Platonic Activities

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NANAMIC
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In this session we will:

1. Look at the properties of Platonic Solids
2. Suggest ten activities with Platonic Solids
3. Seek comments and further ideas for activities
4. Look together at approaches and sample solutions

Activities	
1. Identify the Platonic solids	6. Examine the cube's geometry
2. Identify what is a valid cube net	7. Classify Platonic solids
3. Count the ways to number a cubic die	8. Apply Euler's formula
4. Count the ways to colour a Rubik's cube	9. Examine the 4th Dimension
5. Count the ways to paint a tetrahedron	10. Look at Platonic dice

Platonic Solids – what are they?

Platonic Solids have:

1. Identical (congruent) regular polygon faces
2. identical (congruent) vertices

For example, a cube

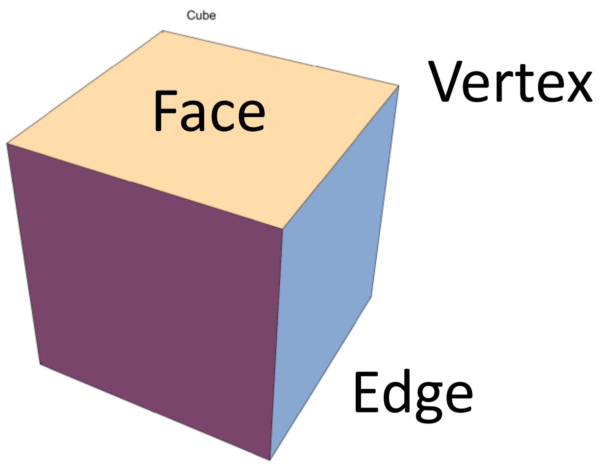
1. Each face is an identical square

2. Three faces meet at each vertex

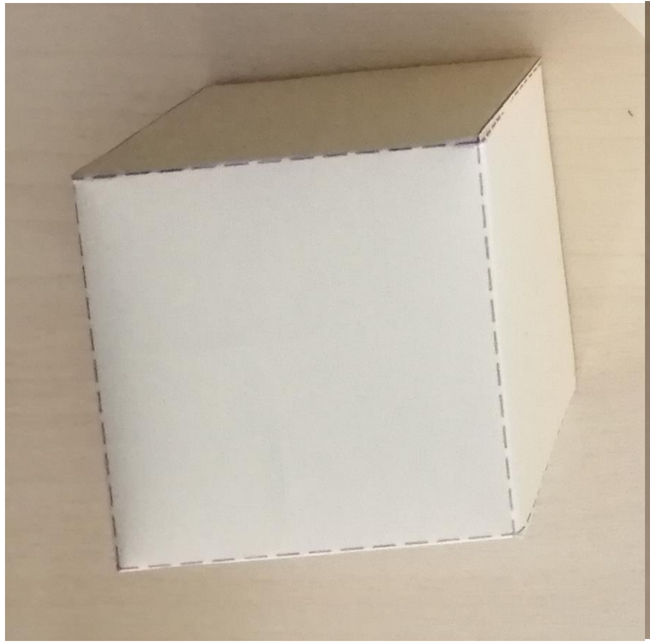
In general

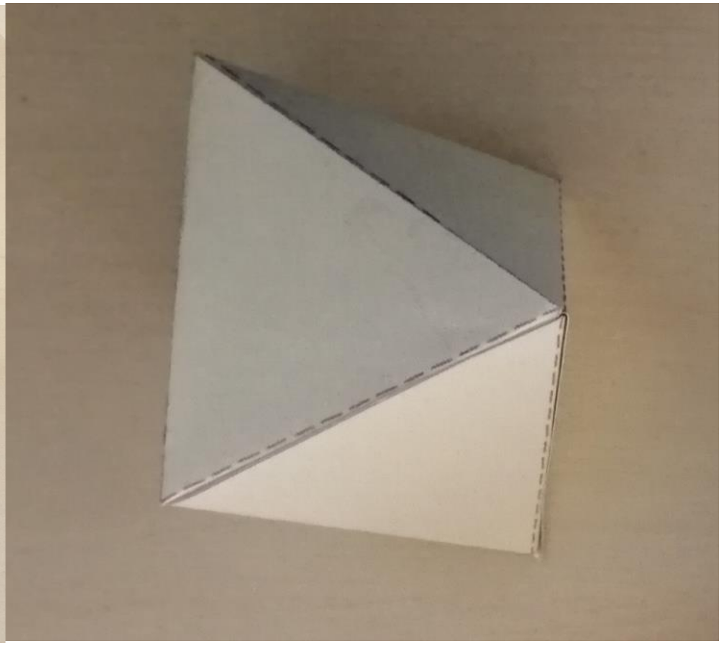
1. Each face is an identical regular polygon.

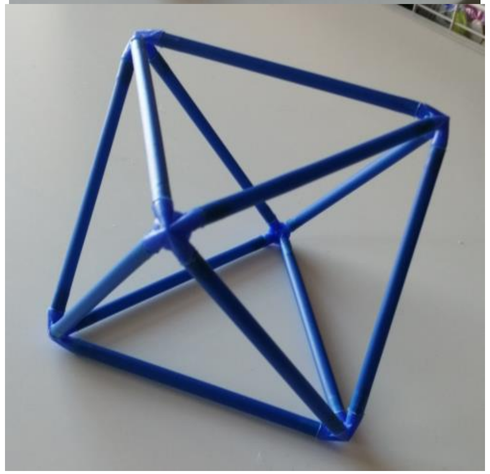
2. An equal number of faces meet at each vertex.



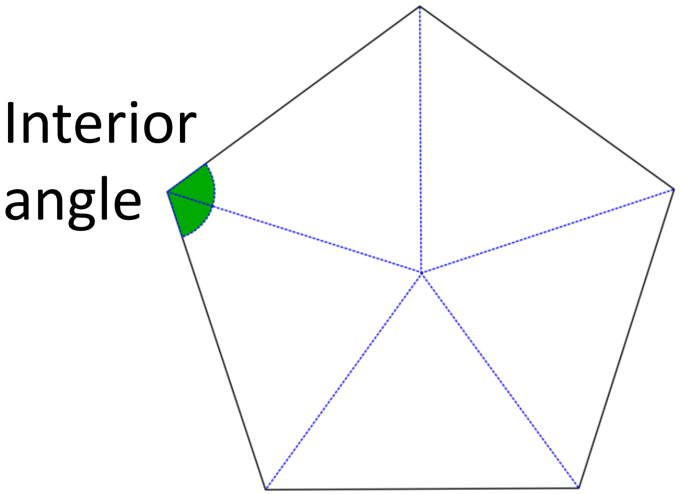
Model building – using nets and ‘straws’







Regular polygons - identical edges and identical vertices

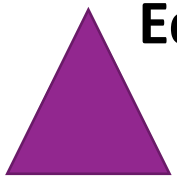
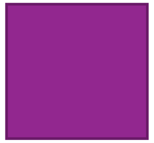
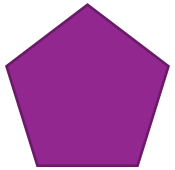
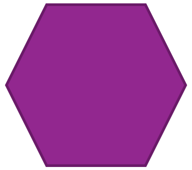


Regular Pentagon

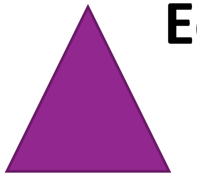
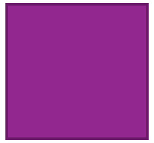
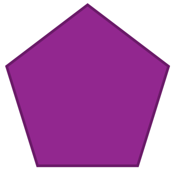
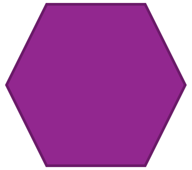
Partition the Regular Pentagon into 5 triangles
The sum of the angles in each triangle is 180°
The total sum of angles in the 5 triangles is 5 x 180°
The angles in the centre total 360°
So, the sum of the interior angles is 5 x 180° - 360°
Each interior angle is 180° - 360°/5 = 180° - 72° = 108°

For a n sided Regular Polygon, the interior angles are
180° - 360°/n

Regular polygons – Interior angles

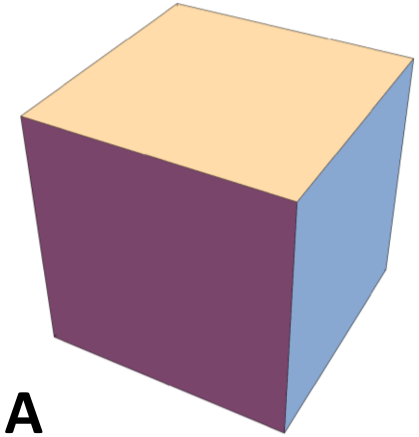
Regular Polygon	360°/n	Interior angle 180° – 360°/n
 **Equilateral Triangle**	$\frac{360°}{3} = 120°$	$180° - 120° = \mathbf{60°}$
 **Square**	$\frac{360°}{4} = 90°$	$180° - 90° = \mathbf{90°}$
 **Pentagon**	$\frac{360°}{5} = 72°$	$180° - 72° = \mathbf{108°}$
 **Hexagon**	$\frac{360°}{6} = 60°$	$180° - 60° = \mathbf{120°}$

How many regular polygons can form vertices?

