Good Mathematicians Can Go Backwards!

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Going Backwards

There is so much opportunity for thinking backwards when we teach – a great learning opportunity and also a problem solving strategy.

working backward

Search Results 996 | **Title Matches 4**

Type
- [x] Resources
- [x] Games
- [x] Articles
- [x] Interactivities

Age
- [x] All ages
- [ ] 3-5
- [ ] 5-7
- [ ] 7-11
- [ ] 11-14
- [ ] 14-16
- [ ] 16+

Challenge Level
- [x] ★
- [x] ★★
- [x] ★★★

Working Backwards at KS1
Age 5 to 7
The lower primary tasks in this collection could each be solved by working backwards.

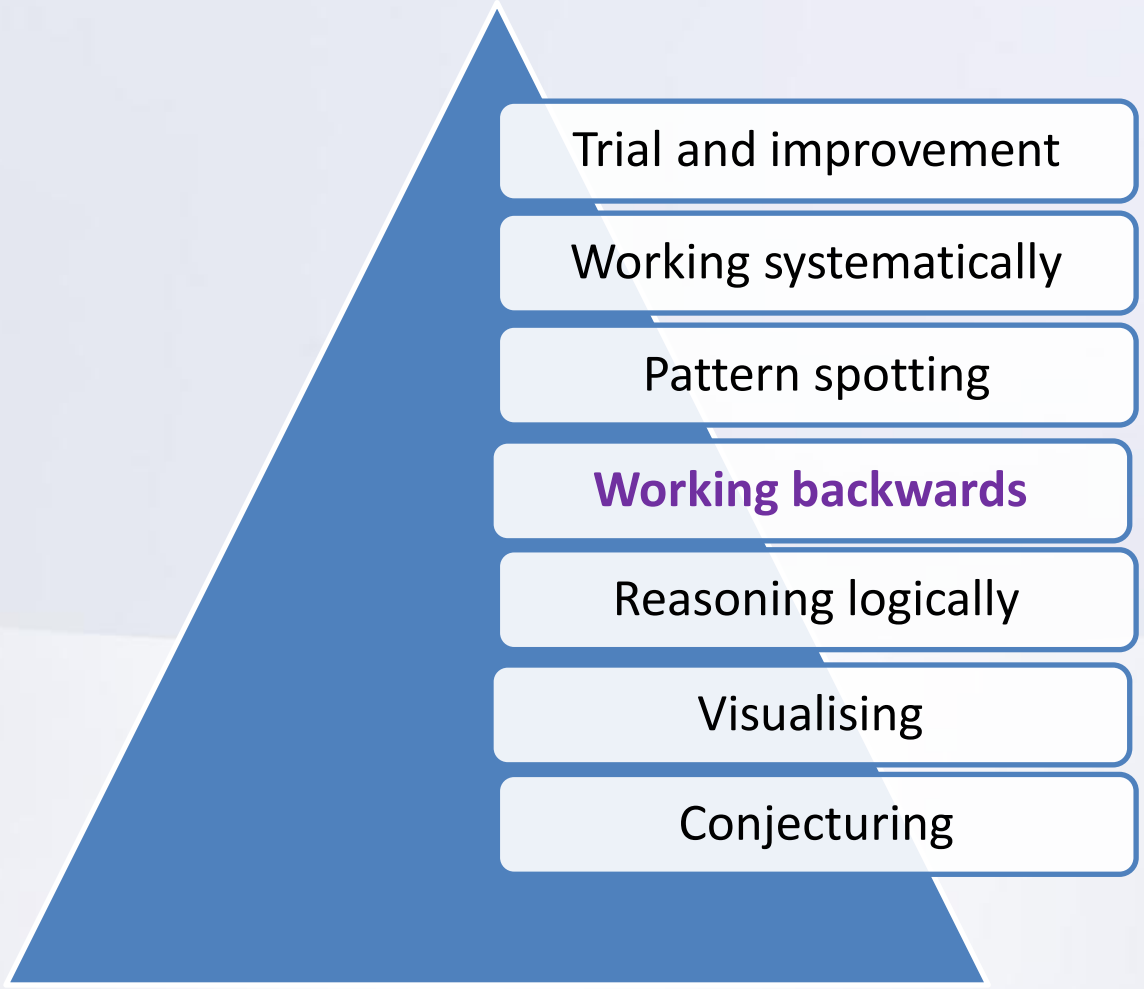
Working Backwards at KS2
Age 7 to 11
The upper primary tasks in this collection could each be solved by working backwards.

Working Backwards to Move Forwards
Age 11 to 18
Challenge Level ★

Working Backwards, Leaping Forwards live
Age 11 to 18
Challenge Level ★
Working backwards can help you to make great leaps forwards!

See from Liz Woodham on Nrich: Developing Problem-solving skills

From Nrich



Going Backwards

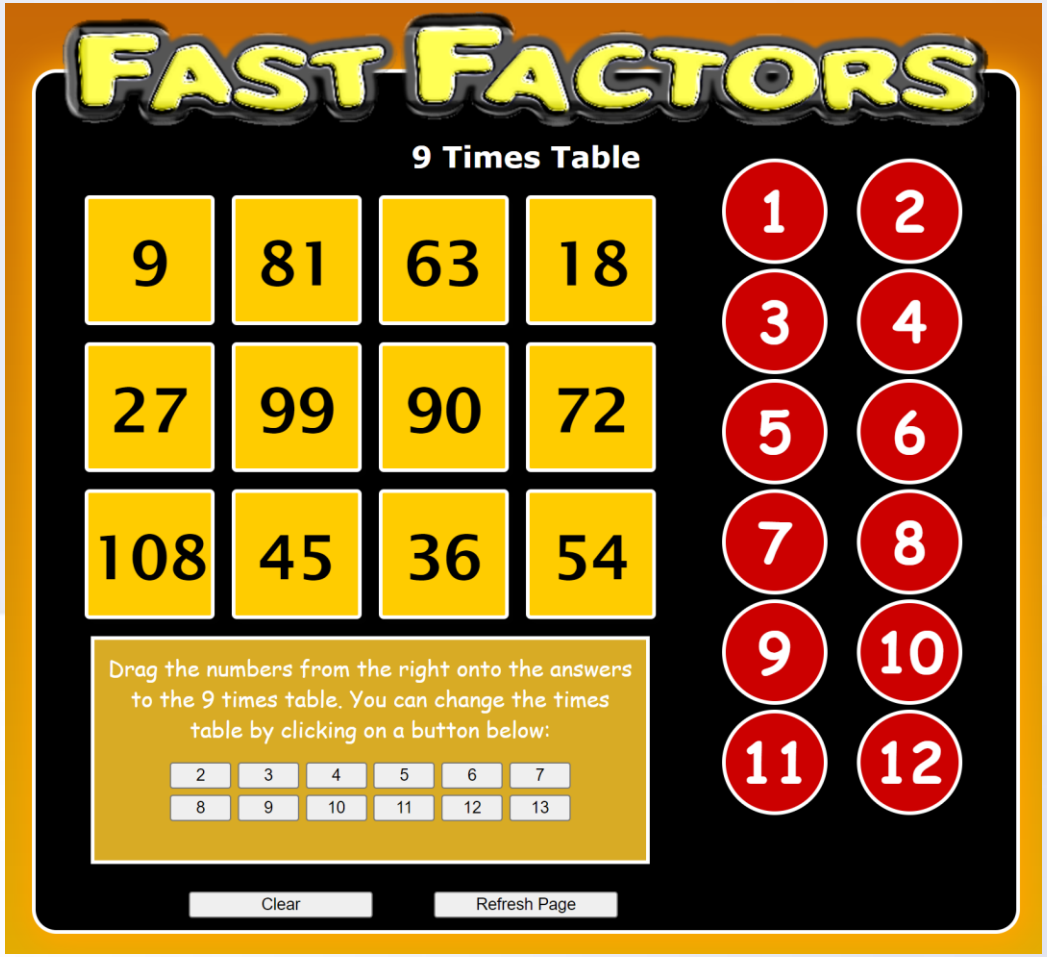
Some ideas and resources for thinking backwards!

All resources discussed here are linked to from colleenyoung.org

Number - Tables

Learn them backwards too.

[Try Transum's Fast Factors]



Settings

Reveals: 10 | 12 | 14 | Unlimited

Level: Simple | Normal

Style: Textured | Flat

Settings will take effect when you apply them...

In the current window

In a resizable pop up window

Missing Multipliers

What is the smallest number of answers you need to reveal in order to work out the missing headers?

Missing Multipliers

Can you work out the missing multipliers?

Hidden behind each cell is the product of the values at the far top and left of it.

10 reveals left

Show the solution

Restart

X	2	3	4	5	6	7	8	9	10	11	12
2	●										
3											
4											
5	●										
6											
7											
8	●										
9											
10											
11	●										
12											

Missing Multipliers

Can you work out the missing multipliers?

Hidden behind each cell is the product of the values at the far top and left of it.

4
reveals left

Show the solution

Restart

X	2	3	4	5	6	7	8	9	10	11	12
5											
2	8	40									
3											
4											
5	5	25									
6											
7											
8											
9											
10											
11											
12											

Missing Multipliers

Can you work out the missing multipliers?

Hidden behind each cell is the product of the values at the far top and left of it.

4
reveals left

Show the solution

Restart

X	2	3	4	5	6	7	8	9	10	11	12
8	40										
5	25										
6		54	60								

Missing Multipliers

Can you work out the missing multipliers?

Hidden behind each cell is the product of the values at the far top and left of it.

4
reveals left

Show the solution

Restart

X	5	9	10	11
8	40			
5	25			
6		54	60	
2		18		22

Key questions

Which numbers, when revealed, make it straightforward to work out the row and column headings?
Which numbers give lots of possibilities for row and column headings?

Is there a strategy for working out the row and column headers in fewer than 10 reveals?

Can you find a way to work out the row and column headers using only 6 reveals?

Missing Multipliers

Can you work out the missing multipliers?

Hidden behind each cell is the product of the values at the far top and left of it.

4 reveals left

Show the solution

Restart

X	5	9	10	11
8	40			
5	25			
6		54	60	
2		18		22

Missing Multipliers

Can you work out the missing multipliers?

Hidden behind each cell is the product of the values at the far top and left of it.

