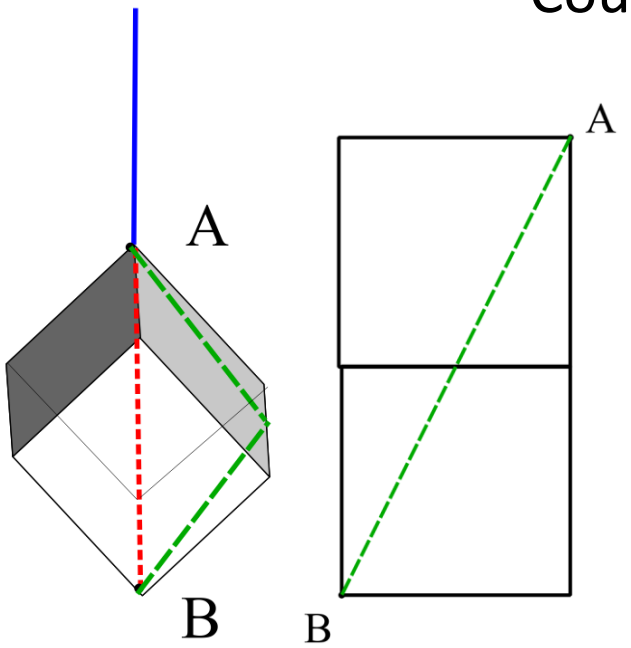


A cube with 5cm edges is suspended by one of its vertices at A.
An insect starting at A wishes to crawl to B
It takes the route shown in red
Could the insect have taken a shorter route?

S5. Shortest distance over a cube.







A cube with 5cm edges is suspended by one of its vertices at A.
An insect starting at A wishes to crawl to B
It takes the route shown in red
Could the insect have taken a shorter route?

The route in red has a length of

$$5 + \sqrt{5^2 + 5^2} = 5 + 5\sqrt{2} = 12.07$$

Examining the net fragment, there is a shorter route:

The route in green has a length of

$$\sqrt{5^2 + 10^2} = 5\sqrt{5} = 11.18$$

Q6. What are the numbers of Vertices, Edges and Faces and how are these numbers connected?

Platonic Solid	**Vertices (V)**	**Edges (E)**	**Faces (F)**
Tetrahedron			
Cube (Hexahedron)			6
Octahedron	6		8
Dodecahedron	20	30	12
Icosahedron	12	30	20

Find the formula connecting V, E and F and use it to check and complete the table

S6. What are the numbers of Vertices, Edges and Faces and how are these numbers connected?

Platonic Solid	**Vertices (V)**	**Edges (E)**	**Faces (F)**
Tetrahedron	*4*	*6*	*4*
Cube (Hexahedron)	*8*	*12*	6
Octahedron	6	*12*	8
Dodecahedron	20	30	12
Icosahedron	12	30	20

Find the formula connecting V, E and F and use it to check and complete the table

In Euler's Formula V – E + F = 2 e.g., for a tetrahedron 4 – 6 + 4 = 2
This means that if we have counted just two of the number of Vertices(V), Edges(E) and Faces(F) then we can calculate the missing count, so, in an Octahedron as V = 6 and F = 8, then 6 – E + 8 = 2, i.e., the number of edges, E = 12

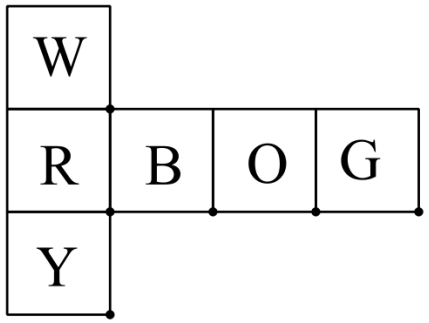
Q7. How many Rubik's Cubes?

Opposite sides of a Western six-sided Rubik's cube are White and Yellow (W + Y); Blue and Green (B + G); Red and Orange (R + O). Note that adding Yellow to the first colour gives the opposite colour. How many different Rubik's cubes are possible colouring in this way?