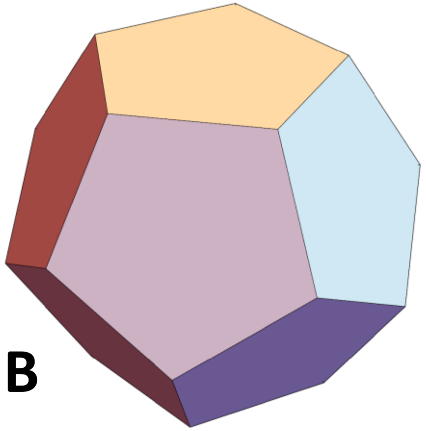
Regular Polygon	Interior angle	Each vertex will need 3 or more totalling less than 360°
 **Equilateral Triangle**	60°	180°, 240°, 300°, ~~360°~~
 **Square**	90°	270°, ~~360°~~
 **Pentagon**	108°	324°, ~~432°~~
 **Hexagon**	120°	~~360°~~

Platonic Solids – identical regular polygon faces and identical vertices

1. Cube (Hexahedron)
2. Tetrahedron
3. Octahedron
4. Truncated Icosahedron
5. Icosahedron
6. Dodecahedron
7. Triangular Bipyramid



A



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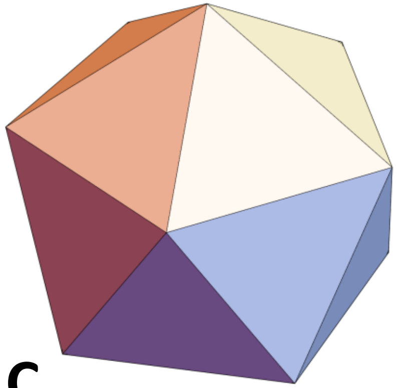

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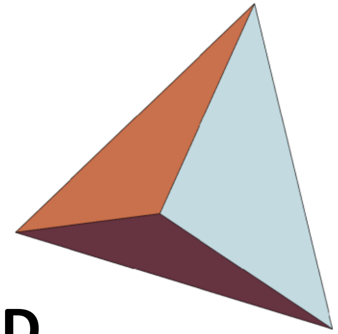

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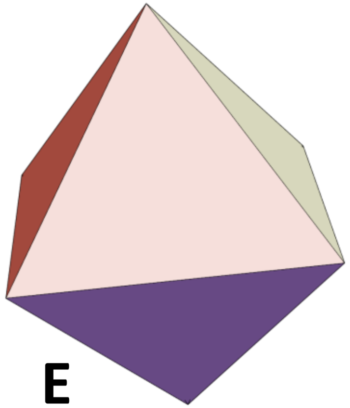

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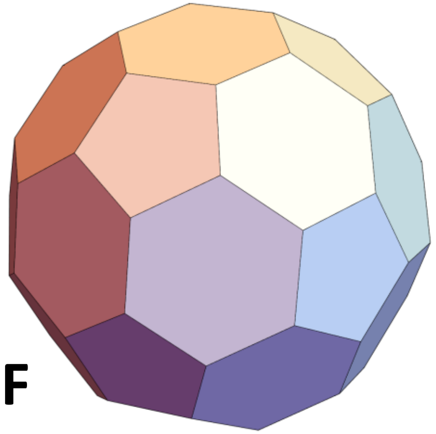
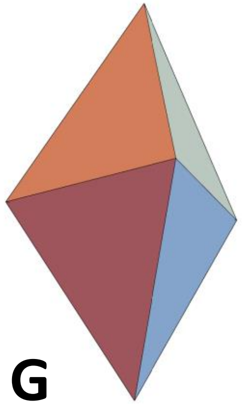


F

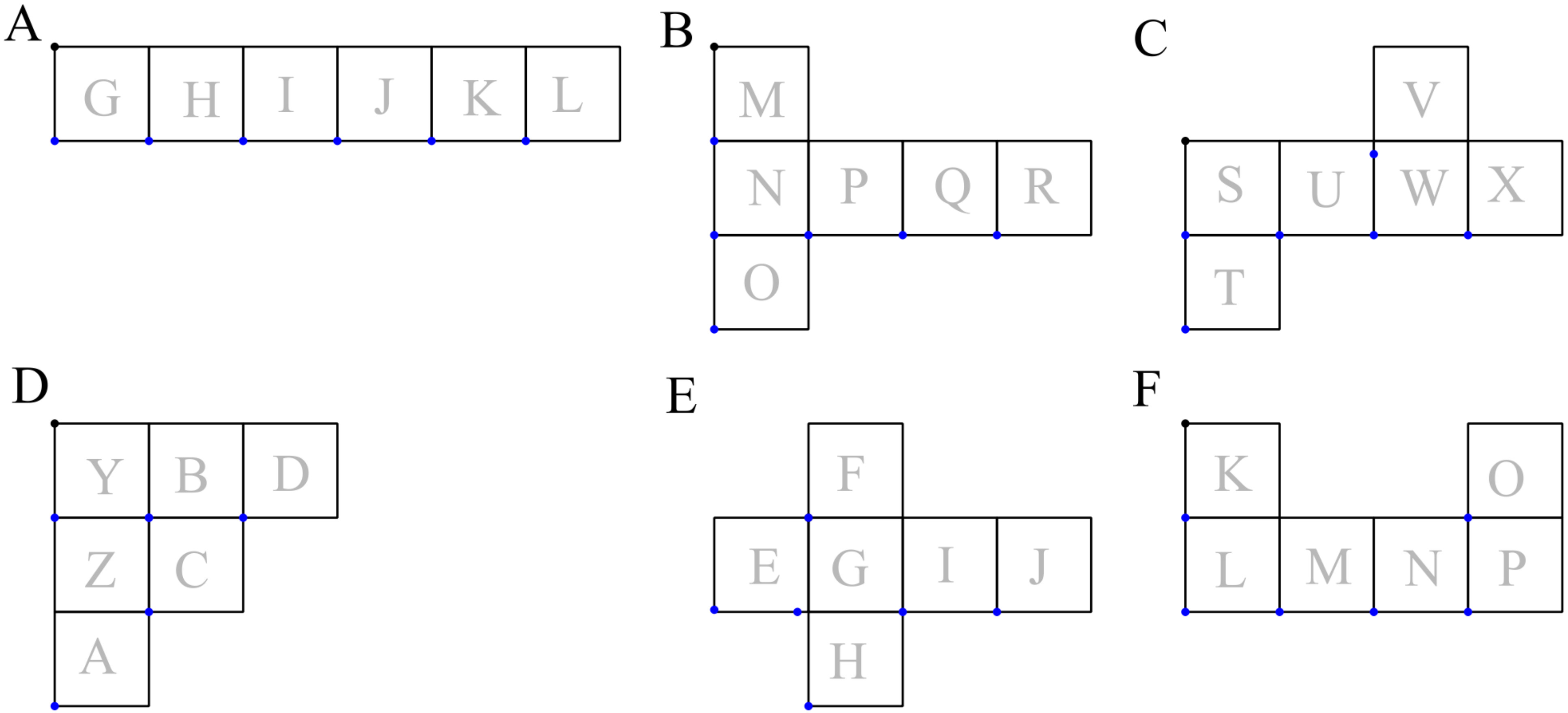


G

Which of these are not Platonic Solids?

Match 3D shape and name e.g., A1

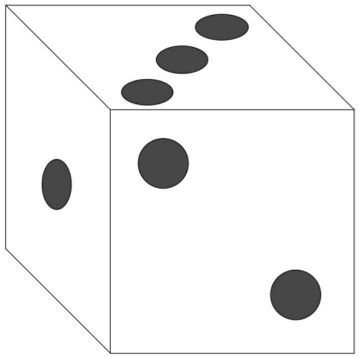
Cube nets

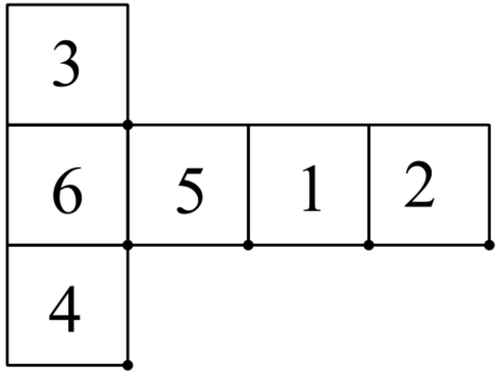


Which of these nets do not form cubes?

How many different dice?

Opposite sides of a six-sided die sum to 7 i.e., 6 is opposite 1, 5 is opposite 2, and 4 is opposite 3. How many possible dice are there?