4 reveals left

Hide the solution

Restart

X	5	9	10	11
8	40	72	80	88
5	25	45	50	55
6	30	54	60	66
2	10	18	20	22

Missing Multipliers

Can you work out the missing multipliers?

Hidden behind each cell is the product of the expressions at the far top and left of it.

X	x	-1	-2	-3	-4	-5	5	0	1	2	3	4	5
		$x+-2$	$x+2$	$x+5$	$x+4$								
x	$x+4$	x^2+2x-8											
-1	$x+-1$	x^2-3x+2			x^2+4x-5								
-2													
-3	$x+-3$		x^2-1x-6					x^2+x-12					
-4													
-5													
0													
1													
2													
3	$x+3$		x^2+5x+6										
4													
5													

Show the solution

Restart

Missing Multipliers

Can you work out the missing multipliers?

Hidden behind each cell is the product of the expressions at the far top and left of it.

X	$x+-2$	$x+2$	$x+5$	$x+4$
$x+4$	x^2+2x-8	x^2+6x+8	$x^2+9x+20$	$x^2+8x+16$
$x+-1$	x^2-3x+2	x^2+x-2	x^2+4x-5	x^2+3x-4
$x+-3$	x^2-5x+6	x^2-1x-6	$x^2+2x-15$	x^2+x-12
$x+3$	x^2+x-6	x^2+5x+6	$x^2+8x+15$	$x^2+7x+12$

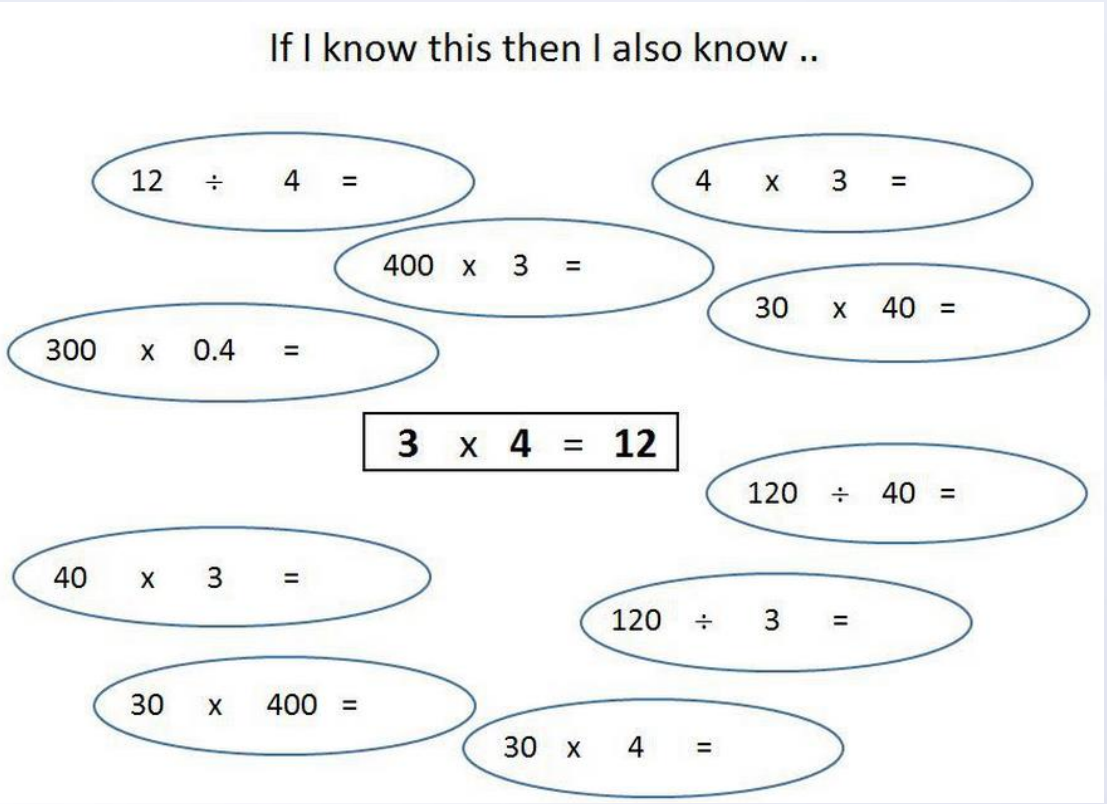
Hide the solution

Restart

Number Operations

Manipulate numerical expressions

	Given that	$25.6 \times 32 = 819.2$
(a)	work out	$\frac{81.92}{32}$
(b)	work out	0.256×320



Number - Rounding

A pop concert has a crowd of 2000 people rounded to 1 significant figure.

A rock concert has a crowd of 2000 people rounded to 2 significant figures.

Work out the largest possible difference between the exact numbers of the two crowds.

(Total 3 marks)

Look at many examples

Number	1sf
21	20
24	20
25	30
27	30
29	30
29.9999	30

Number	1sf
1499	1000
1500	2000
1501	2000
2000	2000
2400	2000
2499	2000
2500	3000

Number	2sf
1499	1500
1500	1500
1501	1500
2000	2000
2400	2400
2499	2500
2500	2500

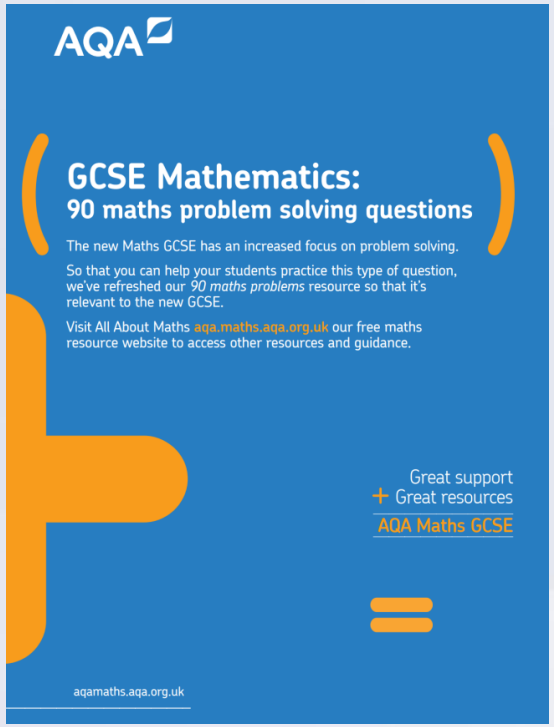
Number	2sf
1501	1500
1900	1900
1949	1900
1950	2000
1960	2000
1990	2000
1949	1900
1950	2000
1500	1500
2100	2100
2050	2100
2049	2000

For example, in C4, the formula is

=ROUND(B4,1-(1+INT(LOG10(ABS(B4)))))

AQA – GCSE Mathematics

90 maths problem solving questions

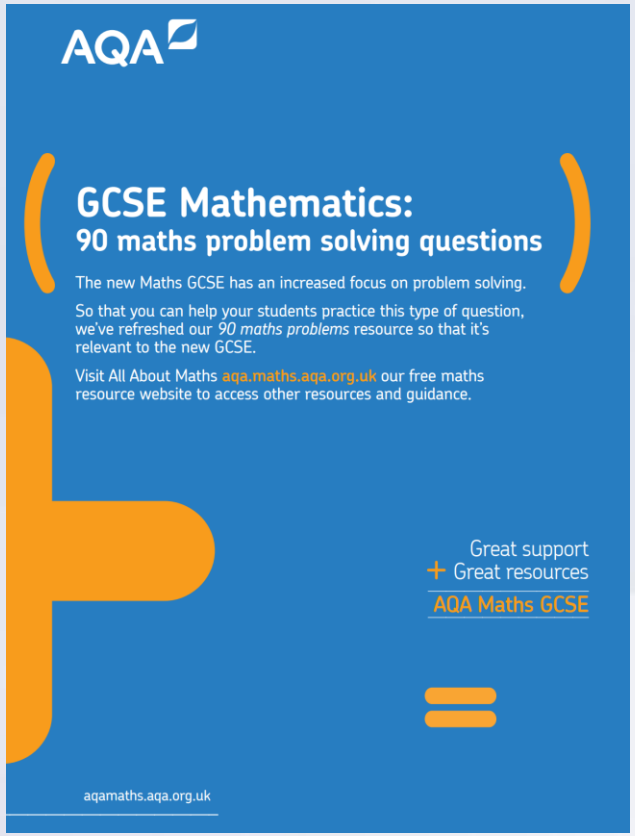


For the purposes of this resource, these five possibilities for action, which can be developed as strategies for the individual, will be labelled as follows:

1. Set out cases
2. Work back familiar; work back unfamiliar
3. Find an example to fit
4. Find key relationships
5. Find mathematical features

AQA – GCSE Mathematics

90 maths problem solving questions



2 To work backwards from a value given in the problem:

- (a) where the inverse is familiar, so just has to be applied but may have to be sustained over a number of steps.
- (b) where the inverse is unfamiliar, so has to be worked out 'from first principles'.

3 To find one or more examples that fit a condition for the answer, and see whether those examples fit with the other conditions in the situation, making adjustments until they do.

AQA – GCSE Mathematics

90 maths problem solving questions

Work back - familiar	Apples ✓	Foundation (easiest)
	Stationery	Foundation
	Boat Hire	Foundation
	Half Take ✓	Either
	Hotel	Either
	Flight cost	Higher
Work back - unfamiliar	Put the Numbers in	Foundation
	Poster ✓	Either
	Bag	Either
	Pointillism	Either
	Scale Factor	Either
	Javelin A	Higher
	Javelin B ✓	Higher (hardest)
	Form	Higher (hardest)
	Square Area	Higher (hardest)

✓

✓

✓ Commentary available
(All problems have full answers)

Apples

Lottie has a bag of apples.

She gives half of them to Fred.