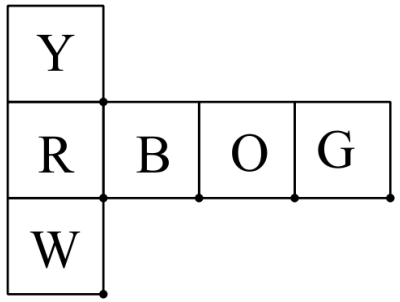
S7. How many Rubik's Cubes?

Opposite sides of a Western six-sided Rubik's cube are White and Yellow (W + Y); Blue and Green (B + G); Red and Orange (R + O). Note that adding Yellow to the first colour gives the opposite colour. How many different Rubik's cubes are possible colouring in this way?

As in the dice problem, there are two ways to colour in this way. [Japanese mass-produced Rubik's cubes had different opposite colours – swap the Blue and Yellow - https://ruwix.com/the-rubiks-cube/japanese-western-color-schemes/ - accessed 230609]

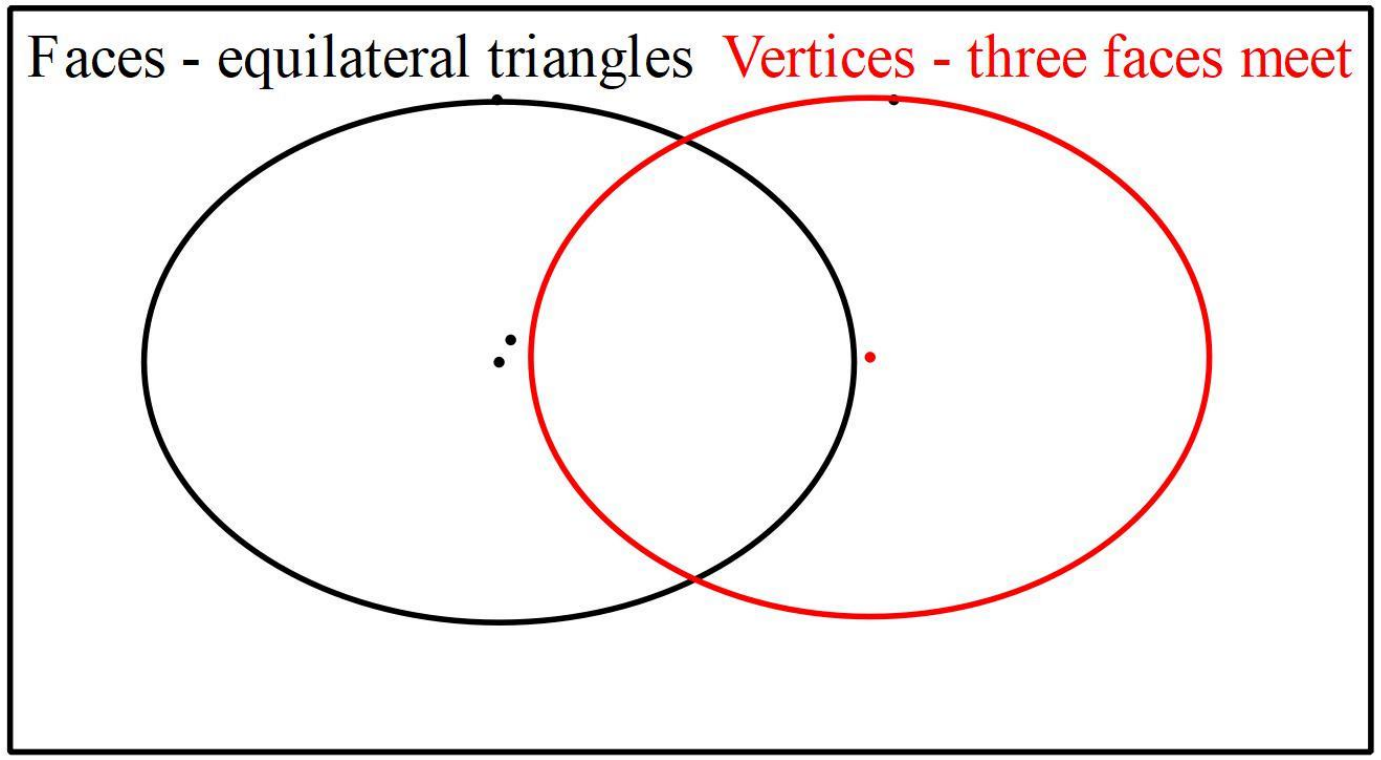


Western



Other possibility