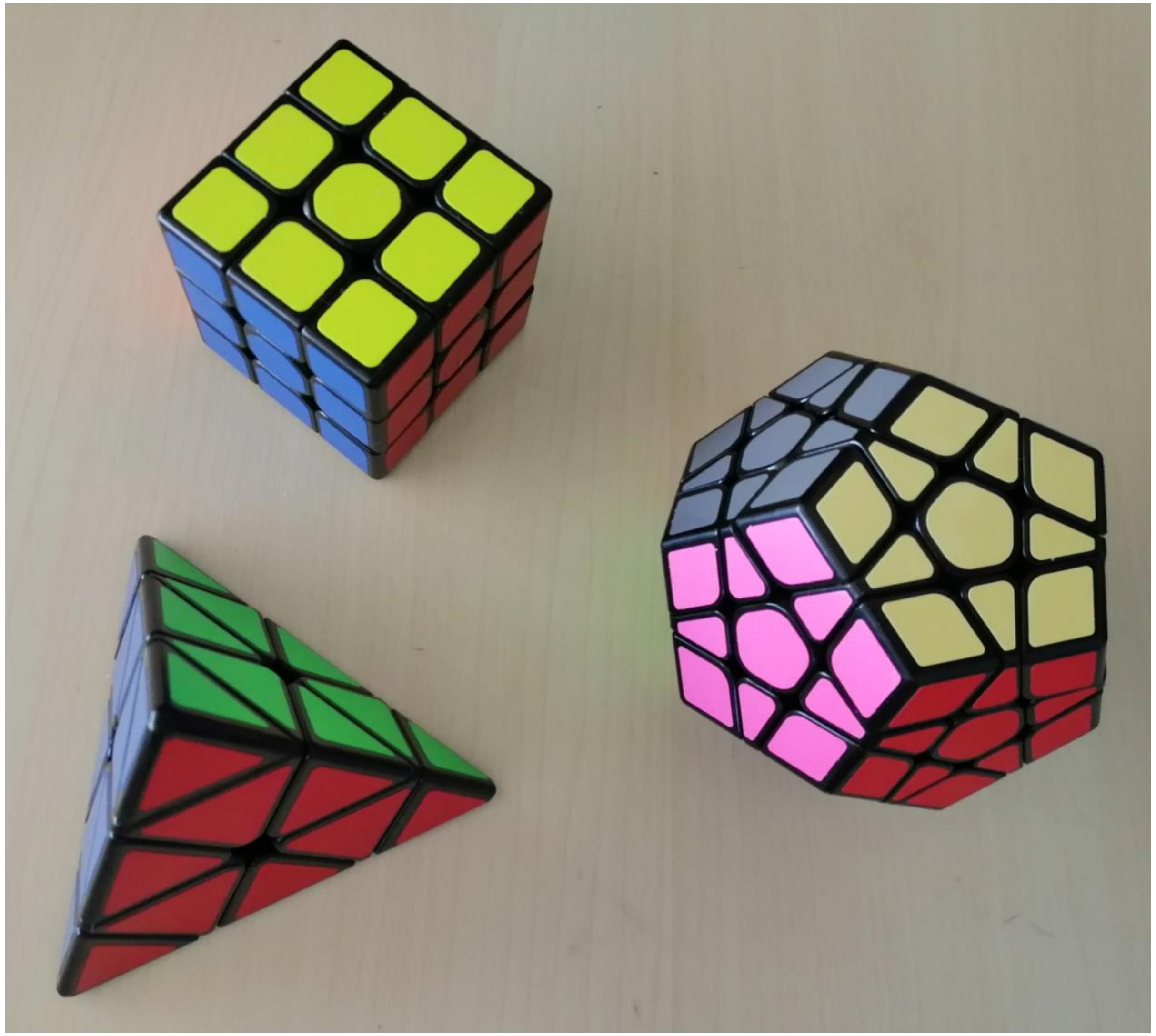




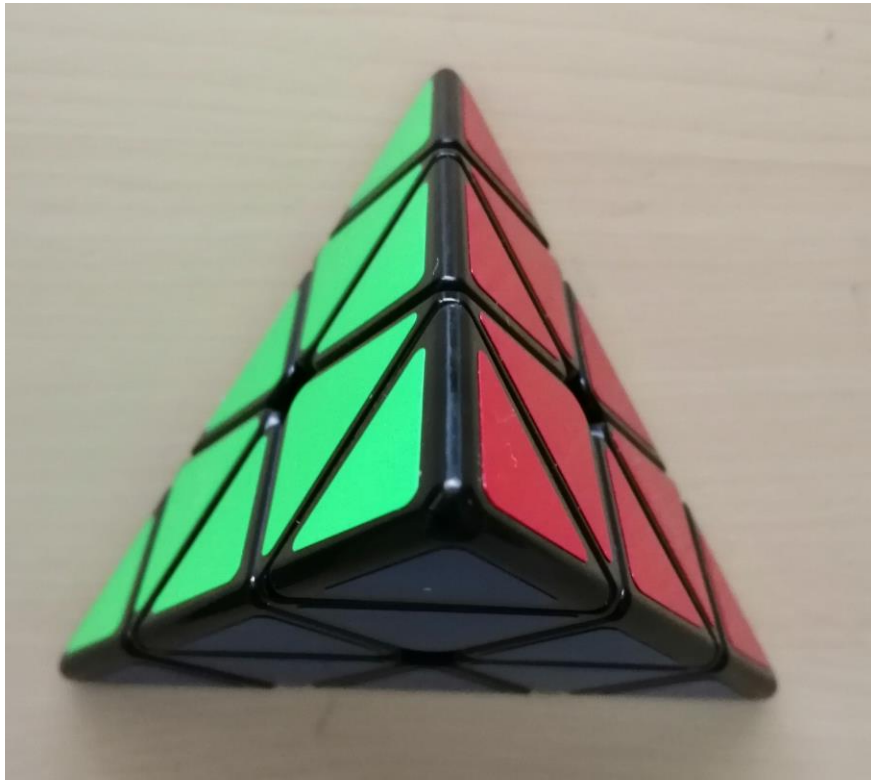
How many Rubik's Cubes?

Opposite sides of a Western six-sided Rubik's cube are White and Yellow (W + Y); Blue and Green (B + G); Red and Orange (R + O). Note that adding Yellow to the first colour gives the opposite colour. How many different Rubik's cubes are possible colouring in this way?

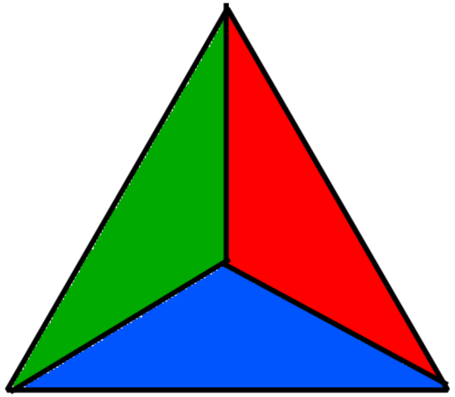
Rubik's cube, tetrahedron and dodecahedron



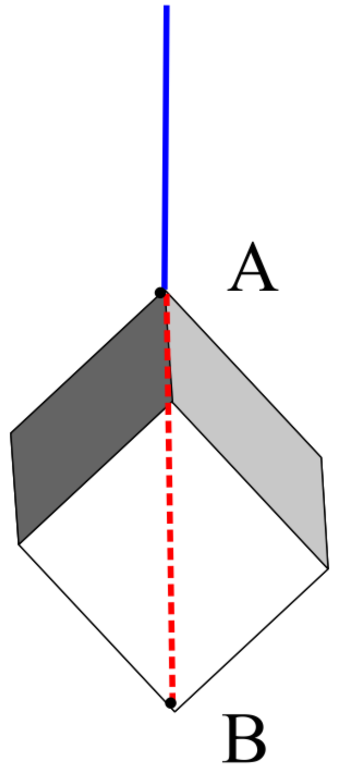
How many ways to paint a tetrahedron?



A tetrahedron is to be coloured
In Red, Green, Blue and Yellow,
with a different colour on each face.
How many ways could this be done?

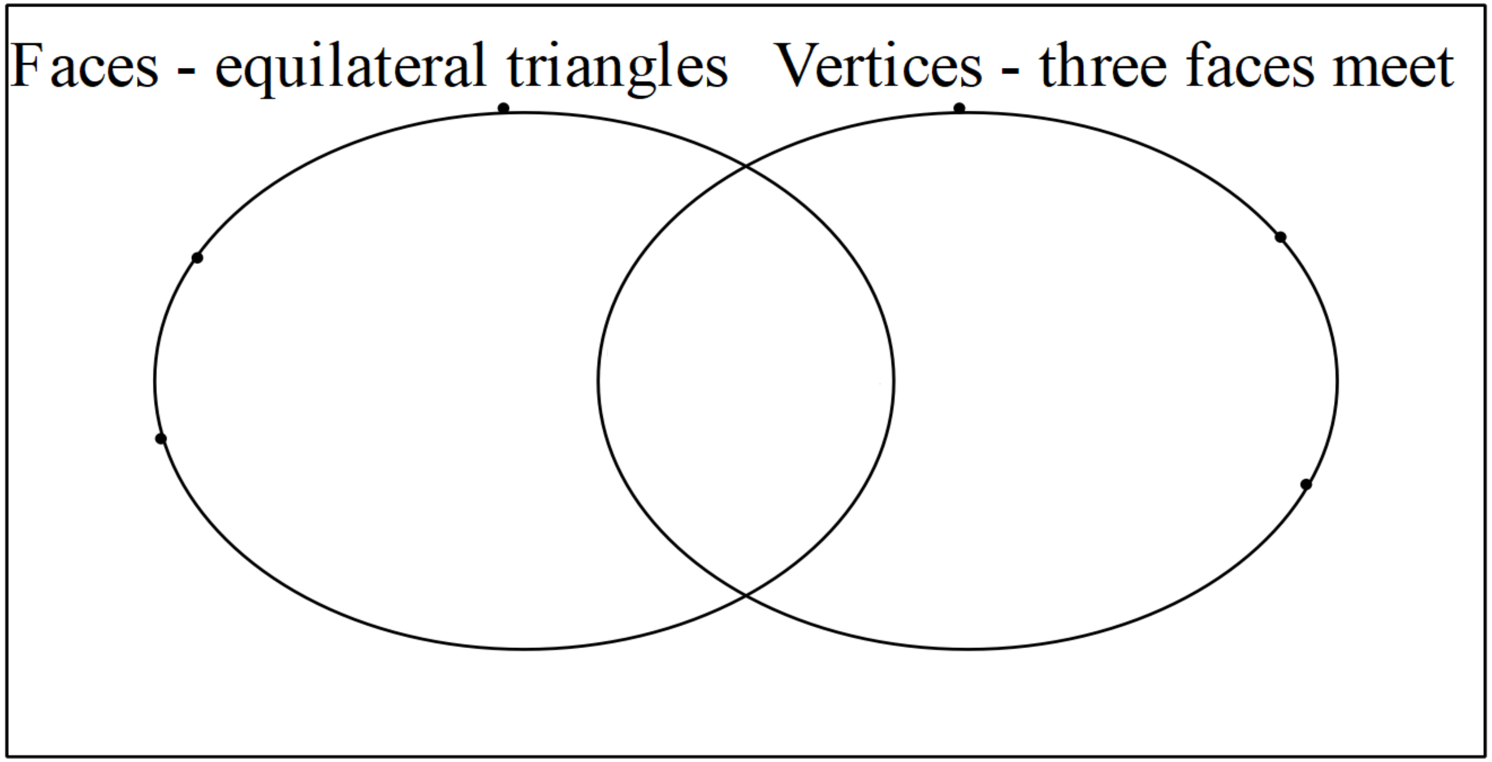


Shortest distance over a cube



A cube with 5cm edges is suspended by one of its vertices at A.
An insect starting at A wishes to crawl to B
It takes the route shown in red
Could the insect have taken a shorter route?

