Developing problem solving approaches for GCSE resit and beyond

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Reflecting on how we solve problems in everyday life

How do we go about solving problems in real life?

Think about a problem that you have ‘solved / fixed’ in the last few days.

Draw a picture of your problem to help the person next to you guess what it was

What did you do to go about solving that problem?

How do we go about solving problems in real life?

My ‘new’ boiler

- this problem matters to me, there is a genuine purpose for fixing it
- some expertise, some experience is needed
- some confidence is needed
- won’t always solve the problem straight away

How do we go about solving problems in real life?

The trumpet

- this problem matters to me
- tried to solve it ourselves
- had to admit defeat and seek outside expertise
- finding solutions to genuine problems takes time and patience

What's the same? What's different about solving a problem in the classroom?

- The problem may well not matter to them
- There is often a time pressure
- They have very little confidence in their ability to tackle the problem
- They have very little experience of sitting with the problem, of waiting for ideas to form, of feeling like they can try something
- They know that their teacher is likely to (i) simplify the problem (ii) be instructive about what method to use (iii) show them the solution quite quickly

How can we begin to create a classroom culture where students are expected to solve problems?

www.rme.org.uk

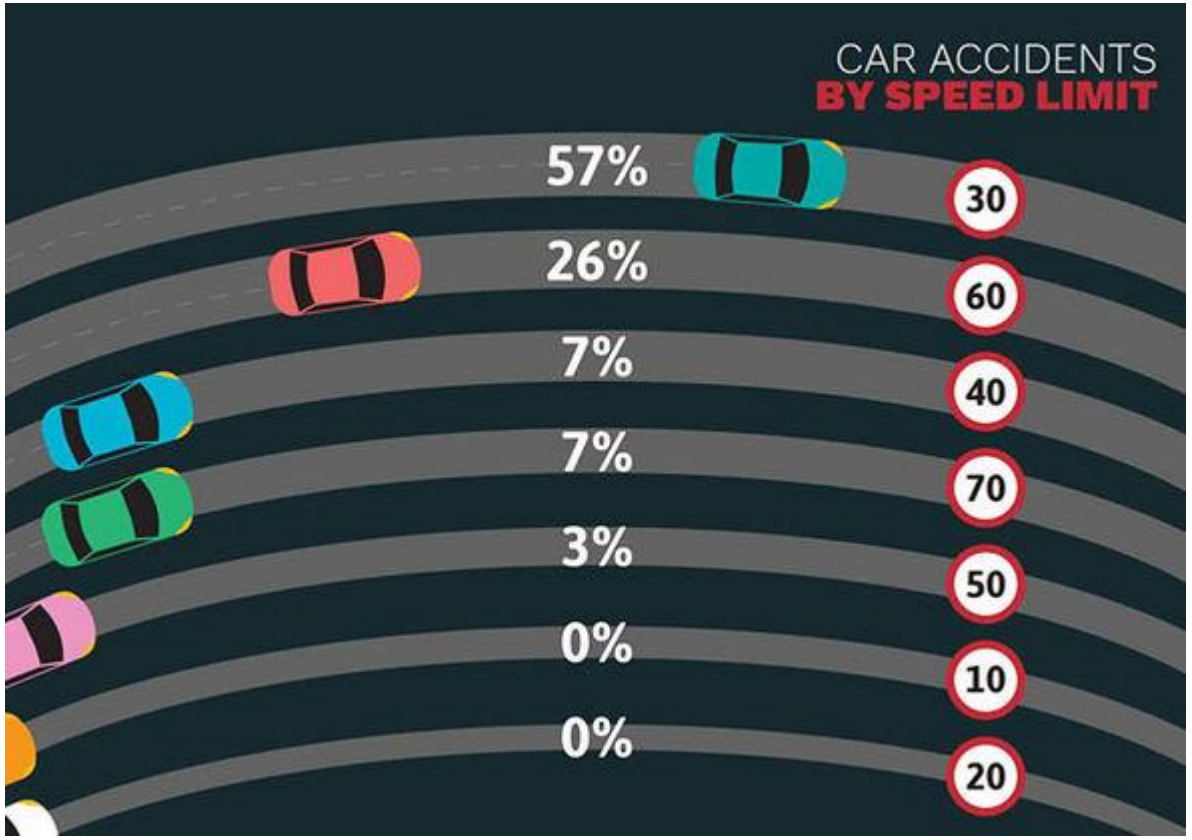
Realistic Maths Education – a ‘bottom up’ approach to learning maths which makes use of relatable contexts where learners are prompted to bring their own informal approaches to solve

Modelling the use of ‘Say what you see’ and ‘What else do you know?’