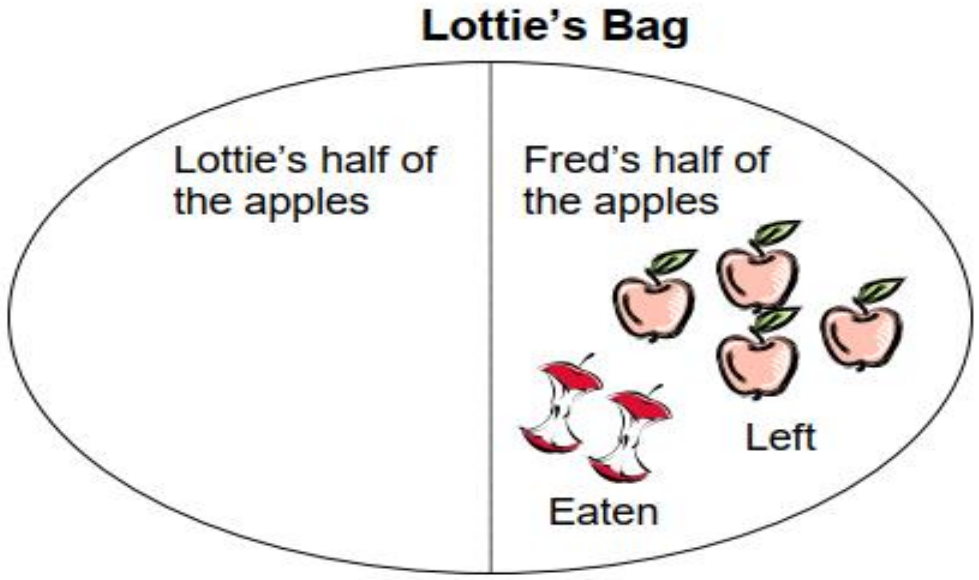
Fred eats two and then has four left.

How many apples did Lottie have at the start?

Answer .....

Finding the answer

1. Fred finished with four apples, and had eaten two, so working backwards he must have initially been given six. Therefore, Lottie must have had twelve (double six) apples to begin with.
2. An alternative is to 'model' the situation using labelled representations of the quantities, for example:



and work it out from there.

Follow up

Variants of the question which become increasingly (but gradually) more complicated are readily invented.

For example:

Lottie has bag of apples. She gives a quarter of her apples to Fred. Fred eats half of these and has 3 left. How many apples did Lottie have to start with?

Lottie has a bag of apples. She gives a third of her apples to Fred. Fred eats 2 apples and gives what he has left to Maisy. Maisy eats 3 apples and has 2 left. How many apples did Lottie have to start with?

Apples

Lottie has a bag of apples.

She gives half of them to Fred.

Fred eats two and then has four left.

How many apples did Lottie have at the start?

Problem solving classification	*Work back - familiar*
Content area classification	*Number in correct context*
Tier	*Foundation (easiest)*
Answer	*12*

About the question

For many students, this is an 'obvious' question, which does not require any working out at all – they would 'just know'. For students who are problem solving at this level, however, it will be a challenge to create and retain a mental model of the situation that would enable a solution to be found, so will have to reason it out step by step.

Problem solving approaches

The reverse operation of subtraction (from some being eaten) is adding, and the reverse of halving is doubling. Working back from the final situation to the starting point may need to be suggested, and possibly supported at each step.

Challenges / issues

Some students may just try a starting number and see whether it 'works' but those who are problem solving at this level may not know how to strategically improve on their first attempt thereby making this method inefficient (at best). A more structured approach should be encouraged.

Apples

Lottie has a bag of apples.

She gives half of them to Fred.

Fred eats two and then has four left.

How many apples did Lottie have at the start?

Half Take

Marcus thinks of a number between 25 and 35
He divides the number by 2 and then subtracts 0.5

He takes his answer, divides it by 2 and then subtracts 0.5

He repeats this process a number of times and gets zero.

What number did he start with?

Answer .....

Problem solving classification	*Work back – familiar*
Content area classification	*Fractions and decimals*
Tier	*Either*
Answer	*31*

About the question

The problem has the appearance of more familiar inverse problems: 'I think of a number and do something with it; this is the answer, what number did I first think of?' As such, it is deceptively accessible - until it is realised that the number of steps in the process is not stated.

Problem solving approaches

The most constructive approach to a solution is one based on inverting the series of operations. Some may be tempted towards the seemingly easier option of trying each number from 25 to 35 in turn but this will mostly lead to a lengthening string of decimals that overshoots the zero target and may well cause many to give up.

Challenges / issues

The challenges for the students may be two-fold. One is recognising that there is a better way of solving the problem than trying numbers until one works. The other is in implementing the better method (inverting) ie, understanding how to invert the operations and inverting the sequence of their application.

Half Take

Marcus thinks of a number between 25 and 35
He divides the number by 2 and then subtracts 0.5

He takes his answer, divides it by 2 and then subtracts 0.5

He repeats this process a number of times and gets zero.

What number did he start with?

Finding the answer

The most direct solution strategy is to repeatedly undo what has been done starting from zero. It may be helpful to represent the mathematics diagrammatically:

number → divide by 2 → subtract 0.5 → next number

and from that work out the reverse process:

previous number ← ? ← ? ← number

This should help students construct an inverse sequence from zero, repeatedly adding 0.5 and doubling the answer, giving the sequence 0, 1, 3, 7, 15, 31, 63 etc.

Reference to the question, and the range of Marcus' starting number, gives the answer.

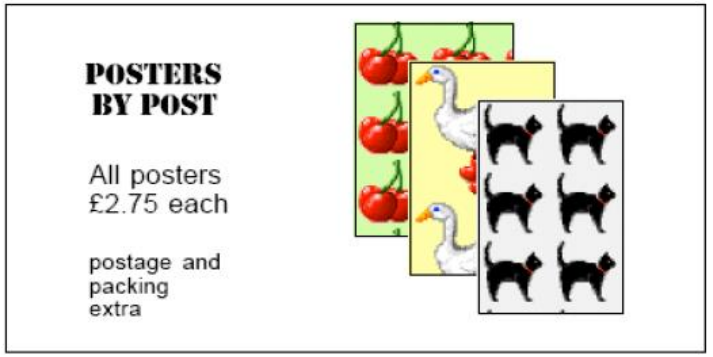
Follow up

Similar problems using different values such as starting with a number between 360 and 370, dividing by 3 and subtracting one-third leading to a zero. What common features do the problems have? (multiplier and divisor have to be the same value, value before 0 must be 1). For higher tier students this could be an introduction to iteration, which is a topic in the new specification (exams from June 2017). The formula for the given problem is

$u_{n+1} = \frac{u_n}{2} - 0.5$ and the

inverse formula is $u_{n+1} = 2(u_n + 0.5)$

Poster



Posters cost £2.75 each
You have to pay postage and packing charges as well.
These are:

Postage and packing	
1 to 10 posters	£3.25
11 to 20 posters	£6.00
21 to 30 posters	£8.75
over 30 posters	£11.50

Zeke has £50

How many posters can he get by post if he spends £50?

Problem solving classification	*Work back – unfamiliar*
Content area classification	*Number in context*
Tier	*Either*
Answer	*16*

About the question

This question requires students to have a good grasp of number and money. It also involves careful reading of a large amount of information, and correct interpretation of the information in the tables in order to recognise that the postage costs cannot be ignored.

Problem solving approaches

This question is the reverse of what is usually asked in typical money problems, usually finding the cost given the quantity. Here the cost is given and number of posters has to be calculated. A useful starting point might be to encourage working forward with a given number of posters in order to gain a fuller understanding of the nature of the question.

Challenges / issues

The two challenges here are that the number of posters is not given and that the postage costs must be accounted for. Furthermore (for example), the postage costs themselves may be misunderstood as the price of the posters, with some thinking that 11 or 20 posters can be bought for £6.00. Some students ignore the postage altogether since this makes for a far easier, though incorrect, solution.

Finding the answer

1. One way to work through this problem is to estimate how many posters could be bought and work forward through the problem adjusting the number of posters until the correct numbers are found. Students could begin by diving £50 by £ 2.75 to provide an approximate starting point, or recognise by mental calculation that 10 posters are too few and 20 posters are too many, and then work inwards from there.

2. Alternatively, once it is known that 10 posters is too few, and 20 is too many, the postage must be £6, so 44 divided by 2.75 gives the number of posters.

3. The formal version of this is to represent the problem using symbols, for example, with p = number of posters and n = postage and packing costs

$$(2.75)p + n = 50$$

There must be more than 10 but fewer than 20 posters, so $n = £6.00$

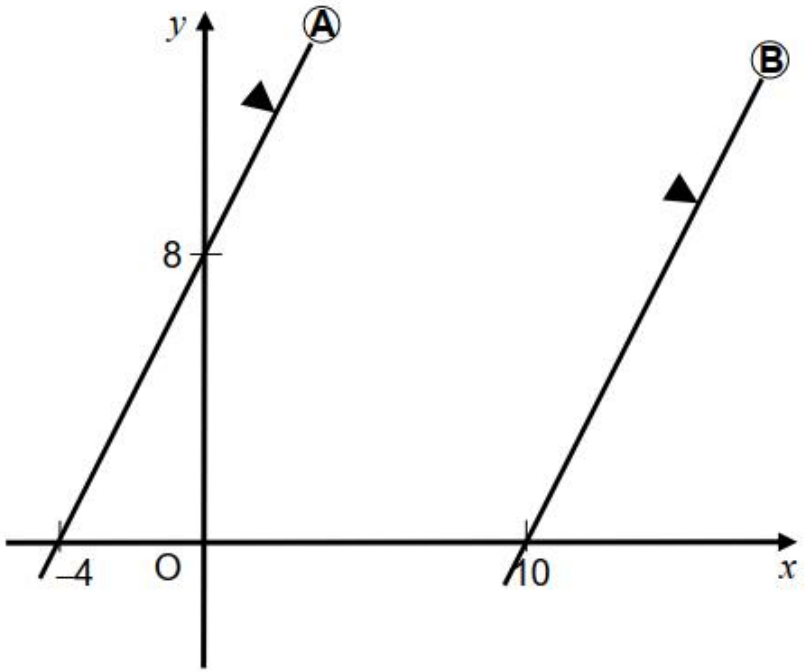
$$(2.75)p + 6 = 50$$

$$p = (50\text{-}6)/2.75$$

$$p = 16$$

Javelin B

The lines A and B are parallel



What is the equation of line B?

Problem solving classification	*Work back – unfamiliar*
Content area classification	*Linear graphs*
Tier	*Higher (hardest)*
Answer	$y = 2x - 20$

About the question

This problem is about equations of straight lines and the important mathematical qualities that are shared between parallel lines, and those properties that are different. Clearly, students need a reasonable understanding of slope, intercept and of how to form an equation from them.