Classifying Platonic Solids – similar and different



The Platonic Solids
Tetrahedron
Cube (Hexahedron)
Octahedron
Dodecahedron
Icosahedron

Complete the Venn diagram with the names of the five Platonic solids

What are the numbers of Vertices, Edges and Faces and how are these numbers connected?

Platonic Solid	Vertices (V)	Edges (E)	Faces (F)
Tetrahedron		6	4
Cube (Hexahedron)	8	12	
Octahedron	6		8
Dodecahedron		30	12
Icosahedron	12	30	

Find the formula connecting V, E and F and use it to check and complete the table

Imagining the 4th Dimension

The four corner points (vertices) of a square wire frame are at coordinates {(0,0), (0,1), (1,0), (1,1)}. The shape has 4 vertices and 4 lines (edges)

The 8 vertices of a cube wire frame are at coordinates {(0,0,0), (0,0,1), (0,1,0), … (1,1,1)}. The shape has 8 vertices, 12 edges and 6 squares (faces),

It might be imagined that the vertices of a 4D-cube wire frame are at coordinates {(0,0,0,0), (0,0,0,1), (0,0,1,0), … (1,1,1,1)}. How many vertices, edges, faces and cubes does this 4D-cube have and conjecture how these numbers of vertices, edges, faces and cubes might be related?

Platonic dice

The dice on the left are made from Platonic solids. The cubic die can be used to randomly select between six alternatives, in this case between 1, 2, 3, 4, 5 and 6.

1. How many alternatives for each die?

2. Why is the way you read the score on the tetrahedral die different from the others?

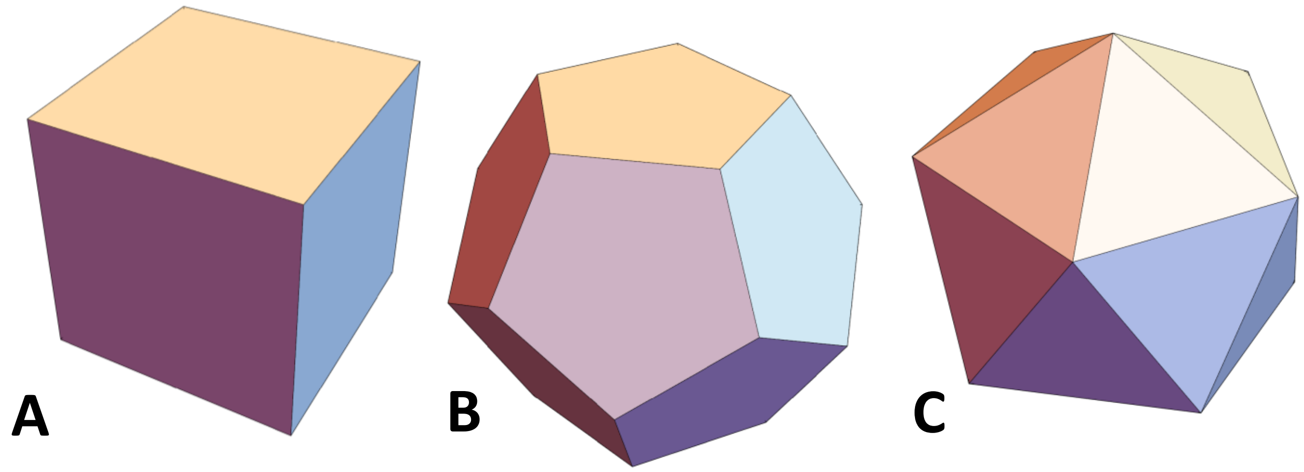
3. What do you notice about the two dice below?

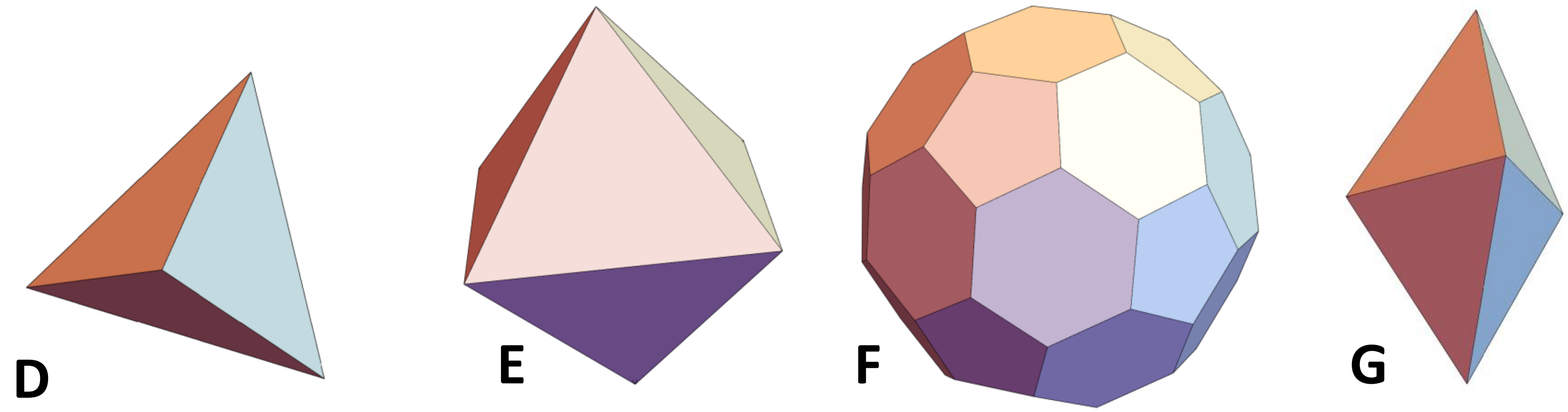
Welcome comments and further ideas for Platonic activities

Approaches to activities and sample solutions

Platonic Solids – identical regular polygon faces and identical vertices

1. Cube (Hexahedron)
2. Tetrahedron
3. Octahedron
4. Truncated Icosahedron
5. Icosahedron
6. Dodecahedron
7. Triangular Bipyramid

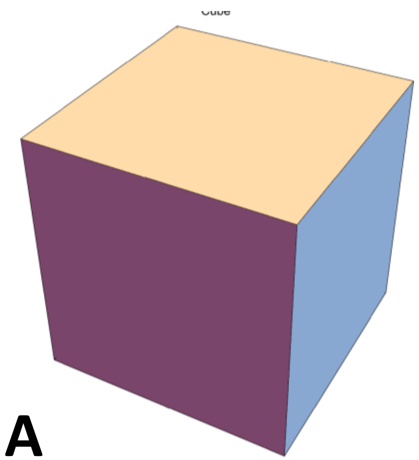




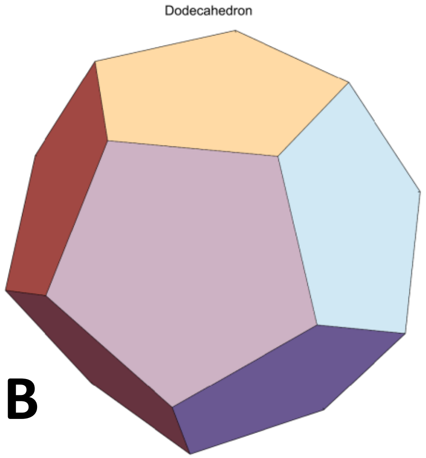
Which of these are not Platonic Solids?

Match 3D shape and name e.g., A1

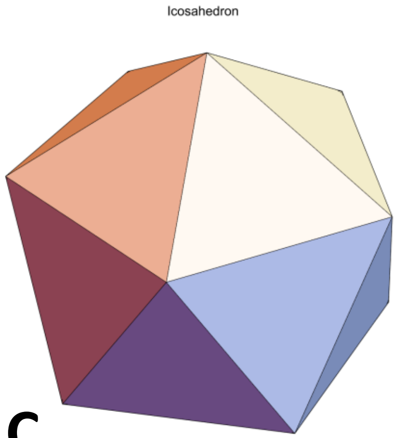
Platonic Solids – identical regular polygon faces and identical vertices



A



B



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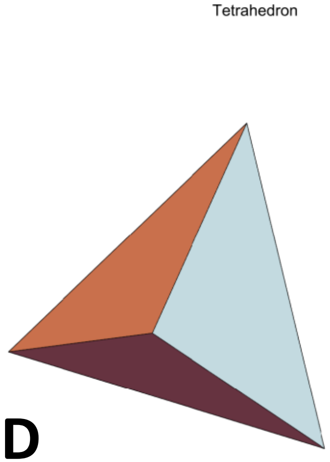

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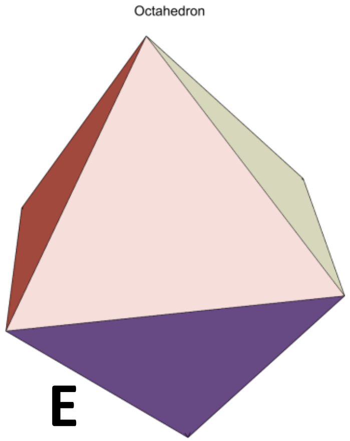
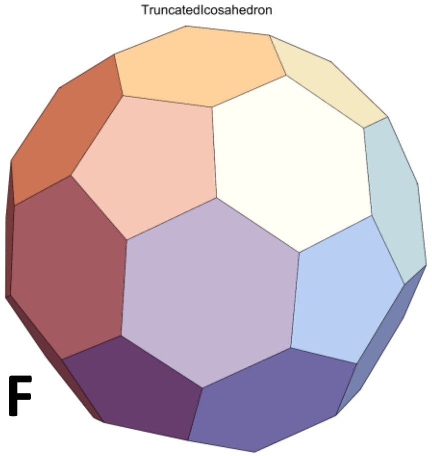
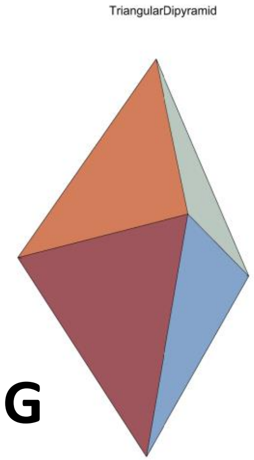