By Accident

C1.

C1.

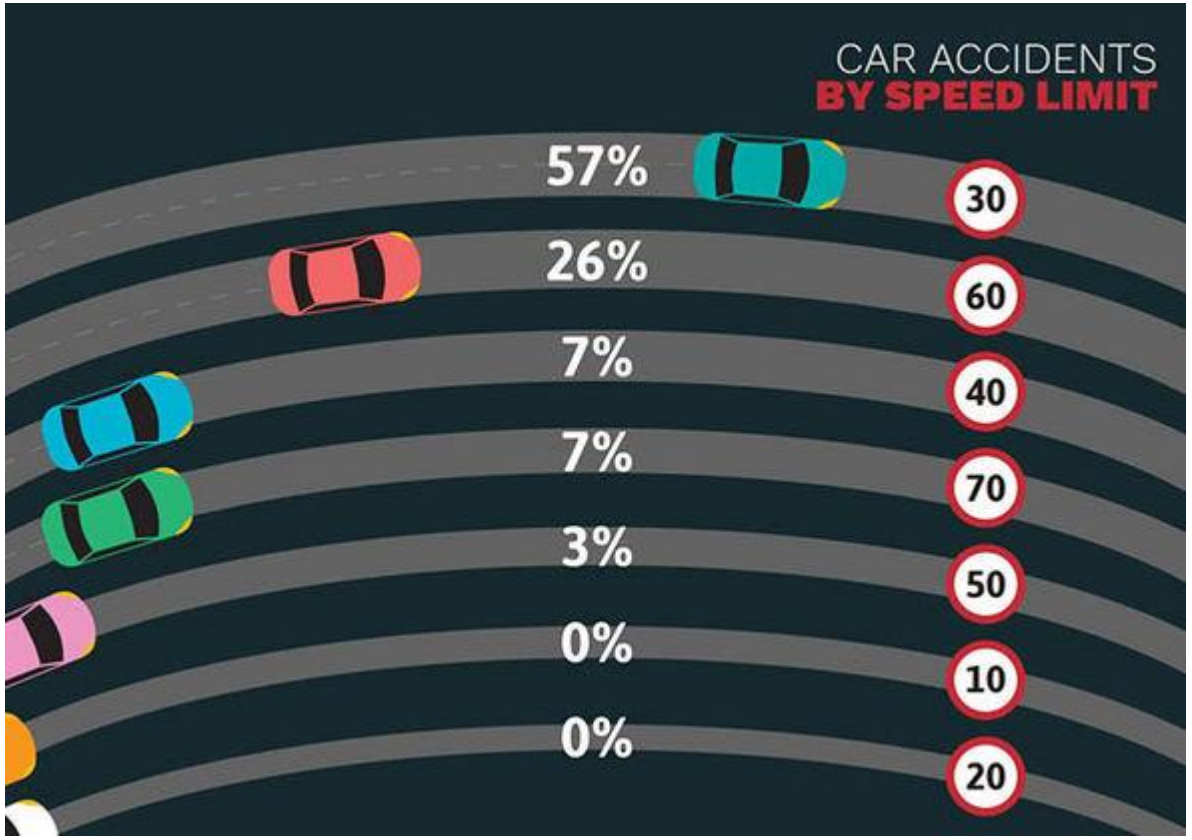


a) Say what you see in the picture.

b) Represent the same information in a different way.

By Accident

C1.



a) Say what you see in the picture.

b) Represent the same information in a different way.

C1. Delegate responses:

30 miles an hour appears to be the most dangerous speed

They are in percentage order

Only 7% of cars travelling over 70 miles an hour had an accident

If you drive at 10 or 20 mph you won't have an accident

It's the speed limit

It's 7% safe if you drive at 70mph

Say what you see

Type of counter	red circle	green circle	red square	green square
Number of counters	16	26	11	7

- What else do you know?