Problem solving approaches

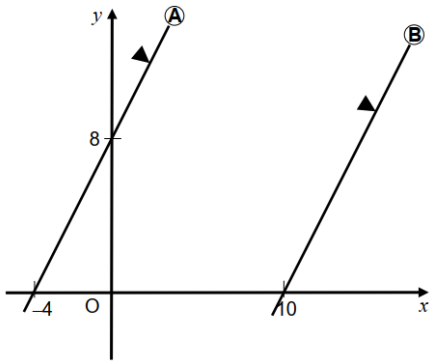
The problem relies on students being able to construct the equations of straight lines from diagrams of the lines. Thus, the question is unusual in being the reverse of what is normally asked of students – sketching a line given its equation. To tackle the problem students need to be aware of what information determines the equation of a line, and of how they can go about finding such data from that presented on the graph.

Challenges / issues

Students confident with the $y = mx + c$ formulation for the equation of a straight line still have some work to do since m is not given explicitly in the question for either line. In addition to this, the intercept of line A is shown but that for B is not.

Javelin B

The lines A and B are parallel



What is the equation of line B?

Finding the answer

The two pieces of information that determine the equation of line B (slope and intercept) need to be found. The fact that parallel lines have the same slope needs to be understood so that the gradient of line B can be inferred from that of line A. Then the increase in y over increase in x formulation for gradient can be applied to the triangle containing the intercepts for line A. This gives 8/4 = 2 and so $m = 2$ for line B.

The intercept for line B can be found in (at least) two distinct ways:

1. Solving an equation for $y = 2x + c$ through the point (10, 0).
2. An alternative method, involving proportionate thinking based on the known gradient (2), would be to note that on line B at (10, 0) moving 10 units left must be equivalent to moving 20 units down so that the intercept of B must be at (0, –20).

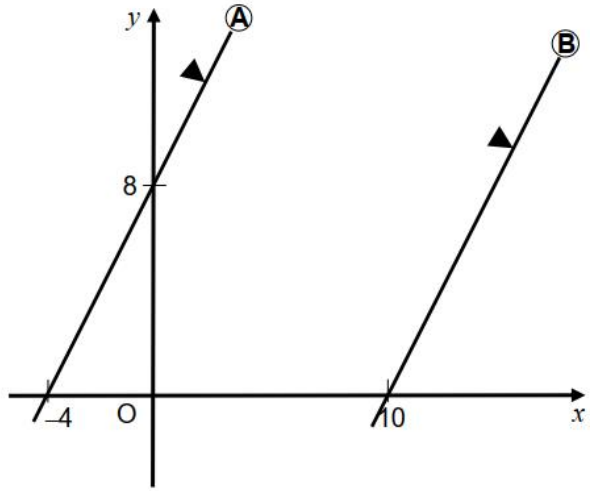
Follow up

Other related problems using parallel lines can be constructed, with negative slope an additional complication that could be included.

AQA

Javelin B

The lines A and B are parallel



What is the equation of line B?

AQA Javelin B

Save

1 $(-4,0),(0,8)$

Label:

2 $y-0=\frac{(8-0)}{0--4}(x--4)$

3 $y=2x+8$

4 $y=2x+c$

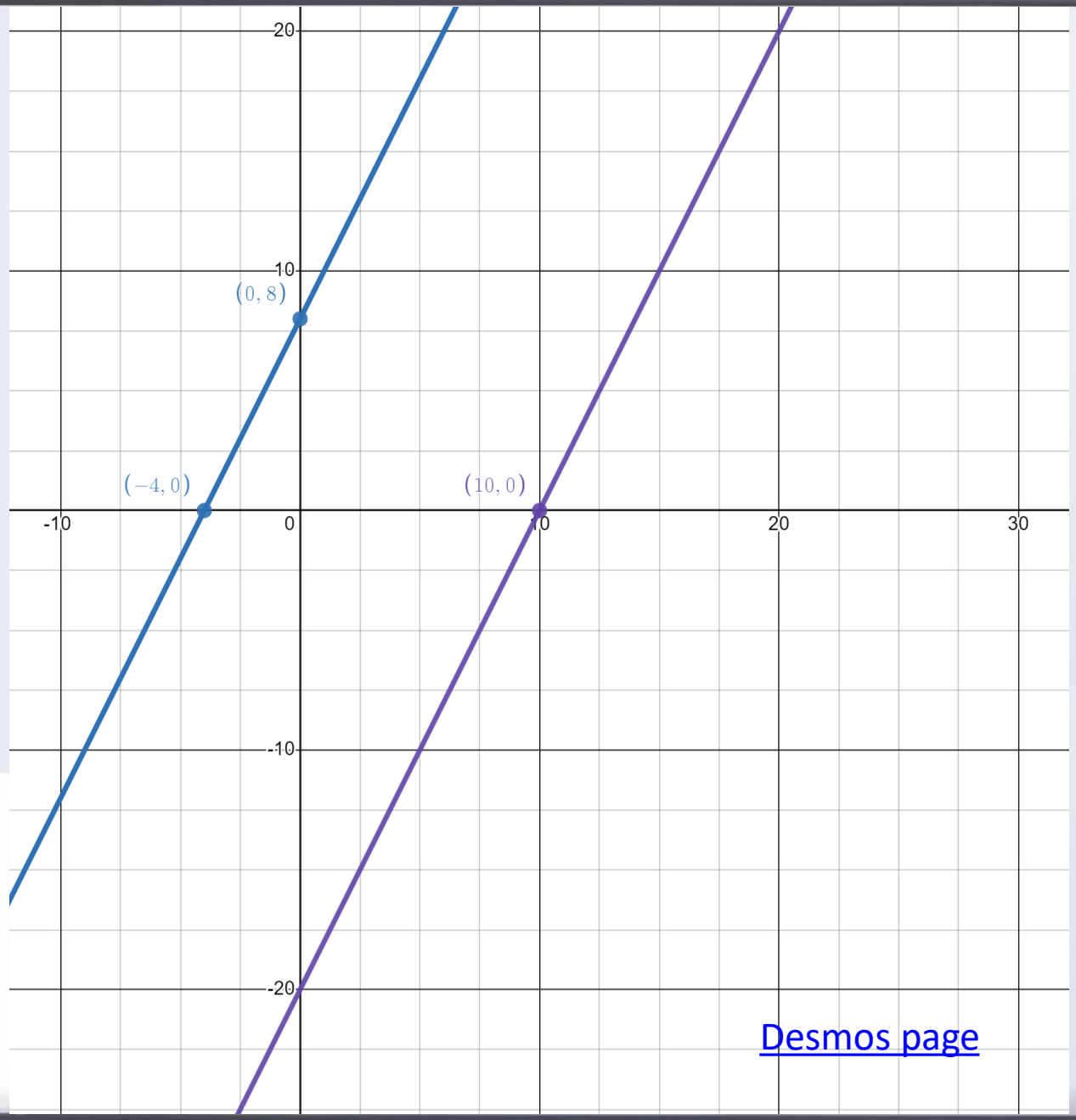
5 $c=-20$

-30

6 $(10,0)$

Label:

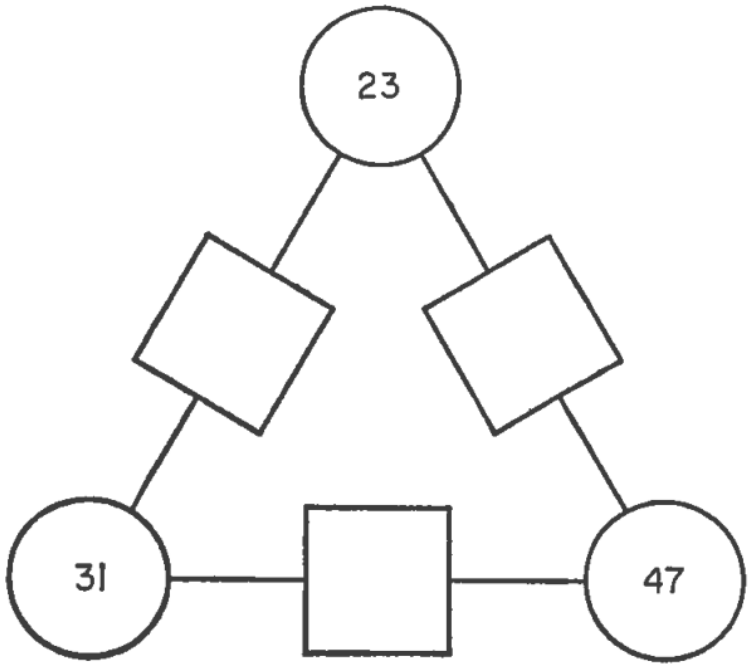
7



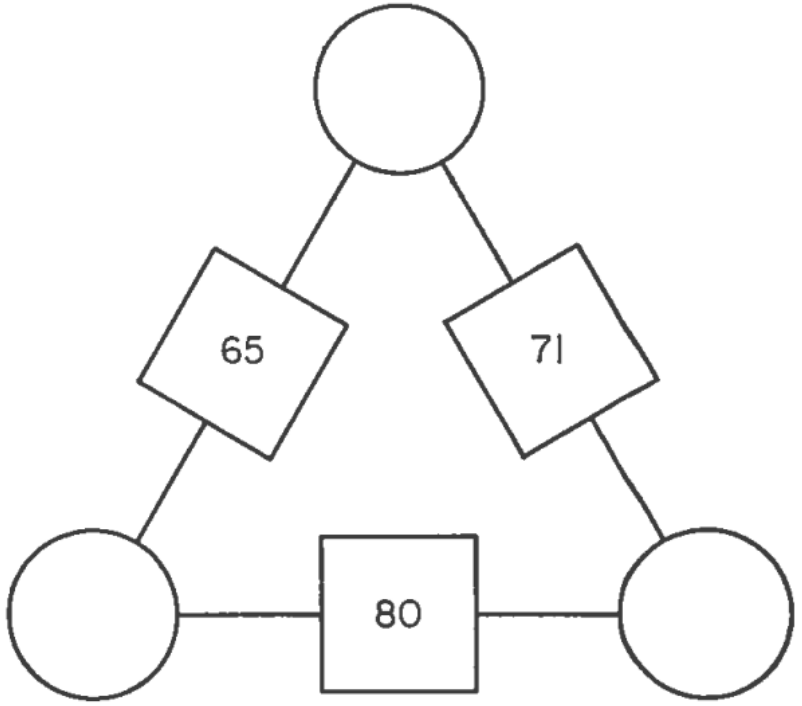
Desmos page

Arithmogons

Level 1: Direct computational practice



Level 2: An arithmetical problem

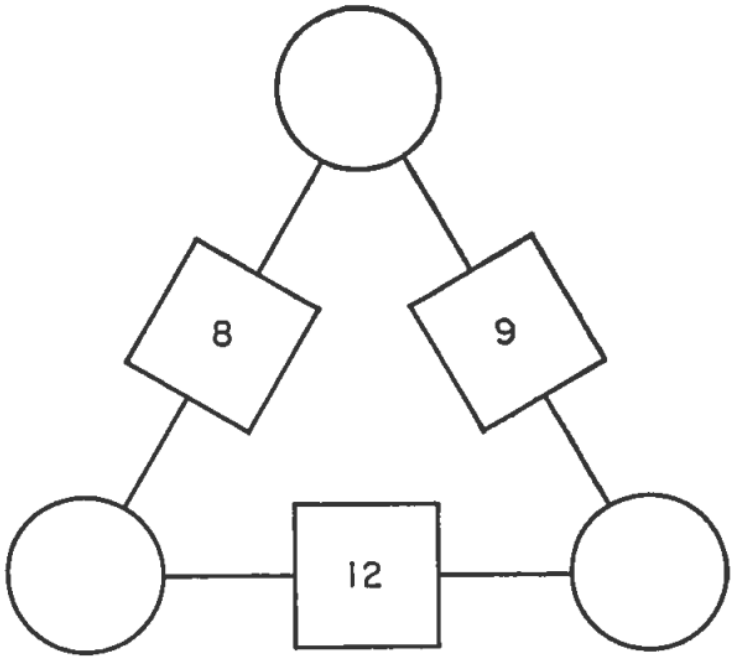


Level 3: The problem as before, but can't we do better than trial and error? Yes. **Let's invent algebra.**

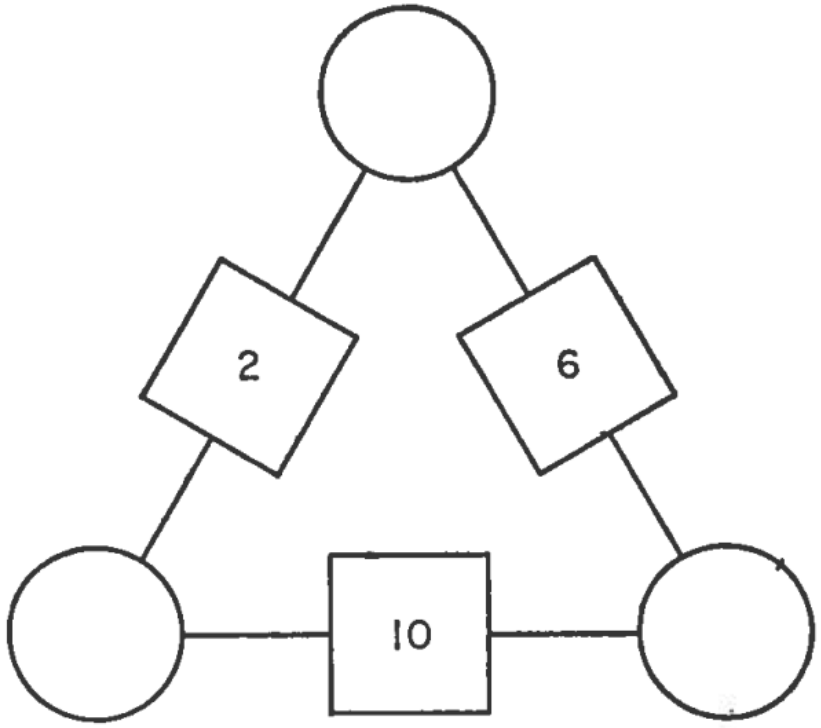
ATM, Mathematics Teaching, 70 – March 1975
Arithmogons Alistair McIntosh & Douglas Quadling

Arithmogons

Level 4: The need for new numbers



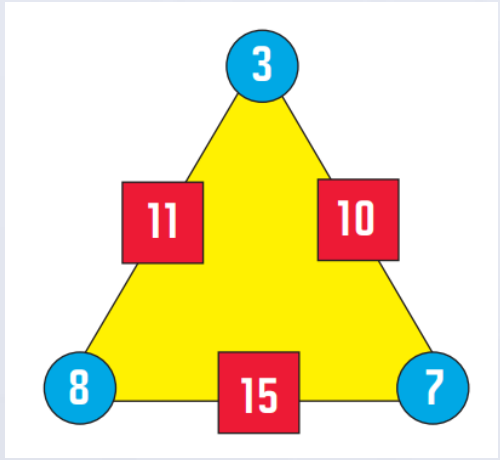
To solve this we need fractions.

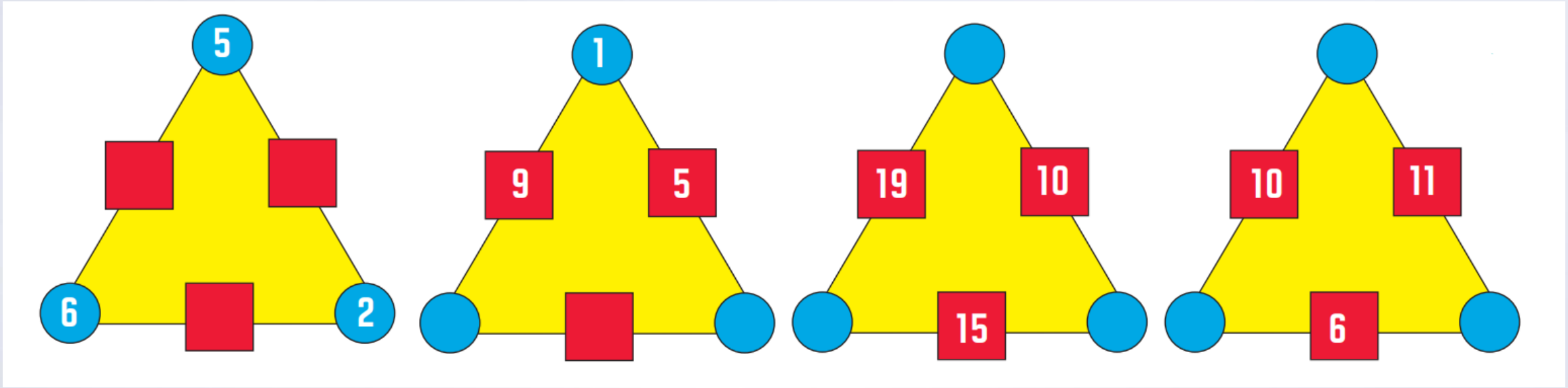


To solve this we need negative numbers.

ATM, Mathematics Teaching, 70 – March 1975
Arithmogons Alistair McIntosh & Douglas Quadling