E



F



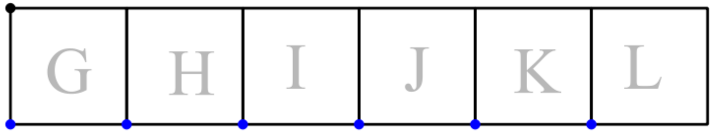
G

A1. Cube (Hexahedron)
B6. Dodecahedron
C5. Icosahedron
D2. Tetrahedron
E3. Octahedron
F4. Truncated Icosahedron
G7. Triangular Bipyramid

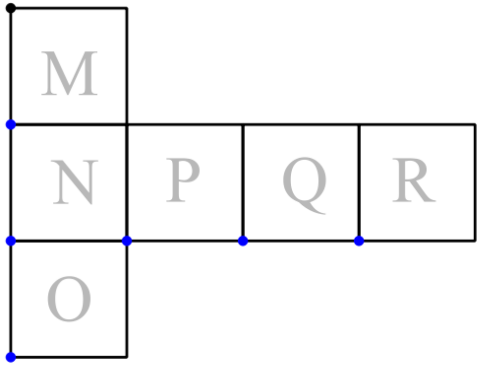
The following are not Platonic solids.
F – different faces
12 x 5 + 20 x 6 sided
G – different vertices
3 x 4 + 2 x 3 faces

Cube nets

A



B



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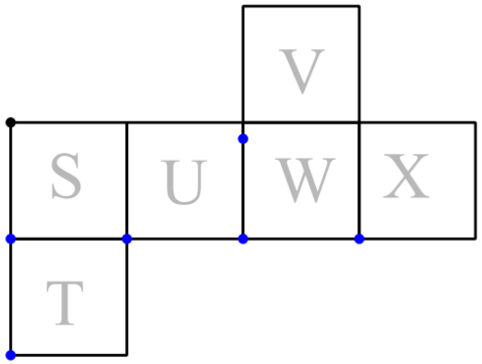
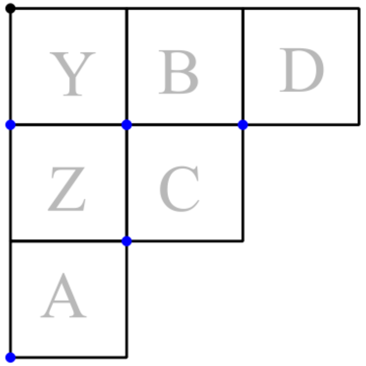
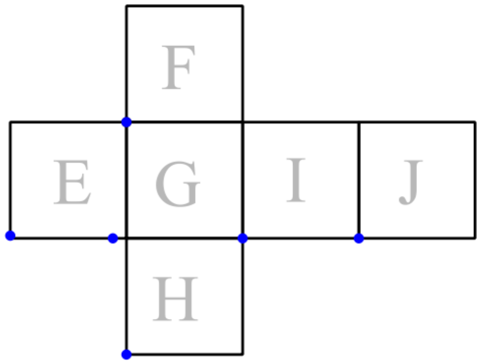


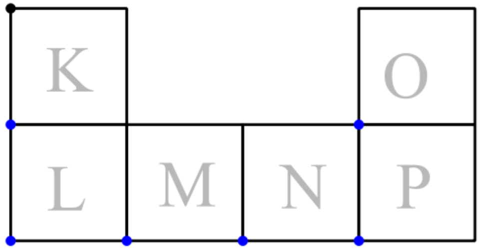
D



E

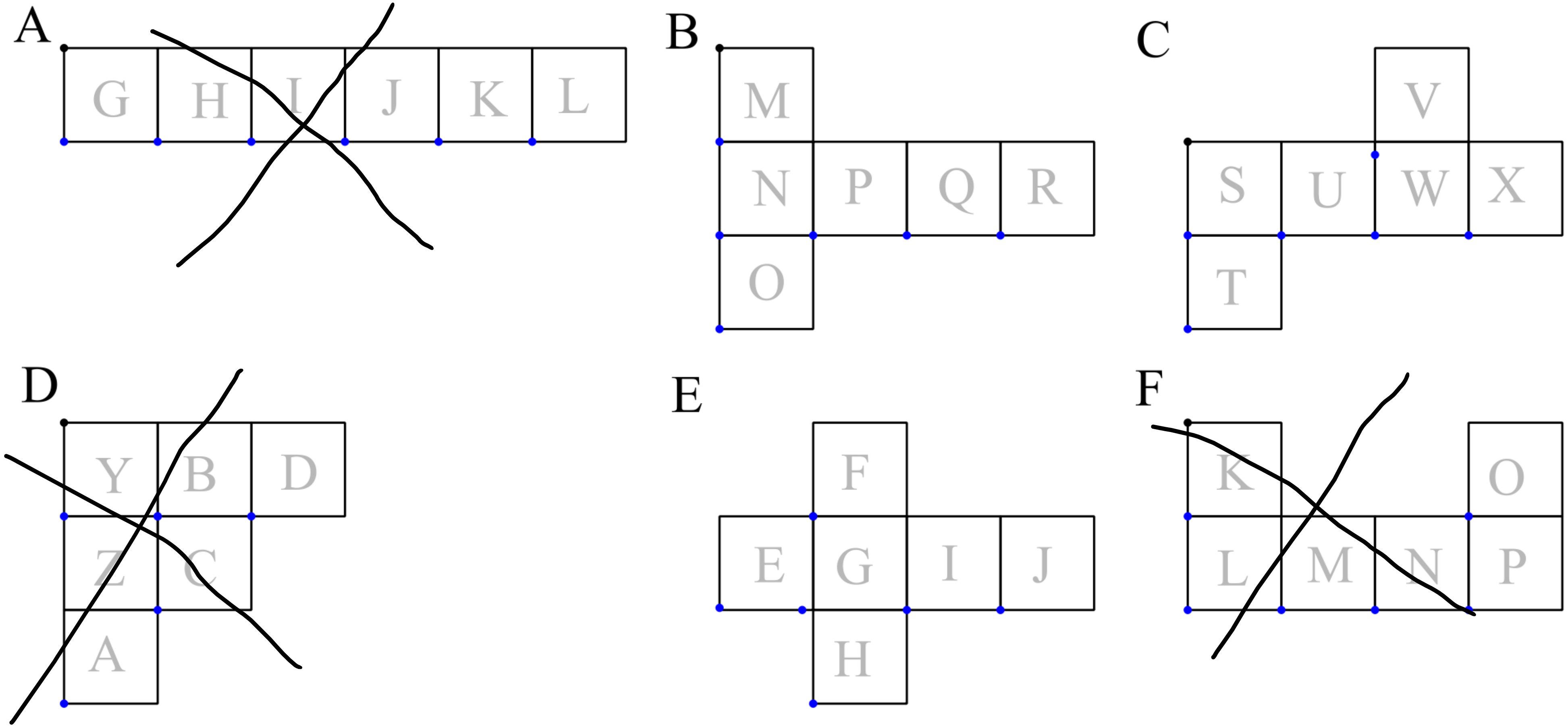


F



Which of these nets do not form cubes?

Cube nets



Which of these nets do not form cubes?

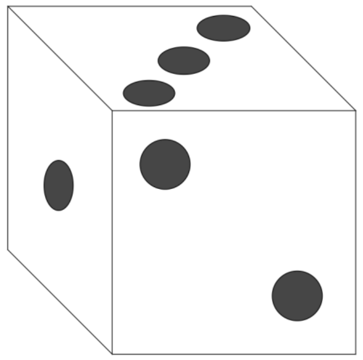
Platonic solids from their nets

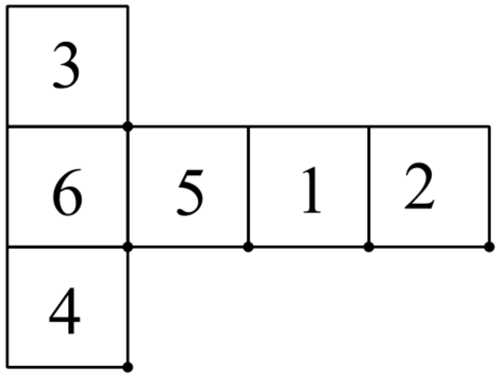
https://www.mathsisfun.com/platonic_solids.html

Spotlight

How many different dice?

Opposite sides of a six-sided die sum to 7 i.e., 6 is opposite 1, 5 is opposite 2, and 4 is opposite 3. How many possible dice are there?

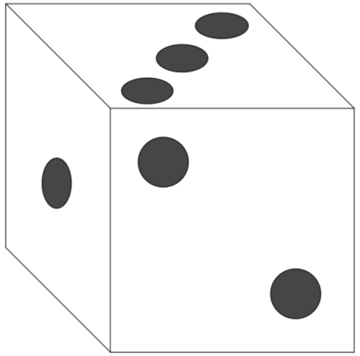




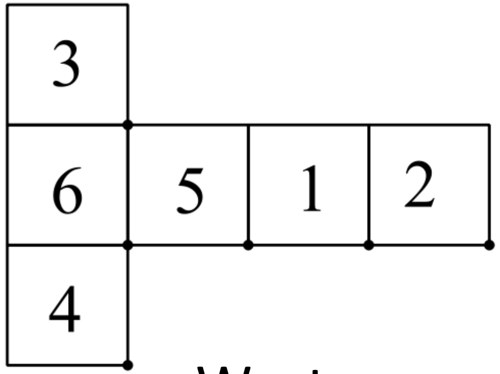
Spotlight

How many different dice?