7 There are four types of counter in a bag.

The table shows the number of each type of counter in the bag.

Type of counter	red circle	green circle	red square	green square
Number of counters	16	26	11	7

There are more green counters than red counters.
How many more?

.....

(Total for Question 7 is 2 marks)

Say what you see

In one hour a machine can make

600 nuts

or

720 bolts.

At 3 pm the machine starts working.

It makes 900 nuts and then changes to making bolts.

- .

The original question

AQA 2019 GCSE MATHEMATICS
Foundation Tier Paper 3 Calculator

23 In one hour a machine can make

600 nuts

or

720 bolts.

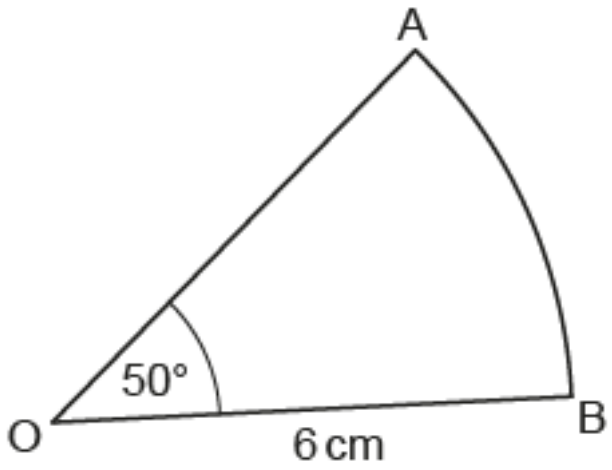
At 3 pm the machine starts working.

It makes 900 nuts and then changes to making bolts.

How many **bolts** will the machine make by 8 pm?

[4 marks]