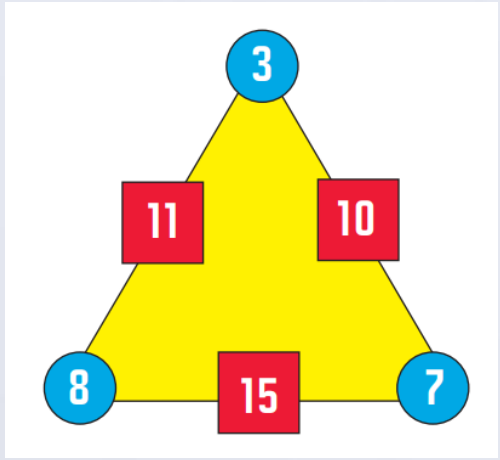
Arithmagons

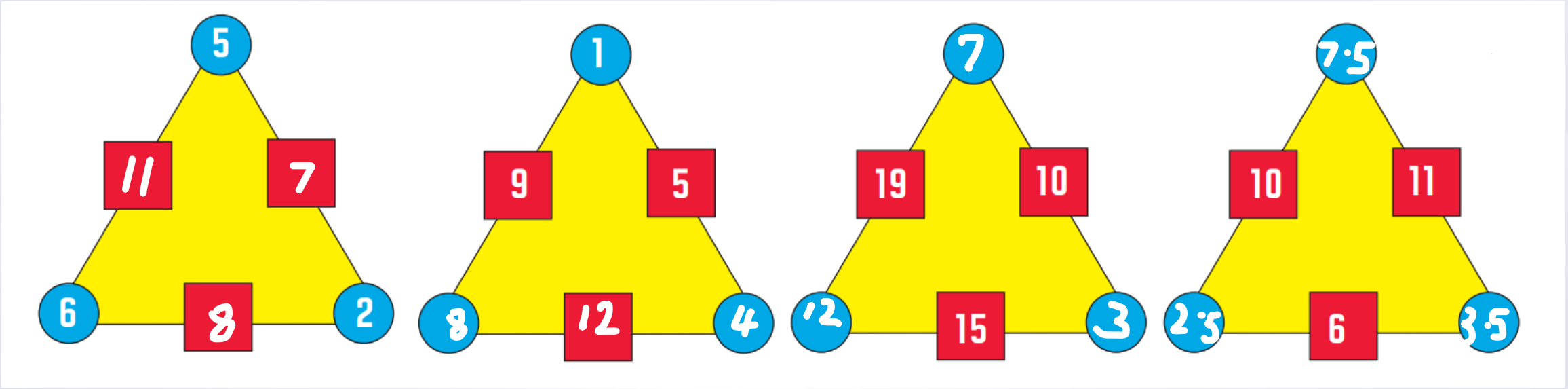




From Arithmagons – a lesson plan by Colin Foster in Teach Secondary

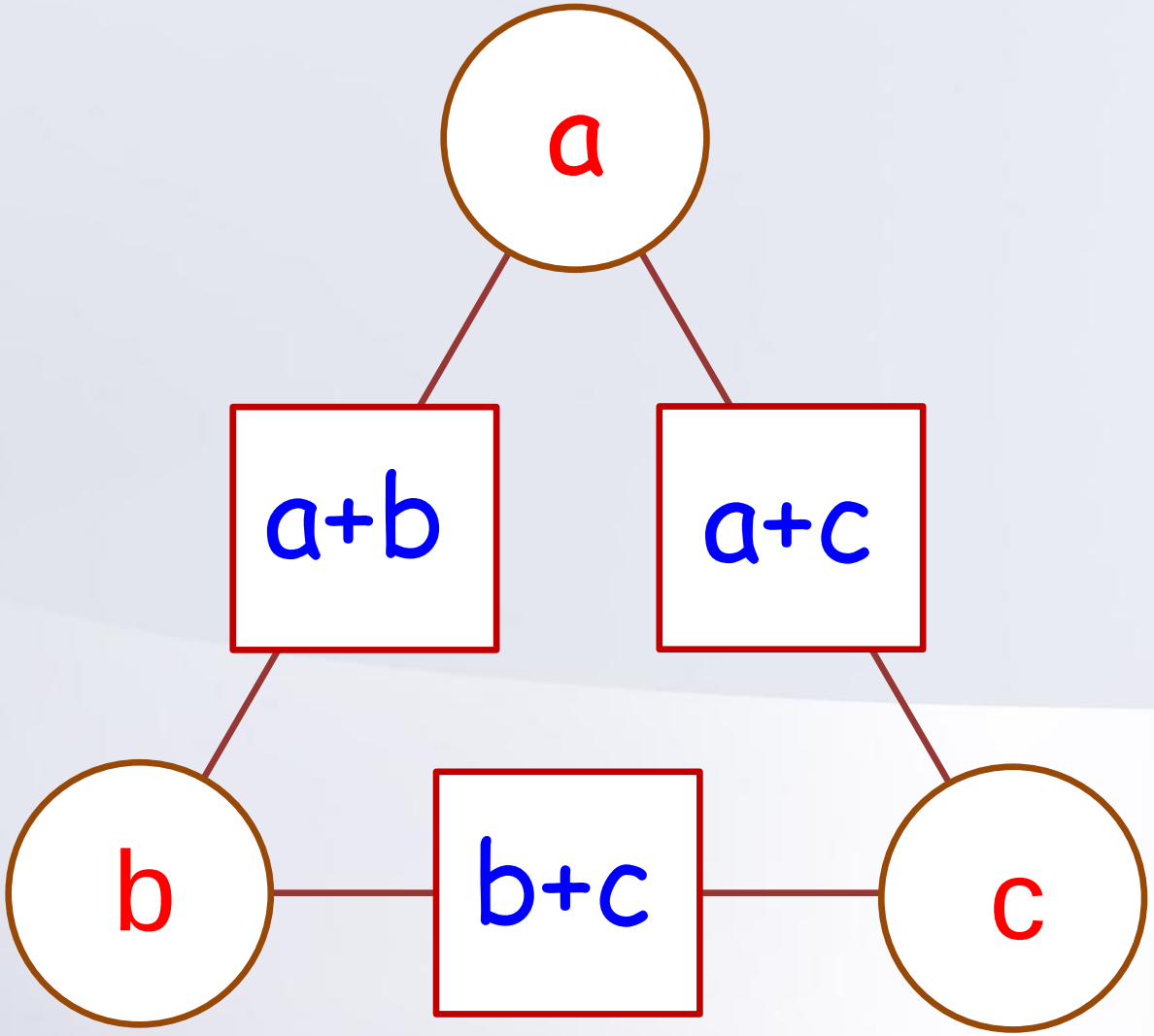
Arithmagons





From Arithmagons – a lesson plan by Colin Foster in Teach Secondary

Algebra - Arithmagons



Sum of circles

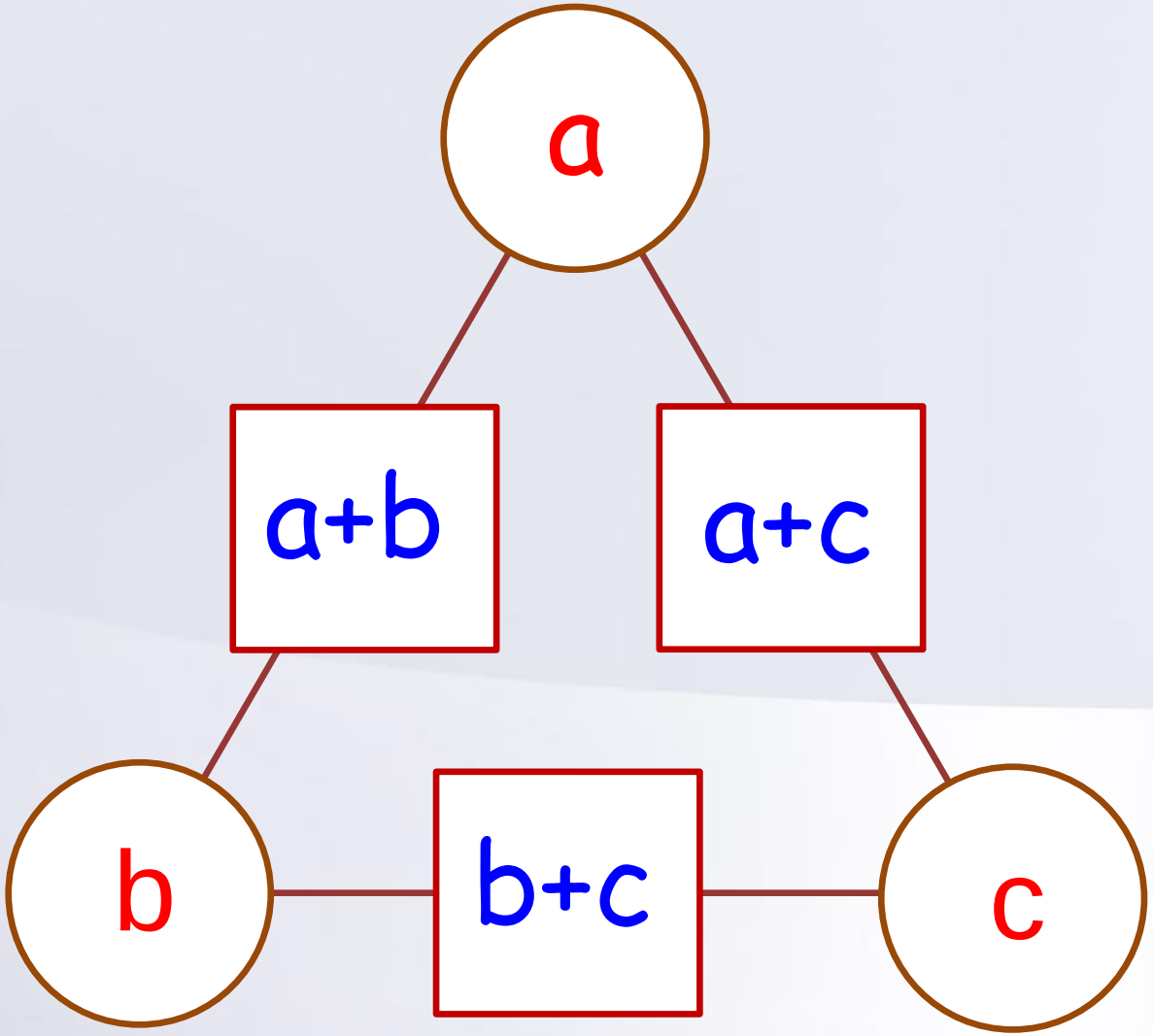
$a + b + c$

Sum of squares

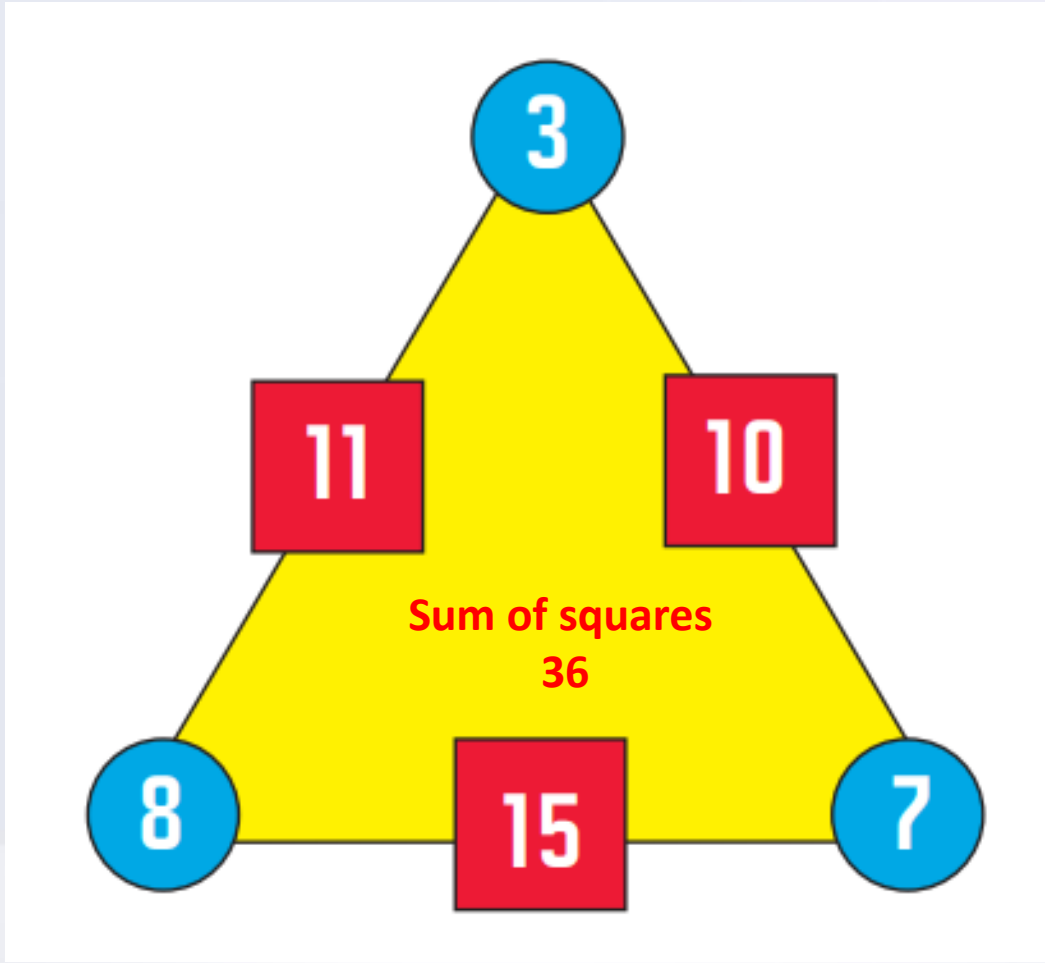
$a + b + b + c + a + c =$

$2(a + b + c)$

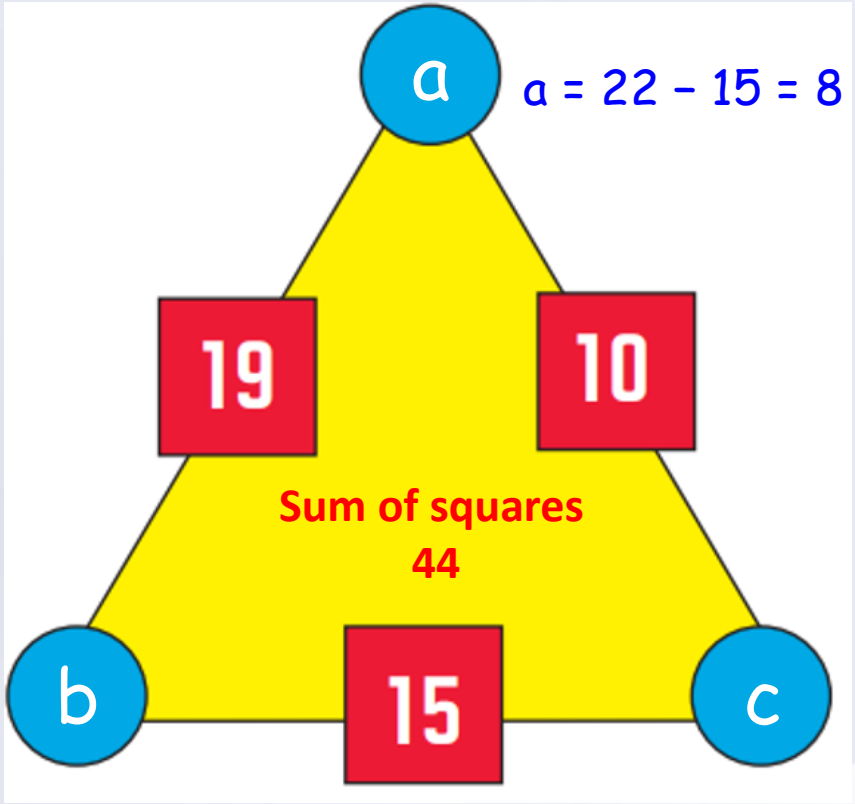
Algebra – Arithmagons



The sum of a circle and the square opposite = a + b + c