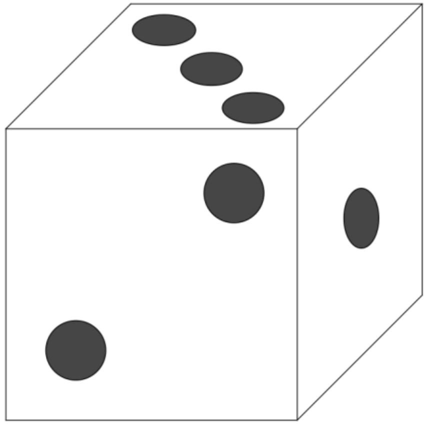
Opposite sides of a six-sided die sum to 7 i.e., 6 is opposite 1, 5 is opposite 2, and 4 is opposite 3.
How many possible dice are there?
Placing the die with 3 uppermost and 1 facing, the 2 could either be to the right or left.



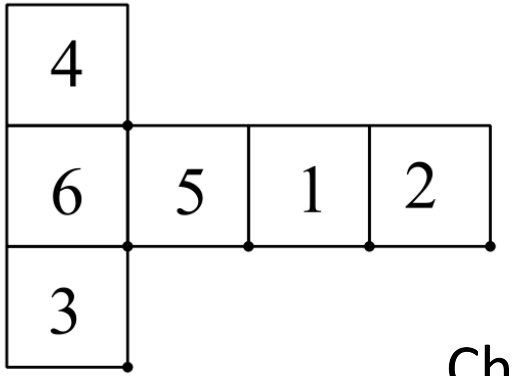
1,2,3 Anticlockwise



Western – right-handed



1,2,3 Clockwise



Chinese – left-handed

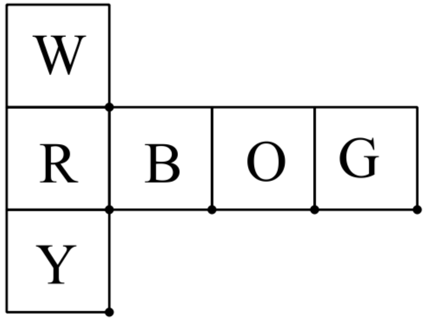
How many Rubik’s Cubes?

Opposite sides of a Western six-sided Rubik’s cube are White and Yellow (W + Y); Blue and Green (B + G); Red and Orange (R + O). Note that adding Yellow to the first colour gives the opposite colour. How many different Rubik’s cubes are possible colouring in this way?

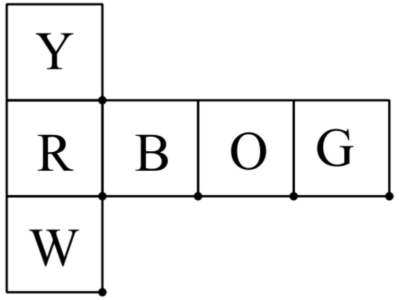
How many Rubik’s Cubes?

Opposite sides of a Western six-sided Rubik’s cube are White and Yellow (W + Y); Blue and Green (B + G); Red and Orange (R + O). Note that adding Yellow to the first colour gives the opposite colour. How many different Rubik’s cubes are possible colouring in this way?

As in the dice problem, there are two ways to colour in this way. [Japanese mass-produced Rubik’s cubes had different opposite colours – swap the Blue and Yellow - https://ruwix.com/the-rubiks-cube/japanese-western-color-schemes/ - accessed 230609]

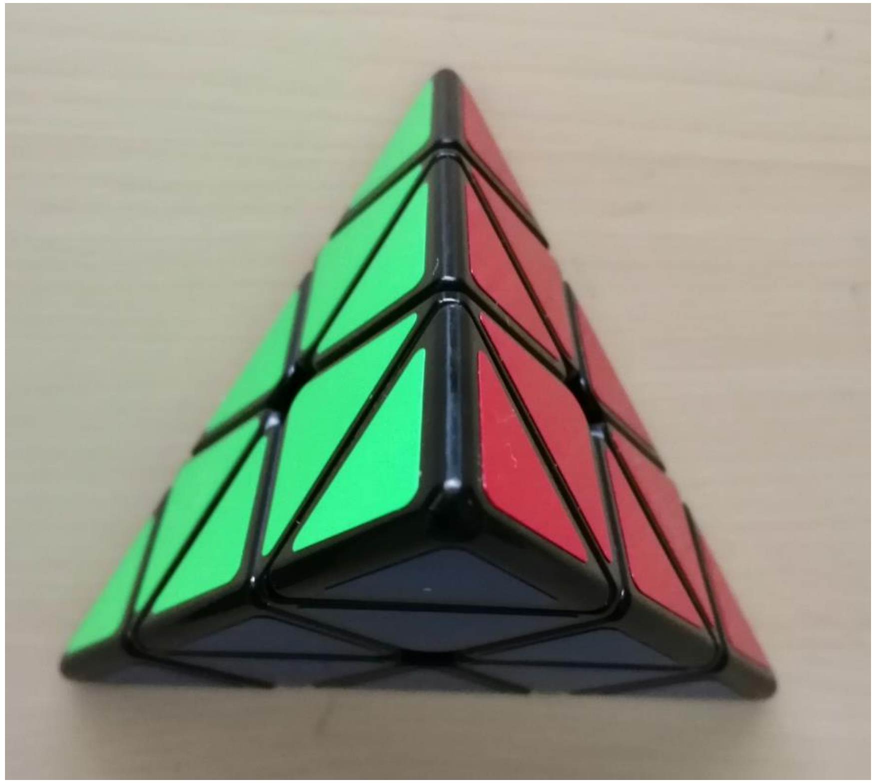


Western

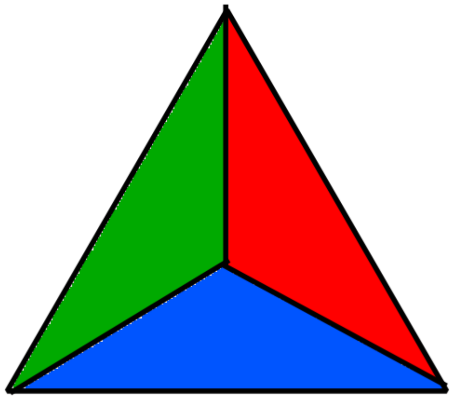


Other possibility

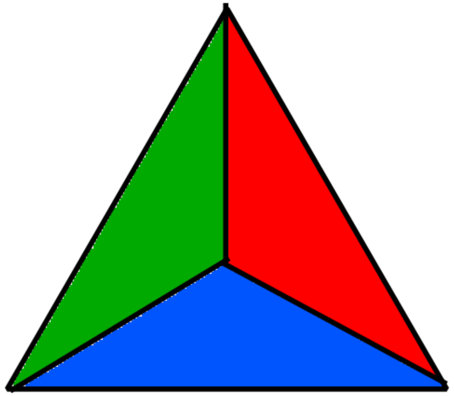
How many ways to paint a tetrahedron?

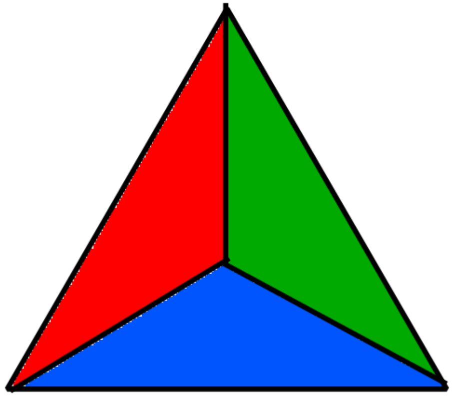


A tetrahedron is to be coloured
In Red, Green, Blue and Yellow,
with a different colour on each face.
How many ways could this be done?



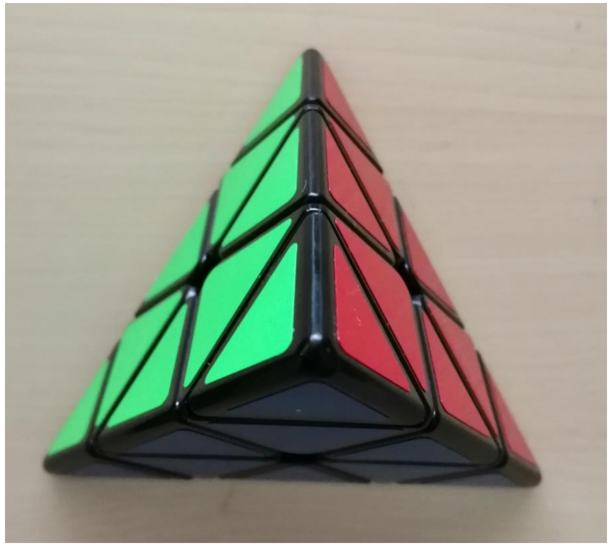
How many ways to paint a tetrahedron?