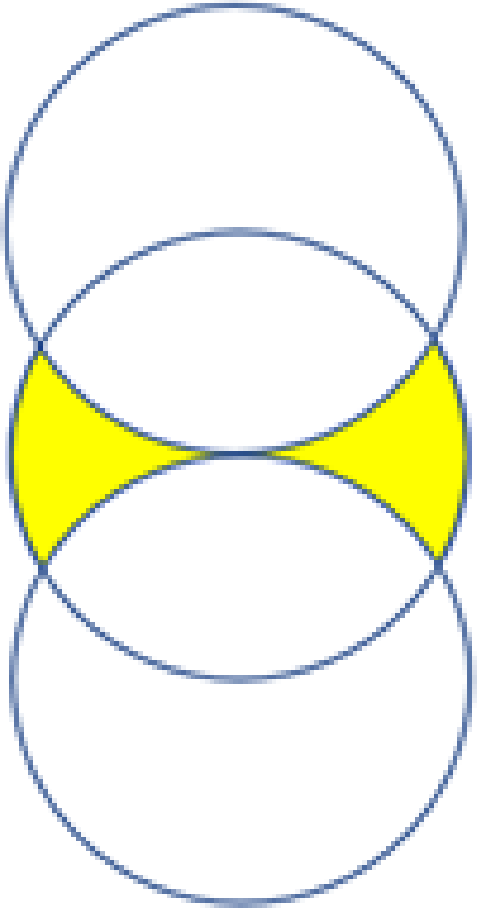
Say what you see



- What else do you know?
- Need the hard edexcel problem plus an A level problem

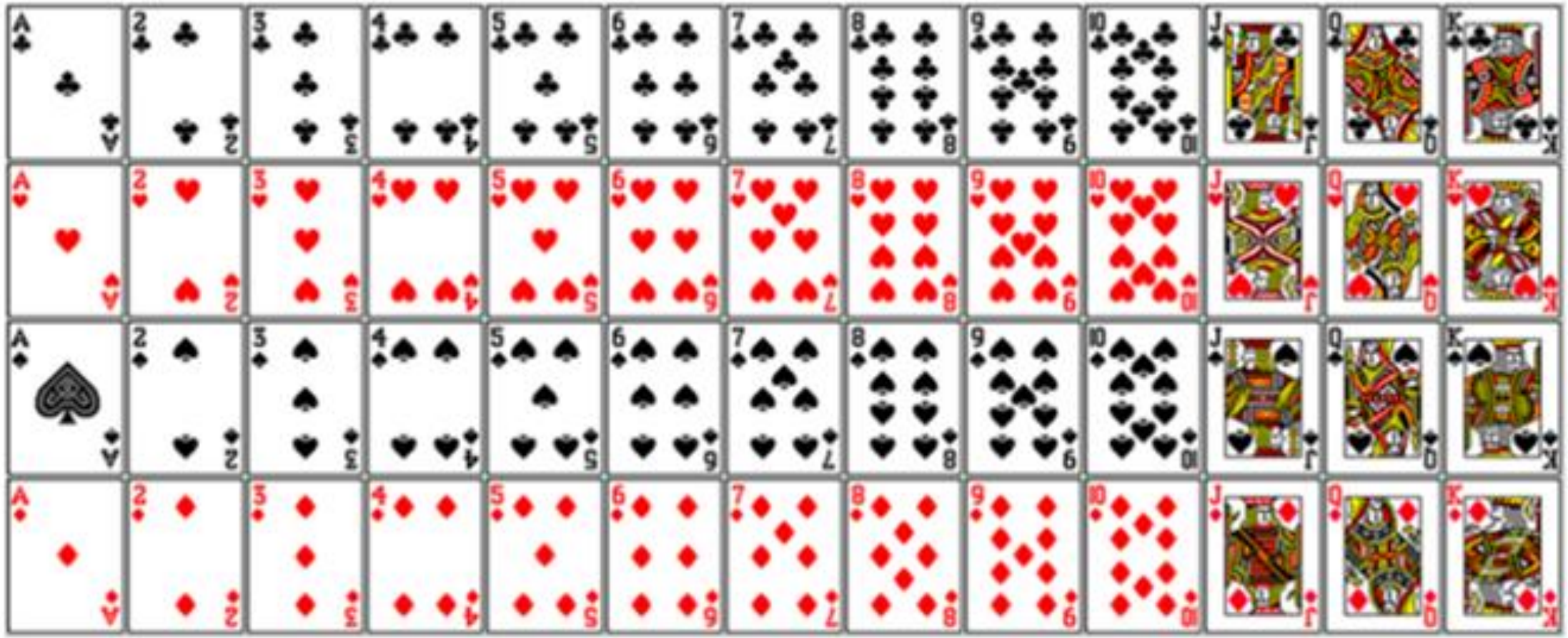
Say what you see

- What else do you know?



The 3 circles are identical. Each one has a diameter of 8cm

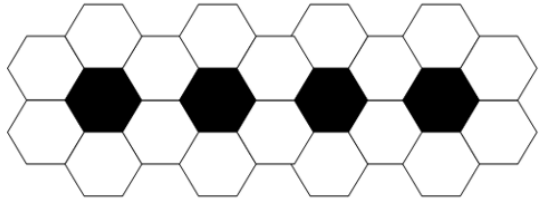
Write down everything you can about this standard pack of cards.



Impact of work on ‘say what you see’ / ‘write down everything you can’.

I think these types of questions inherently make a task more accessible but also more challenging. It creates ‘a low floor but high ceiling’ for pupils, and it forces them to think mathematically.

FLOWERBEDS



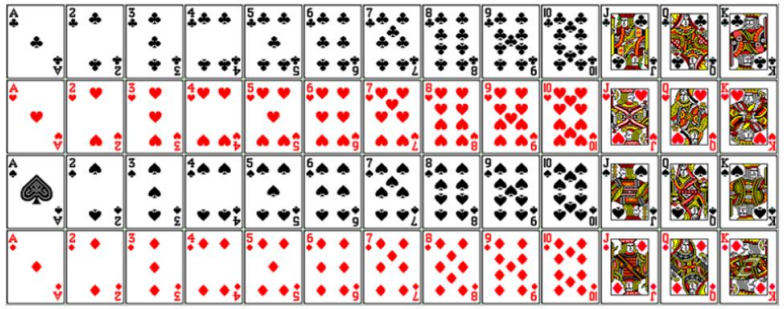
Here are four flowerbeds surrounded by slabs

Write down as much as you can.

Instead of: Work out how many slabs are needed for 100 flowerbeds.

- Students are more likely to be put off and frustrated by this question.
- Everyone can write something down for this problem.
- Open ended gives more time for the teacher (if at the start), and more scope for students to think.