Q8. Classifying Platonic Solids – similar and different

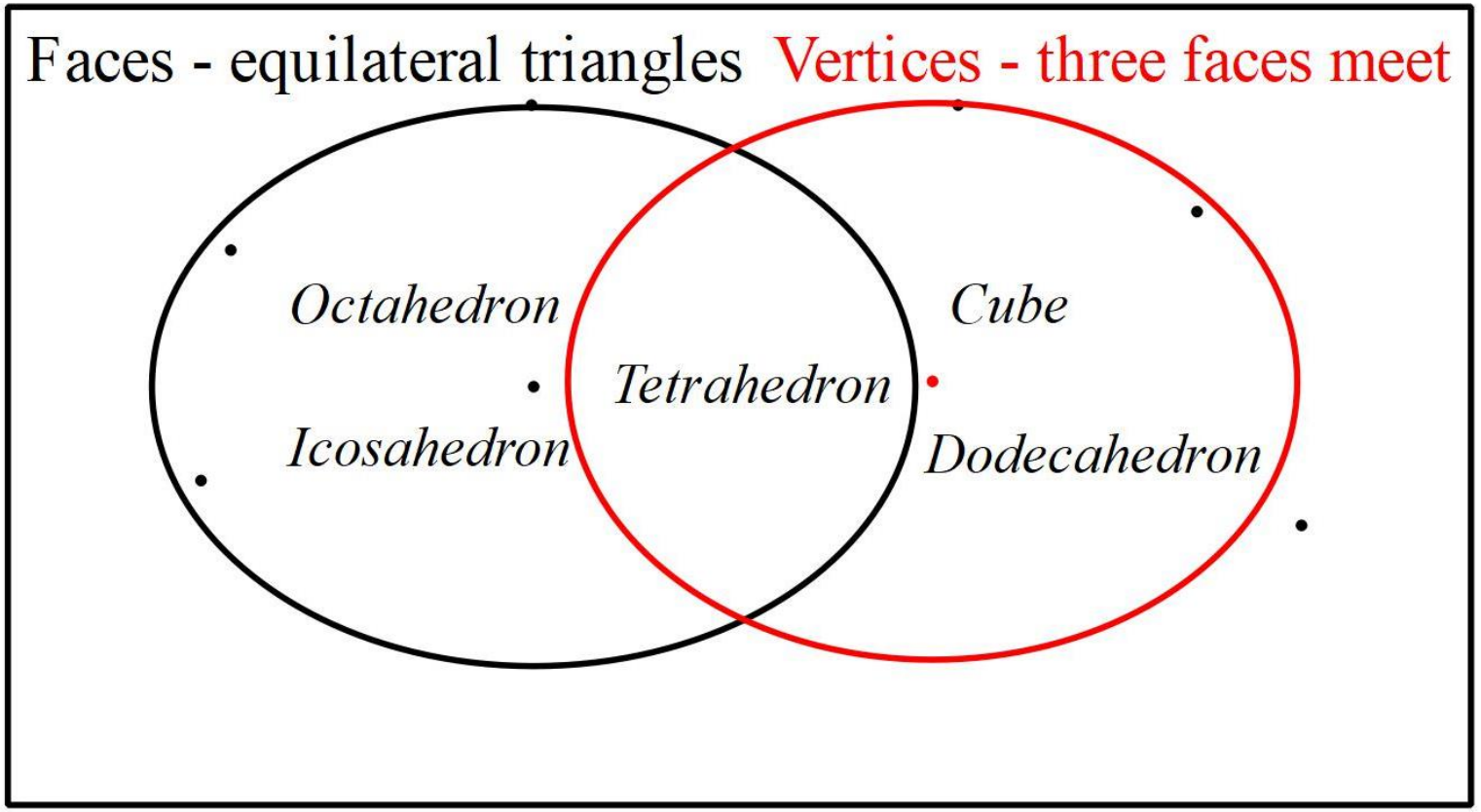


The Platonic Solids

Tetrahedron
Cube (Hexahedron)
Octahedron
Dodecahedron
Icosahedron

Complete the Venn diagram with the names of the five Platonic solids

S8. Classifying Platonic Solids – similar and different



The Platonic Solids

Tetrahedron
Cube (Hexahedron)
Octahedron
Dodecahedron
Icosahedron

Complete the Venn diagram with the names of the five Platonic solids

Q9. Platonic dice

The dice on the left are made from Platonic solids. The cubic die can be used to randomly select between six alternatives, in this case between 1, 2, 3, 4, 5 and 6.