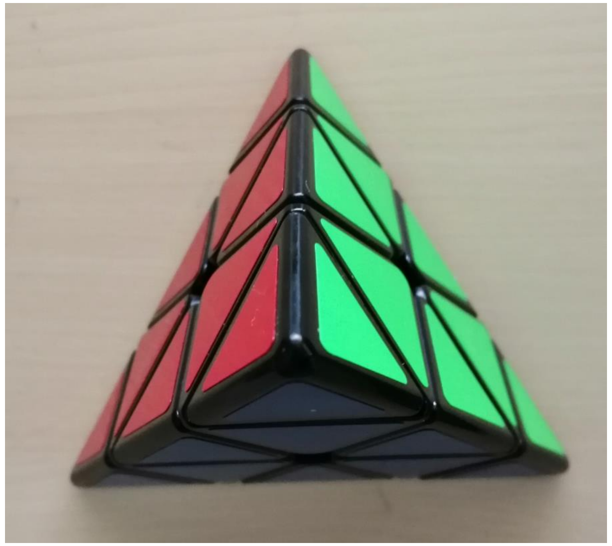




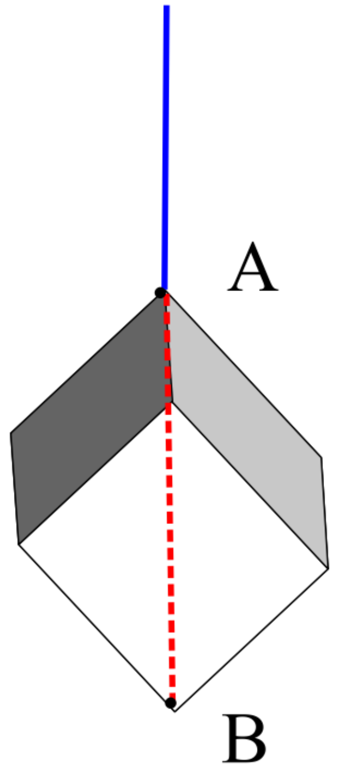
A tetrahedron is to be coloured
In Red, Green, Blue and Yellow,
with a different colour on each face.
How many ways could this be done?

Placing the tetrahedron on its Yellow base with the Blue face facing, the Red face could either be on the right or the left. There are, therefore, two ways to colour.



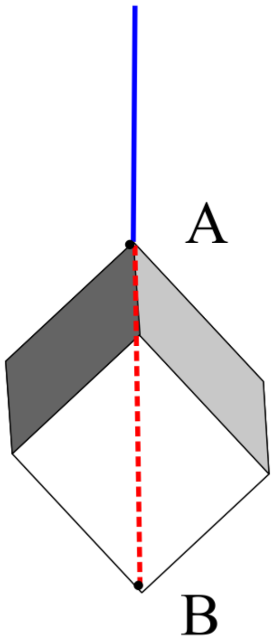


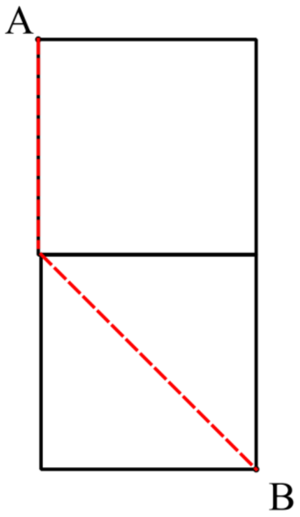
Shortest distance over a cube

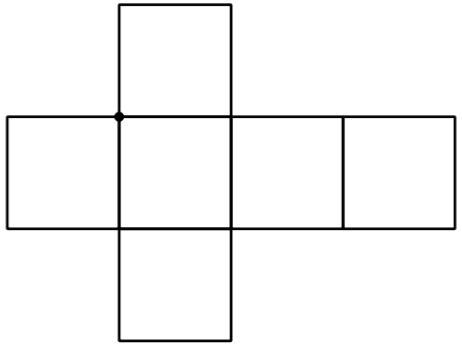


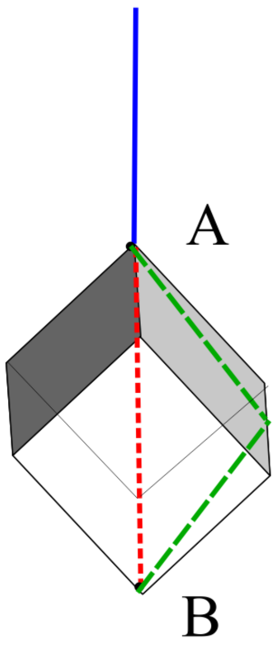
A cube with 5cm edges is suspended by one of its vertices at A.
An insect starting at A wishes to crawl to B
It takes the route shown in red
Could the insect have taken a shorter route?

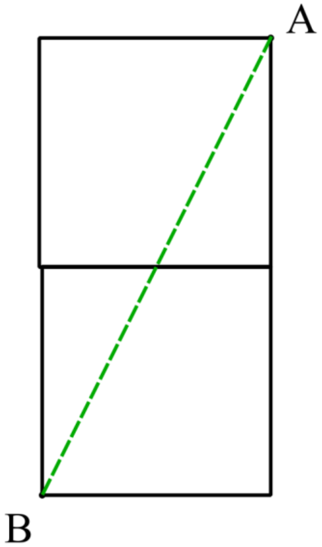
Shortest distance over a cube.











A cube with 5cm edges is suspended by one of its vertices at A.

An insect starting at A wishes to crawl to B

It takes the route shown in red

Could the insect have taken a shorter route?

The route in red has a length of

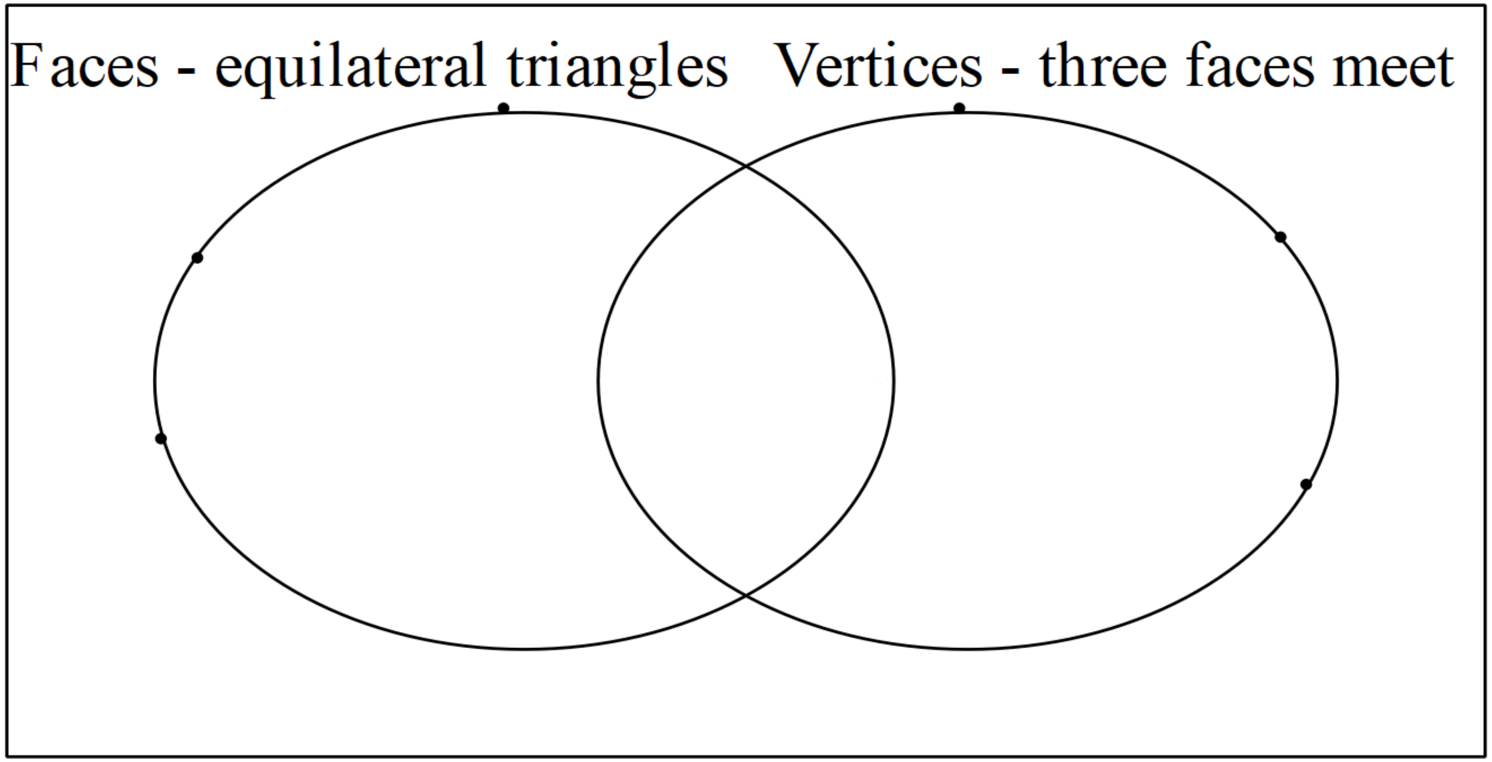
$5+\sqrt{5^2+5^2}=5+5\sqrt{2}=12.07$

Examining the net fragment, there is a shorter route:

The route in green has a length of

$\sqrt{5^2+10^2}=5\sqrt{5}=11.18$

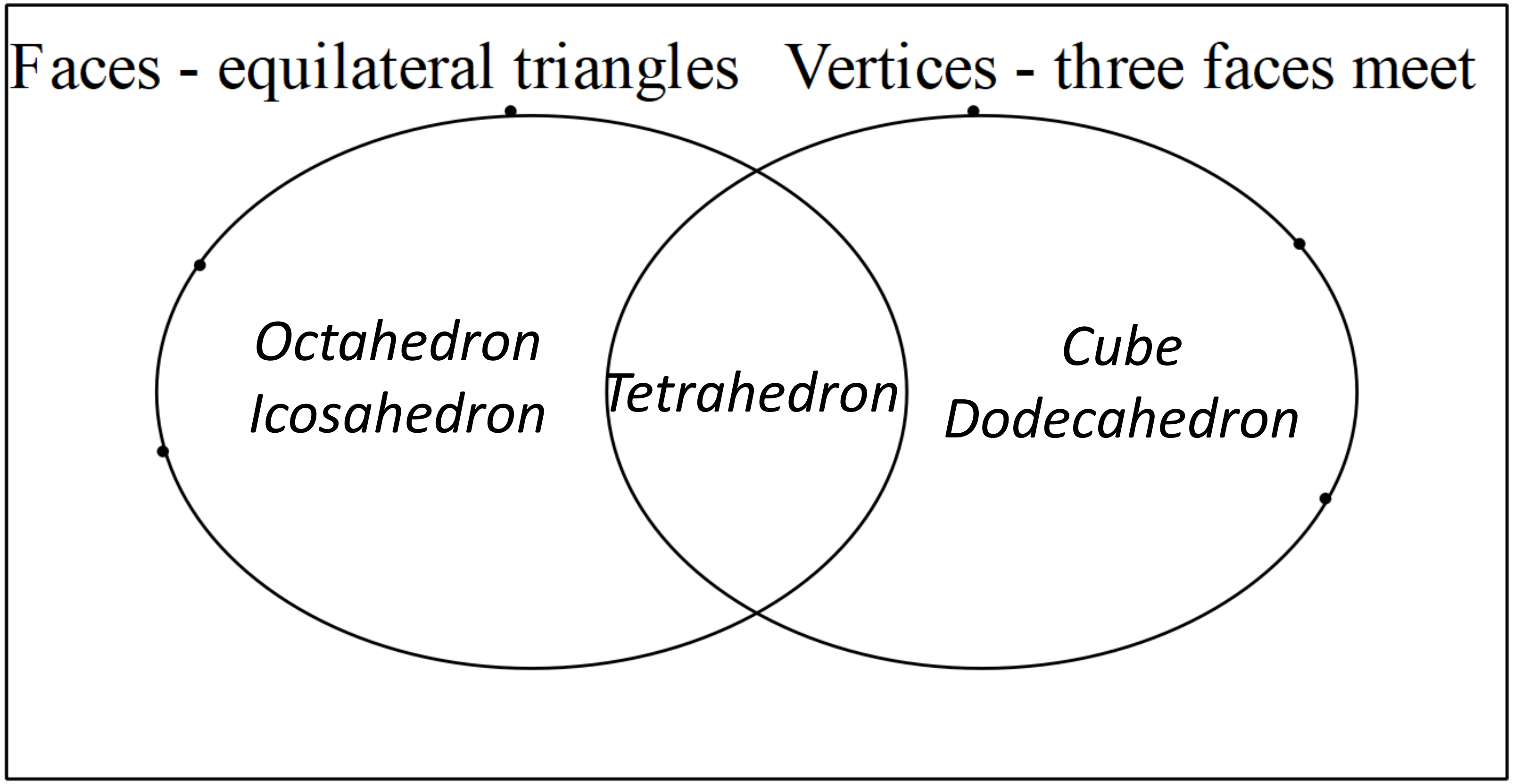
Classifying Platonic Solids – similar and different



The Platonic Solids
Tetrahedron
Cube (Hexahedron)
Octahedron
Dodecahedron
Icosahedron

Complete the Venn diagram with the names of the five Platonic solids

Classifying Platonic Solids – similar and different



The Platonic Solids
Tetrahedron
Cube (Hexahedron)
Octahedron
Dodecahedron
Icosahedron

Complete the Venn diagram with the names of the five Platonic solids

What are the numbers of Vertices, Edges and Faces and how are these numbers connected?

Platonic Solid	Vertices (V)	Edges (E)	Faces (F)
Tetrahedron		6	4
Cube (Hexahedron)	8	12	
Octahedron	6		8
Dodecahedron		30	12
Icosahedron	12	30	

Find the formula connecting V, E and F and use it to check and complete the table

Spotlight

What are the numbers of Vertices, Edges and Faces and how are these numbers connected?

Platonic Solid	**Vertices (V)**	**Edges (E)**	**Faces (F)**
Tetrahedron	*4*	6	4
Cube (Hexahedron)	8	12	*6*
Octahedron	6	*12*	8
Dodecahedron	*20*	30	12
Icosahedron	12	30	*20*

Find the formula connecting V, E and F and use it to check and complete the table

In Euler's Formula $V - E + F = 2$ e.g., for a tetrahedron $4 - 6 + 4 = 2$
This means that if we have counted just two of the number of Vertices(V), Edges(E) and Faces(F) then we can calculate the missing count, so, in an Octahedron as $V = 6$ and $F = 8$, then $6 - E + 8 = 2$, i.e., the number of edges, $E = 12$

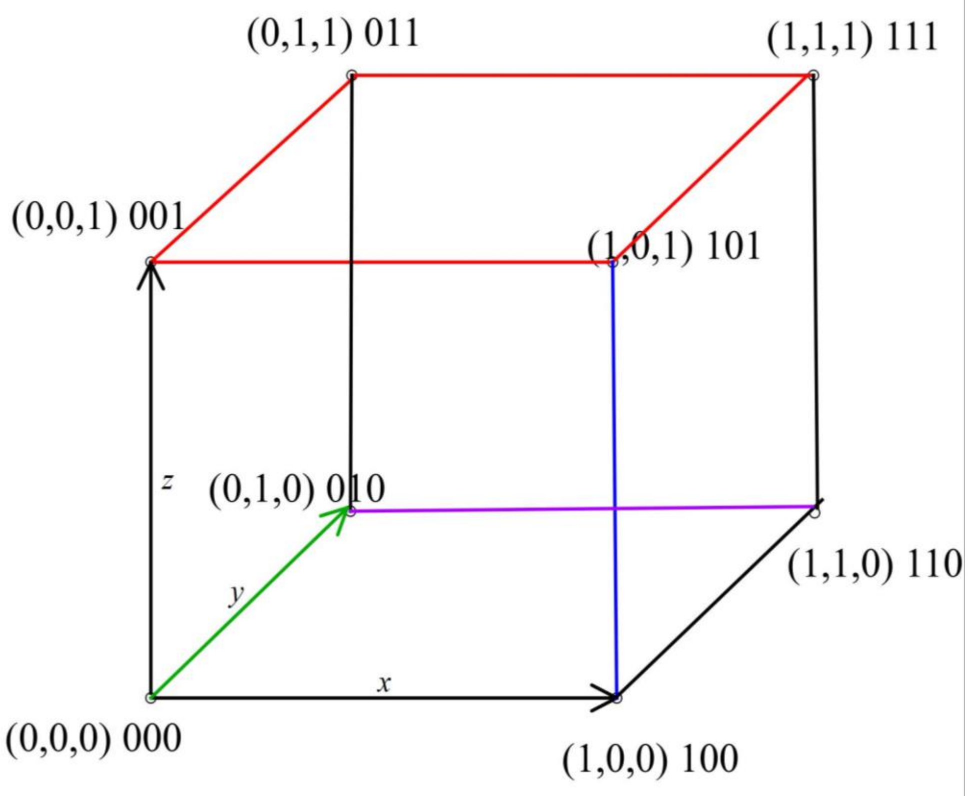
Imagining the 4th Dimension

The four corner points (vertices) of a square wire frame are at coordinates {(0,0), (0,1), (1,0), (1,1)}. The shape has 4 vertices and 4 lines (edges)

The 8 vertices of a cube wire frame are at coordinates {(0,0,0), (0,0,1), (0,1,0), … (1,1,1)}. The shape has 8 vertices, 12 edges and 6 squares (faces),

It might be imagined that the vertices of a 4D-cube wire frame are at coordinates {(0,0,0,0), (0,0,0,1), (0,0,1,0), … (1,1,1,1)}. How many vertices, edges, faces and cubes does this 4D-cube have and conjecture how these numbers of vertices, edges, faces and cubes might be related?

3D Cube



Vertex e.g., 000, 010 (3 fixed coordinates)
Edge e.g., 000+010, 010+110, 100+101 (2 fixed)
Face e.g., 001+011+101+111 (1 fixed coordinate)

V – E + F = 8 – 12 + 6 = 2

4D Tesseract

Number of vertices V (4 fixed coordinates)

$= 2^4 = 16$

e.g. 0000, 0001,…,1111

Number of edges E (3 fixed coordinates)

$= \binom{4}{3} \times 2^3 = 4 \times 2^3 = 32$

e.g. 0000+0001

Number of faces F (2 fixed coordinates)

$= \binom{4}{2} \times 2^2 = 6 \times 2^2 = 24$

e.g. 0000+0001+0010+0011

Number of cubes C (1 fixed coordinate)

$= \binom{4}{1} \times 2^1 = 4 \times 2^1 = 8$

e.g. 0000+0001+0010+0011+0100+0101+0110+0111

V – E + F – C = 16 – 32 +24 - 8 = 0

Platonic dice

The dice on the left are made from Platonic solids. The cubic die can be used to randomly select between six alternatives, in this case between 1, 2, 3, 4, 5 and 6.

1. How many alternatives for each die?
2. Why is the way you read the score on the tetrahedral die different from the others?
3. What do you notice about the two dice below?

Platonic dice

The dice on the left are made from Platonic solids.
The cubic die can be used to randomly select between six alternatives, in this case between 1, 2, 3, 4, 5 and 6.

1. How many alternatives are there for each die?
2. Why is the way you read the score on the tetrahedral die different from the others?
3. What do you notice about the pair of cubic dice?

1. With one alternative for each face: Tetrahedral (4), Cubic (6), Octahedral (8), Dodecahedral (12), Icosahedral (20)
2. The tetrahedral die does not have pairs of opposite faces; one to rest on and the other with a score; it does not roll well.
3. These dice have opposite chirality – right-handed and left-handed. (Western and Chinese)

Platonic Activities – NANAMIC – 27 June 2023

Thank you

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