Write down everything you can about this standard pack of cards.



Instead of: Find the probability of getting a king, or find the probability of getting a king or a queen etc…

This task has a huge scope. From counting cards, and ways of counting, to probability and FDP, to looking at conjecturing and generalising some probability formulas. Crib questioning sheet on next slide.

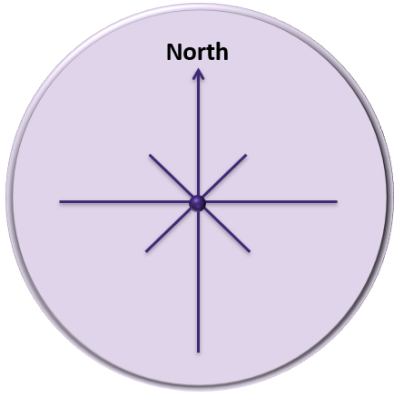
Conjecturing: P(A) = 1 – P(A’)

Conjecturing: P(A or B) = P(A) + P(B)

Conjecturing: P(A or B) = P(A) + P(B) – P(A∩B)
When is it **P(A or B) = P(A) + P(B)?** When they are **mutually exclusive.**

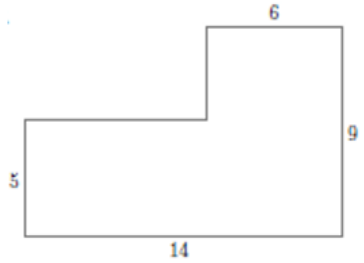
Conjecturing $P(A \cap B)$ formula is for them to discover.

Fill in what you can…



Used as a starter activity on introduction to bearings.

Instead of find the bearing at North West…



Say what you can see:

- I can see 14, 5, 6, 9.
- An L shape.
- Two rectangles stuck together.
- A length of 8.
- A length of 4.

Then prompted where else.

And thus find the area as many ways as you

Open ended, accessible, and scope for deep mathematical thinking.

Adapting exam questions to make use of ‘Say what you see’ and ‘What else do you know’

Which exam paper questions would work well with ‘say what you see’ / ‘what else do you know?’

Please check the examination details below before entering your candidate information

Candidate surname | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 1/Level 2 GCSE (9–1)

Time 1 hour 30 minutes | **Paper reference** **1MA1/1F**

Mathematics

PAPER 1 (Non-Calculator)

Foundation Tier

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may not be used.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Write $\frac{3}{10}$ as a percentage.

.....%

(Total for Question 1 is 1 mark)

2 Write the following numbers in order of size.
Start with the smallest number.

8 −7 −10 1 0 −2

.....

(Total for Question 2 is 1 mark)

3 Write $\frac{9}{100}$ as a decimal.

.....

(Total for Question 3 is 1 mark)

4 Write 327 correct to the nearest ten.

.....

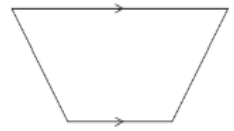
(Total for Question 4 is 1 mark)

5 Write down the value of 7^2

.....

(Total for Question 5 is 1 mark)

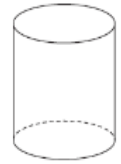
6 (*a*) Write down the mathematical name of this quadrilateral.



.....

(1)

(*b*) Write down the mathematical name of this 3-D shape.



.....

(1)

(Total for Question 6 is 2 marks)

7 £42 is shared equally between 3 friends.

How much does each friend get?

£.....

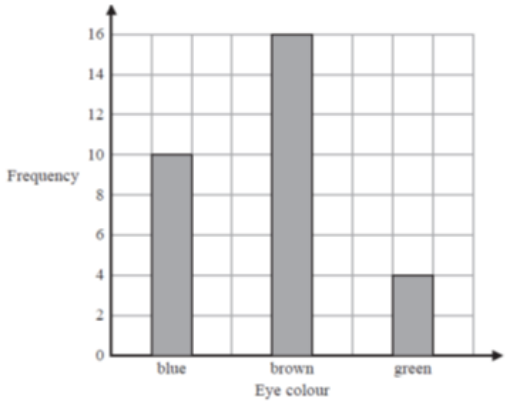
(Total for Question 7 is 2 marks)

8 Grace recorded the eye colour of each of the students in her class.

The frequency table below shows her results.