1. How many alternatives for each die?

2. Why is the way you read the score on the tetrahedral die different from the others?

3. What do you notice about the two dice below?

S9. Platonic dice

The dice on the left are made from Platonic solids.

The cubic die can be used to randomly select between six alternatives, in this case between 1, 2, 3, 4, 5 and 6.

1. How many alternatives are there for each die?
2. Why is the way you read the score on the tetrahedral die different from the others?
3. What do you notice about the pair of cubic dice?

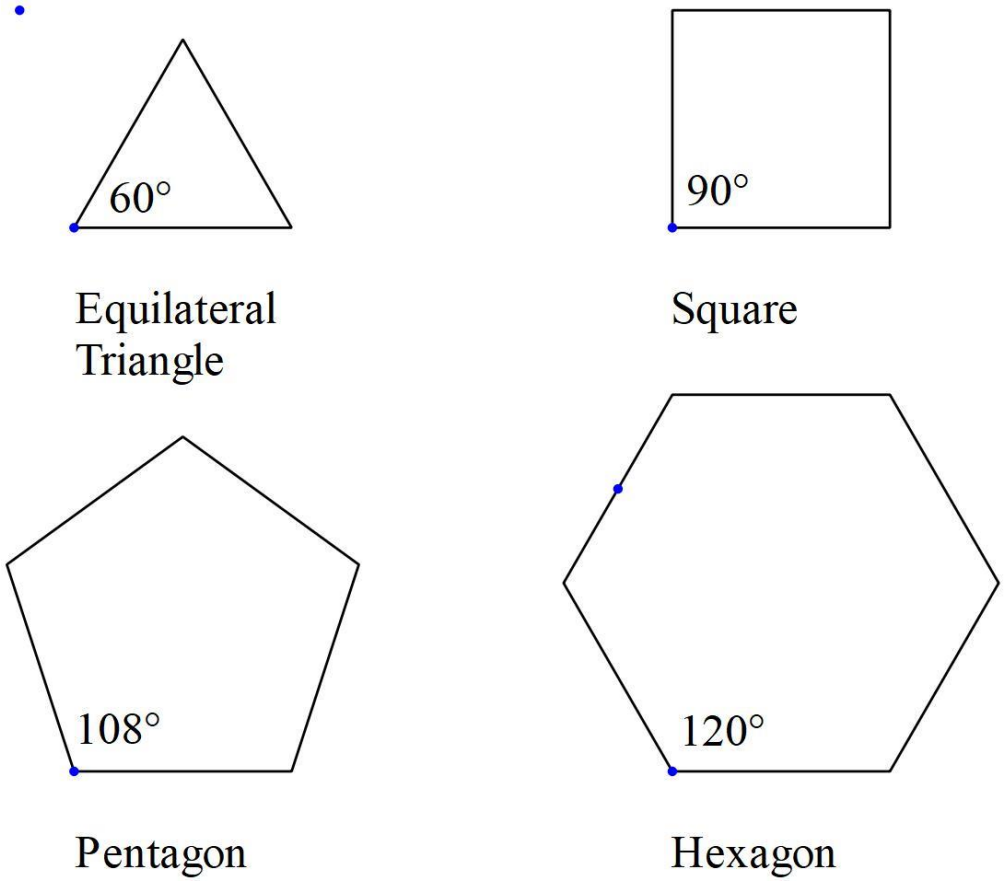
1. With one alternative for each face: Tetrahedral (4), Cubic (6), Octahedral (8), Dodecahedral (12), Icosahedral (20)

2. The tetrahedral die does not have pairs of opposite faces; one to rest on and the other with a score; it does not roll well.