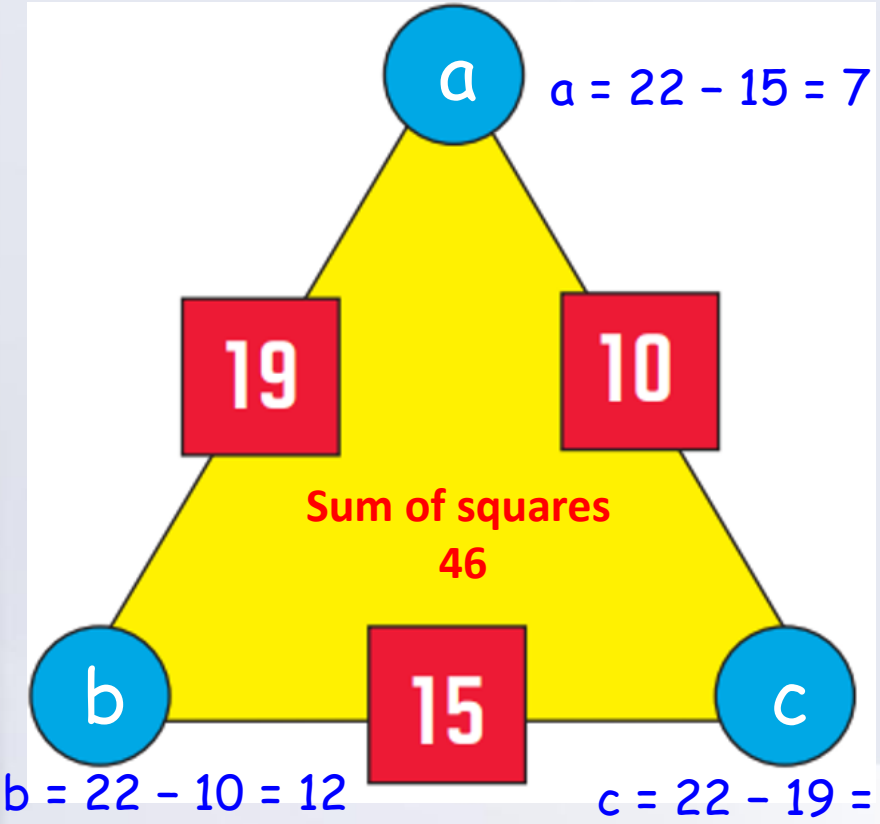
Sum of circles
18



$$a + b + c = 22$$
$$b + c = 15$$

$$a = 22 - 15 = 7$$

Sum of circles
22

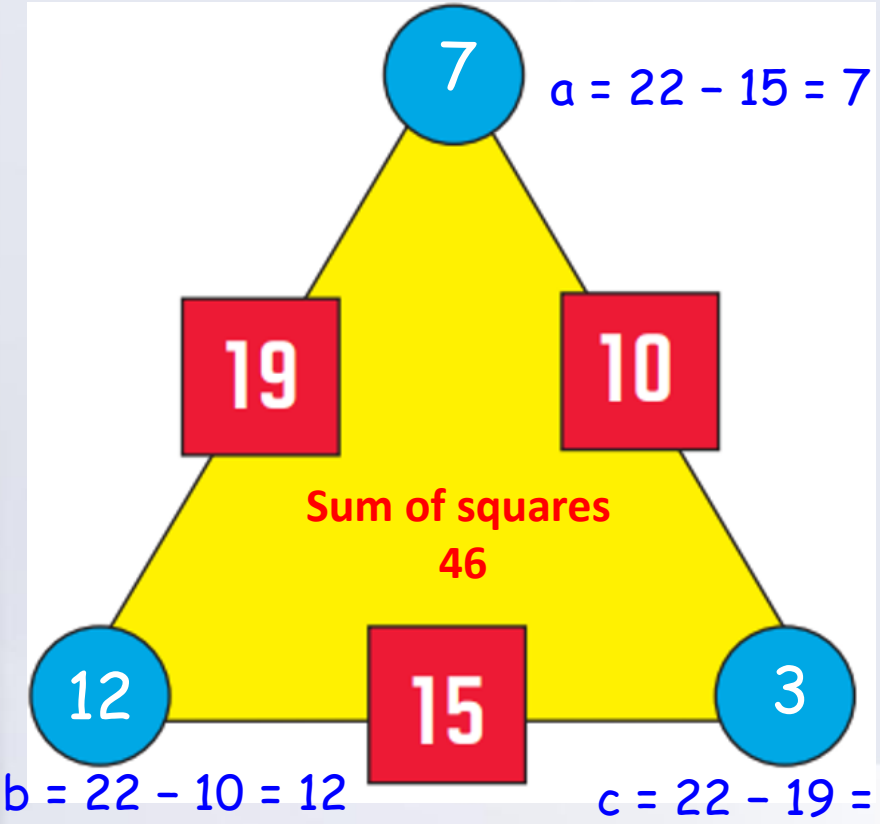


$$a + b + c = 22$$
$$b + c = 15$$

$$a = 22 - 15 = 7$$

Similarly

$$b = 22 - 10 = 12$$
$$c = 22 - 19 = 3$$



$$a + b + c = 22$$
$$b + c = 15$$

$$a = 22 - 15 = 7$$

Similarly

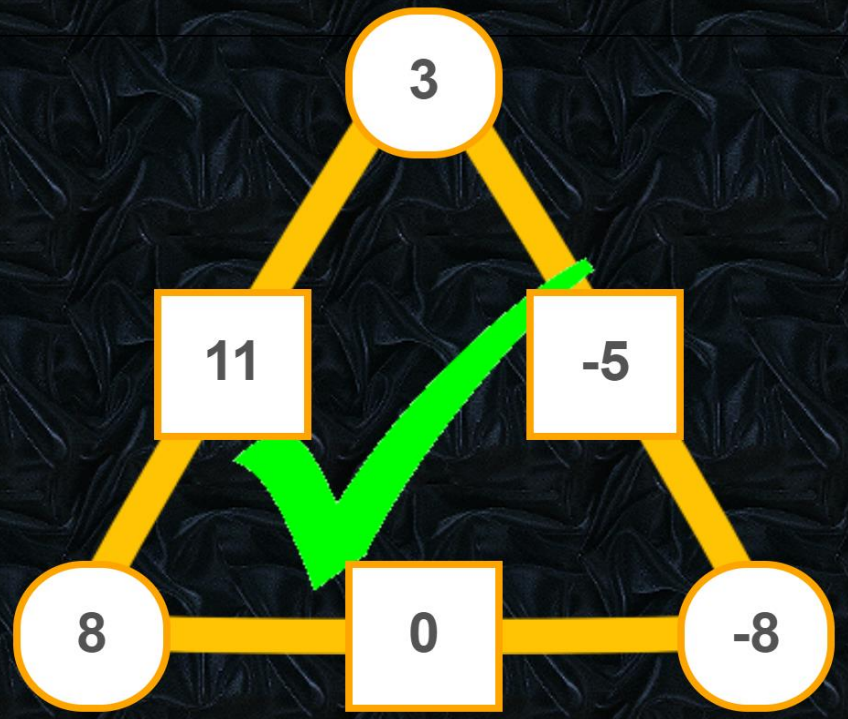
$$b = 22 - 10 = 12$$
$$c = 22 - 19 = 3$$

Arithmagons

Level 1 | Level 2 | Level 3 | Level 4 | Level 5 | Level 6 | Level 7 | Level 8 | Level 9 | Level 10 | More | Help

Type in the missing numbers according to the rule.