Eye colour	Frequency
blue	10
brown	15
green	4

Grace then drew the bar chart below for this information.



Write down one thing that is wrong with this bar chart.

.....

.....

.....

(Total for Question 8 is 1 mark)

9 Danny buys,

1 loaf of bread for £1.20
1 bottle of milk for 70p
2 packets of cheese for £2.30 each packet

Danny pays with a £10 note.

He says,

"I should get £3.30 change."

Is Danny correct?
You must show how you get your answer.

(Total for Question 9 is 3 marks)

10 Rachel records the temperature in her garden at noon each day.

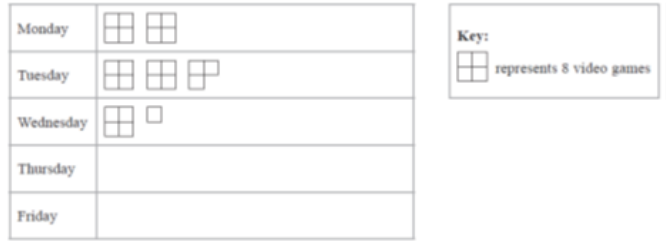
On Monday, the temperature was 5 °C.
On Tuesday, the temperature was 10 ° less than the temperature on Monday.
On Wednesday, the temperature was 3° greater than the temperature on Tuesday.

Find the difference between the temperature on Monday and the temperature on Wednesday.
You must show all your working.

..... °C

(Total for Question 10 is 2 marks)

11 The pictogram shows information about the number of video games sold in a shop on



(*a*) How many video games were sold on Monday?

.....

(1)

More video games were sold on Tuesday than on Wednesday.

(*b*) How many more?

.....

(2)

On Thursday and Friday, a total of 32 video games were sold in the shop.

$\frac{1}{4}$ of these 32 video games were sold in the shop on Thursday.

(*c*) Complete the pictogram for Thursday and Friday.

(3)

(Total for Question 11 is 6 marks)

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12 There are two drama groups in a school.

In one group there are 36 boys and 48 girls.

In the other group, $\frac{3}{7}$ of the students are boys and the rest of the students are girls.

Ann says,

"The ratio of the number of boys to the number of girls is the same for both groups."

Is Ann correct?
You must show how you get your answer.

(Total for Question 12 is 3 marks)

13 A number sequence starts 1 2 4

Emma says that the next term is 7

(*a*) Explain why Emma may be correct.

.....

.....

.....

(1)

Here are the first four terms of the sequence of triangle numbers.

1 3 6 10

(*b*) Find the 8th term of this sequence.

.....

(2)

(Total for Question 13 is 3 marks)

14 3 kg of carrots cost £1.80
2 kg of carrots and 5 kg of potatoes cost a total of £3.45

Work out the total cost of 4 kg of carrots and 2 kg of potatoes.
You must show all your working.

£.....

(Total for Question 14 is 4 marks)

Observations

- ‘Say what you see’ removes the threat of thinking they can’t do the question
- When the image is complex ‘say what you see’ encourages learners to dip their toe in – it develops resilience
- You can’t really be wrong
- ‘Say what you see’ encourages learners to listen to each other
- The teacher’s role is to pause, not to judge, sometimes to scribe