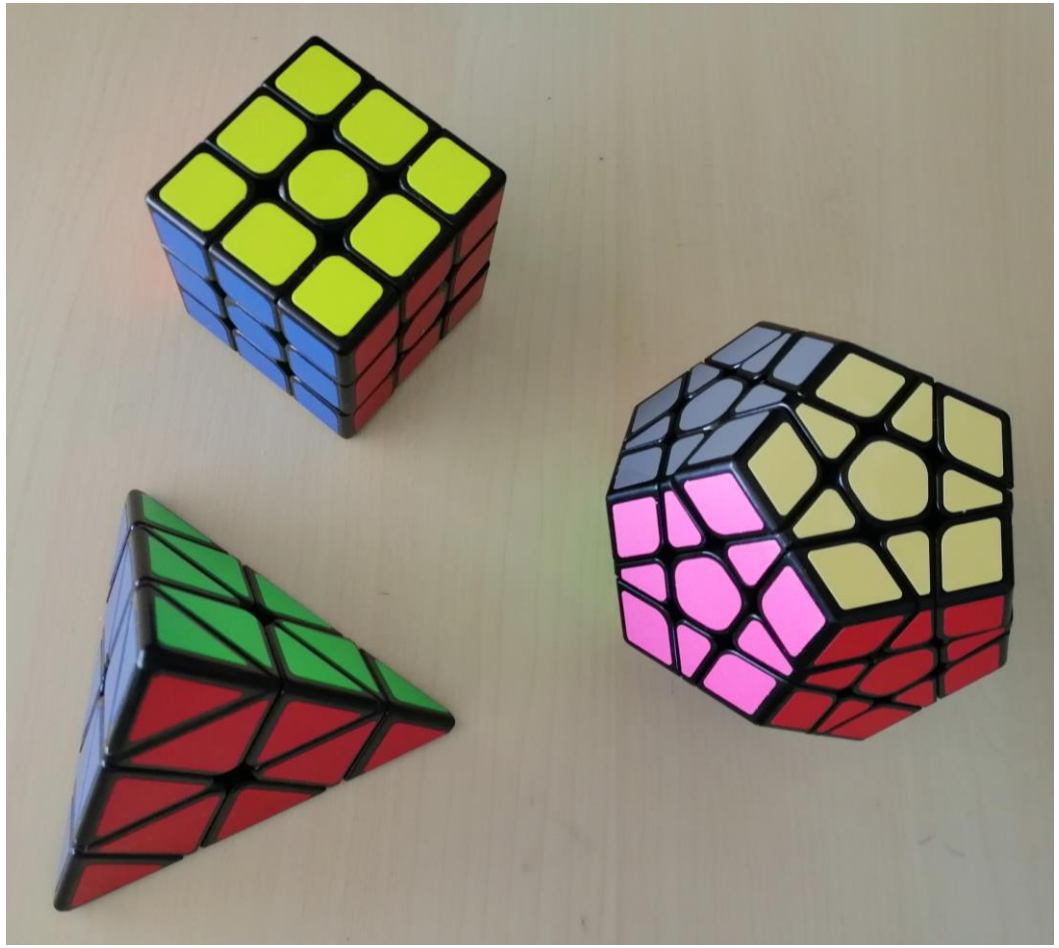
3. These dice have opposite chirality – right-handed and left-handed. (Western and Chinese)

Q10. Platonic Solids have identical (regular polygon) faces and identical vertices – Why only five Platonic solids?

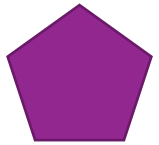
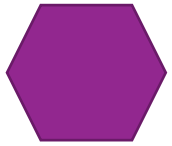
Some regular polygons and their internal angles



Three of the Platonic solids



S10. Why only five Platonic Solids?