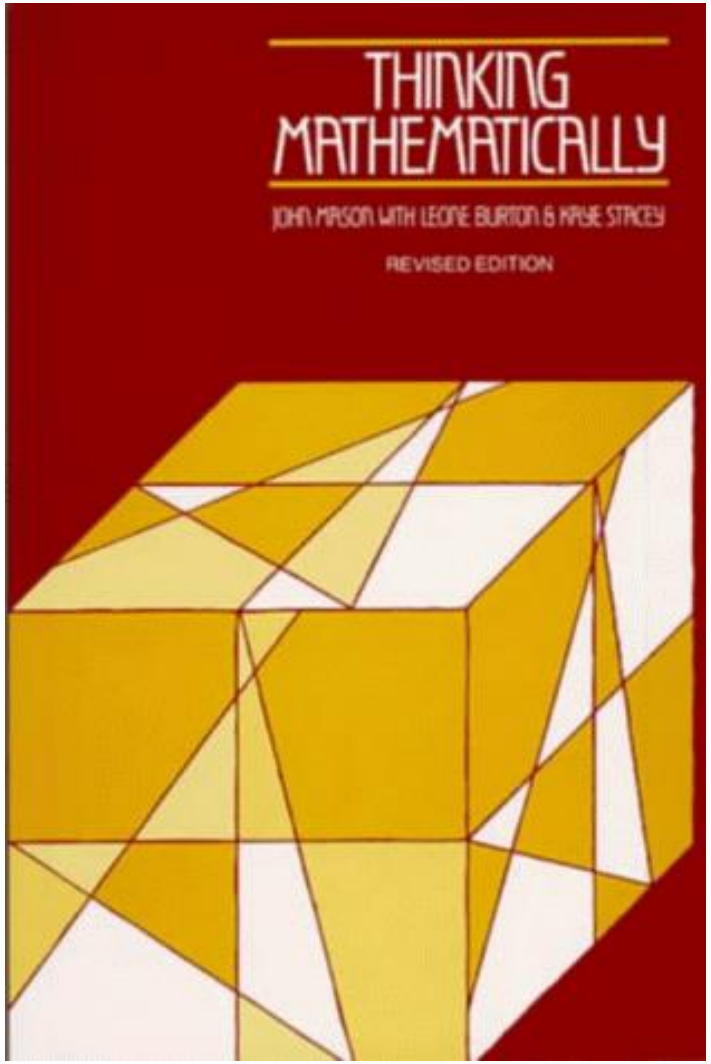
Creating a classroom culture which allows learners to ‘be stuck’ and empowers them with ways to become ‘unstuck’

Problem Solving involves ‘being stuck’

‘…being stuck is an honourable state and an essential part of improving thinking. However to get the most out of being stuck it is not enough to think for a few minutes and then read on… Time taken to ponder the question and to try several approaches is time well spent.’

‘Answers are irrelevant ….concentration on processes rather than answers…’

‘Elegant solutions such as those found in most mathematics texts rarely spring fully formed from someone’s brain. They are more often arrived at after a long and tortuous period of thinking with much modification and changing of thinking along the way …’



What do we do when our learners say they can’t do it?

- It’s really important that they are given time to know they are stuck

- It’s not enough for them to sit there and wait to be told how to do it

Instead:

- Teacher says: ‘If you don’t think you can do the question then please write down why you can’t do it’
- Teacher says: ‘ DRAW SOMETHING’

… and circles round the classroom to check either of these things is happening

The power of a drawing – ‘Draw Something’

Buying ribbon

Louisa is a dress maker. She uses ribbon to edge her garments as a way of making her designs stand out. Louisa buys a sample of the ribbon shown below:

a) It costs her £ 1.16 for 80cm of the spotty ribbon. Draw a picture to represent this information and make it look realistic.

b) What else do you know? Mark on your picture some other quantities of this ribbon, for which you would know the cost.

c) How much would it cost to buy 1 metre of this ribbon?

‘Draw something’

For each of the questions try to draw something as part of your solution

1) Katie got 60% in her exam. If she got 24 marks then what were the total number of marks available
2) Which is closer to 1? 5/7 of 7/5
3) A small pizza has a diameter of 5cm. A large pizza has a diameter of 10cm. If you want as much pizza as possible would you rather have 3 small pizzas or 1 large pizza

Buying ribbon

National Centre for Excellence in the Teaching of Mathematics

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c) How much would it cost to buy 1 metre of this ribbon?

How can we begin to create a classroom culture where students are expected to solve problems?

In today's session we will work with the following ideas:

1) **Remove the problem from the problem**

Goal-free problems, 'say what you see'; use of a scribe; 'what else do you know

2) **Recognise 'being stuck' and practise ways of becoming 'unstuck'**

3) **Who is doing the maths?**

Find ways of shifting the mathematical authority from the teacher to the student. E.g. who reads out the slide? What happens next? Pass the pen: teacher remains neutral

Summary

- ‘Say what you see’ and ‘what else do you know?’ place emphasis on seeing the detail, on thinking creatively. They develop learner confidence and patience

- ‘Being stuck’ is an important state and our learners need experiences which might change how they feel about this, and an awareness of what strategies might help them become unstuck