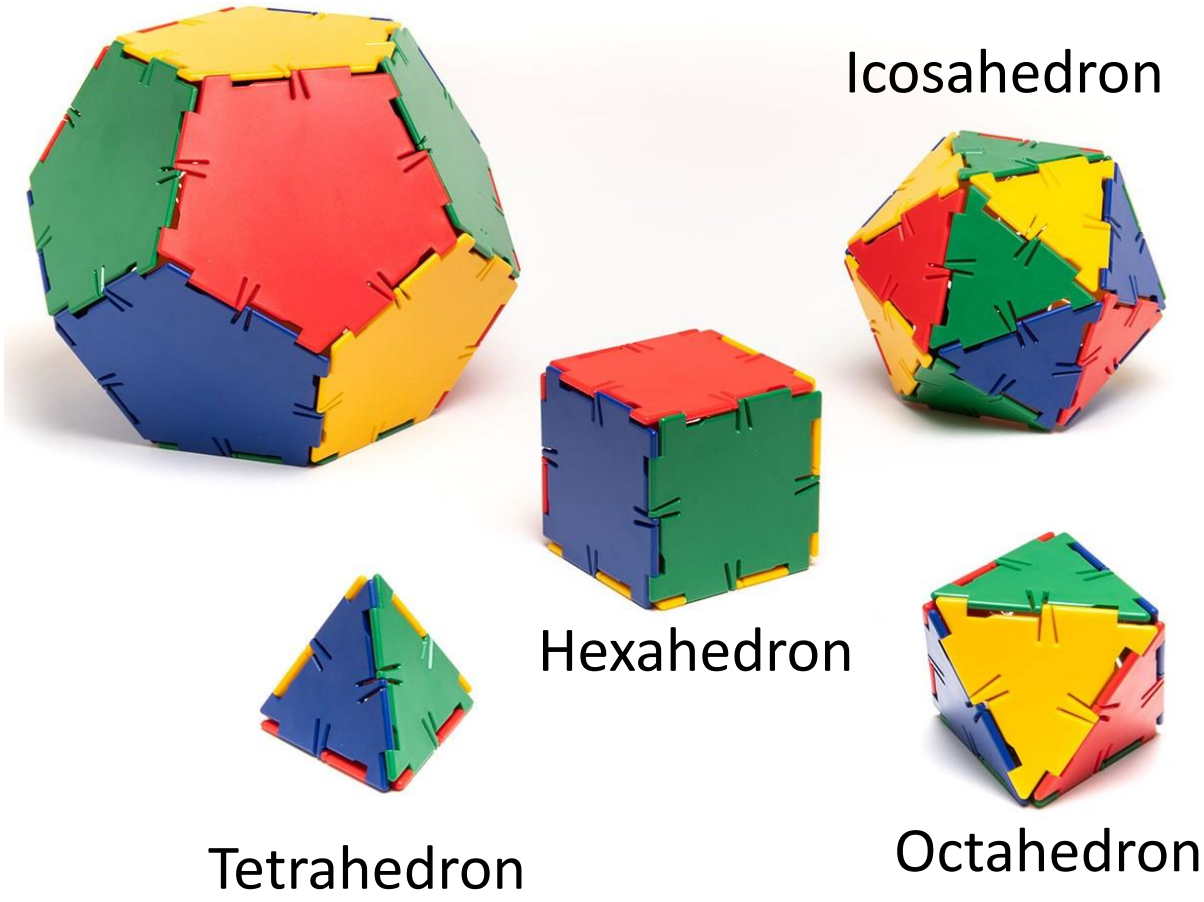
Regular Polygon	Interior angle	Each vertex will need 3 or more totalling less than $360°$
Equilateral Triangle 	$60°$	$3 \times 60° = 180°$ (Tetrahedron) $4 \times 60° = 240°$ (Octahedron) $5 \times 60° = 300°$ (Icosahedron) $6 \times 60° = 360°$
 **Square**	$90°$	$3 \times 90° = 270°$ (Cube) $4 \times 90° = 360°$
 **Pentagon**	$108°$	$3 \times 108° = 324°$ (Dodecahedron) $4 \times 108° = 432°$
 **Hexagon**	$120°$	$3 \times 120° = 360°$

Further ideas for activities welcome

e.g.,

Describe the shapes on a classic football. [12P20H-TI]

The Platonic Solids on the right use four colours to construct each Platonic Solid. Which Platonic Solid needs just two colours to ensure that each edge has a different colour on each side? [T4C3O2D4I3]



Fun with Platonic Solids

Thank you

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With thanks to Polydron
(https://www.polydron.co.uk)