You can earn a Transum Trophy for correctly completing eight arithmagons at any one level.



The numbers in the circles add up to the numbers in the rectangles between them.

✓ Check

→ Next

Positive and negative numbers

	○ ○ ○	○ ○ □	○ □ □	□ □ □
Sum	Level 37	Level 38	Level 39	Level 40
Product	Level 41	Level 42	Level 43	Level 44
Difference	Level 45	Level 46	Level 47	Level 48

Choose Your Own Options:

Single digit numbers in circles

Addition

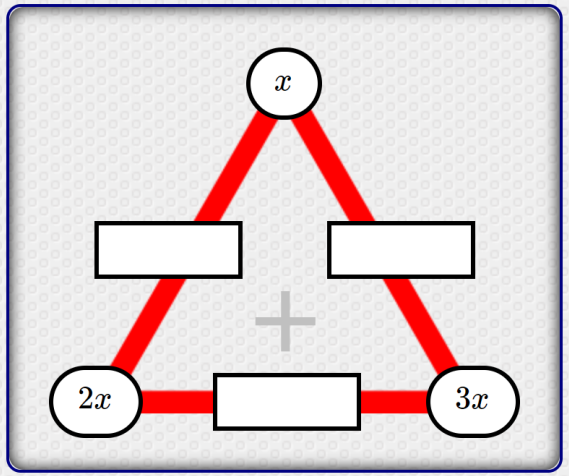
Given 3 circle values only

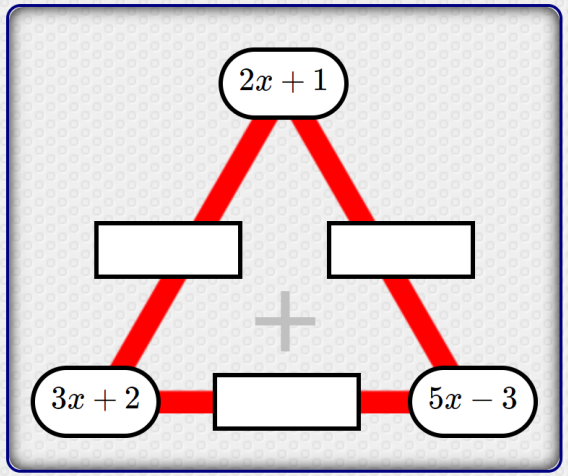
Algebragons

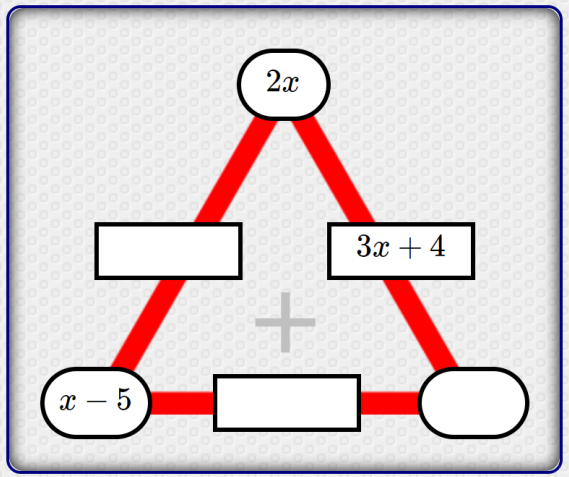
Find the missing expressions in these partly completed algebraic arithmagon puzzles.

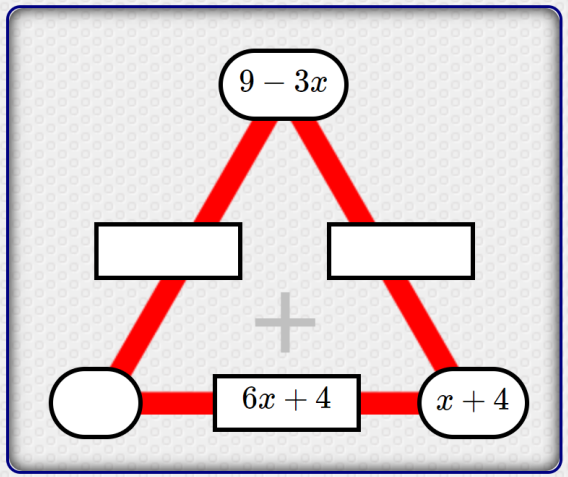
Menu | Arithmagons | Level 1 | Level 2 | Level 3 | Level 4 | Level 5 | Help | More Algebra

Complete the algebragons. The expressions in the rectangles are the sums of the linear expressions in the adjacent circles.







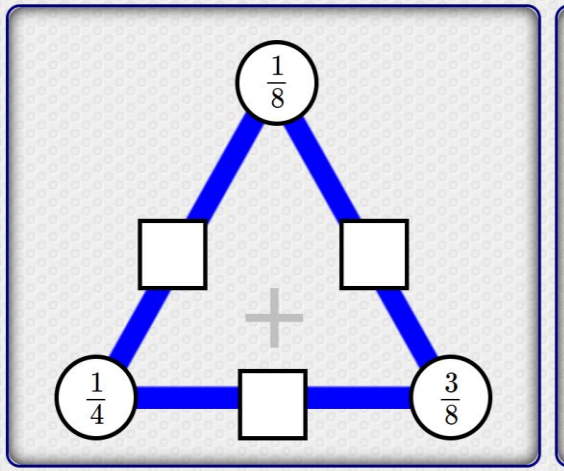


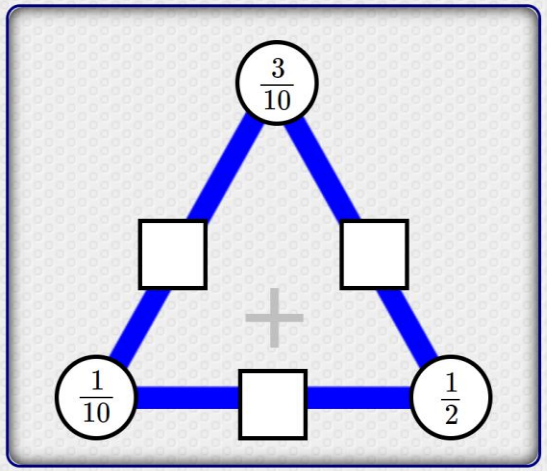
Fractionagons

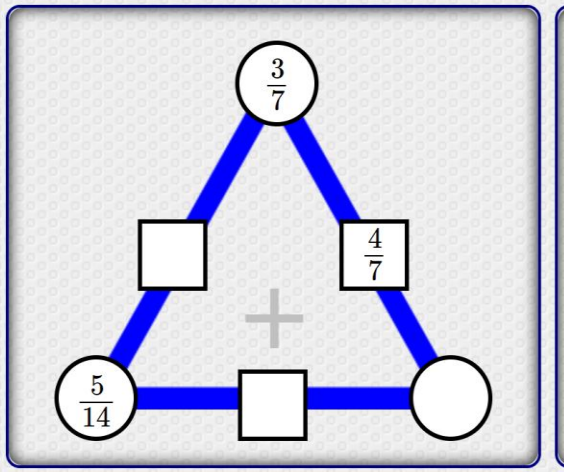
Calculate the missing fractions in these partly completed arithmagon puzzles.

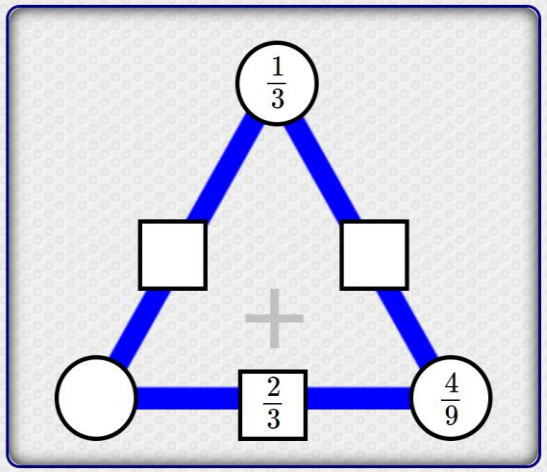
Menu | Arithmagons | Level 1 | Level 2 | Level 3 | Level 4 | Level 5 | Help | More Fractions

Complete the fractionagons so that the fractions in the rectangles are the sums of the fractions at the vertices of the triangles. Use the forward slash / to type a fraction. Give answers in their lowest terms.









Algebra - Factorising

If you can multiply out brackets, you can factorise

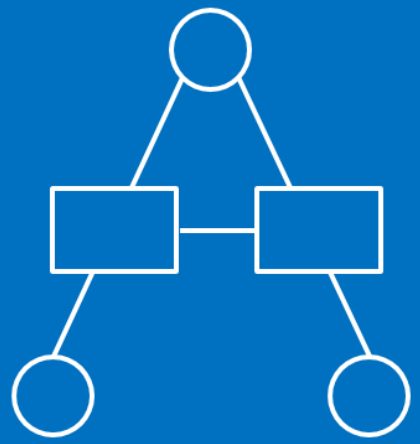
$4(x+y) = 4x+4y$

$6a-6b = 6(a-b)$

$(x+4)(x+2) = x^2+6x+8$

$x^2-5x+6 = (x-2)(x-3)$

Contents



RITHMAGONS

11. Factorising Quadratics

mr barton maths .com

About Arithmagons

- I got inspiration for this series of Maths Arithmagons activities from Jonny Griffith's wonderful RISPs ebook. If you haven't downloaded it, it is available for free here

- Here is why I love them so much
  - 1) They are so simple to create and don't require any special resources
  - 2) Working forwards allows students to consolidate key mathematical concepts
  - 3) Working backwards encourages deep mathematical thinking and creativity, hence effective differentiation
  - 4) They are so versatile they can be used for pretty much any mathematical topic
  - 5) Students can be challenged to create their own for an effective Extension Task

- Give them a go, and if you or your pupils cannot resist the urge to create your own Arithmagons, please let me know

Instructions

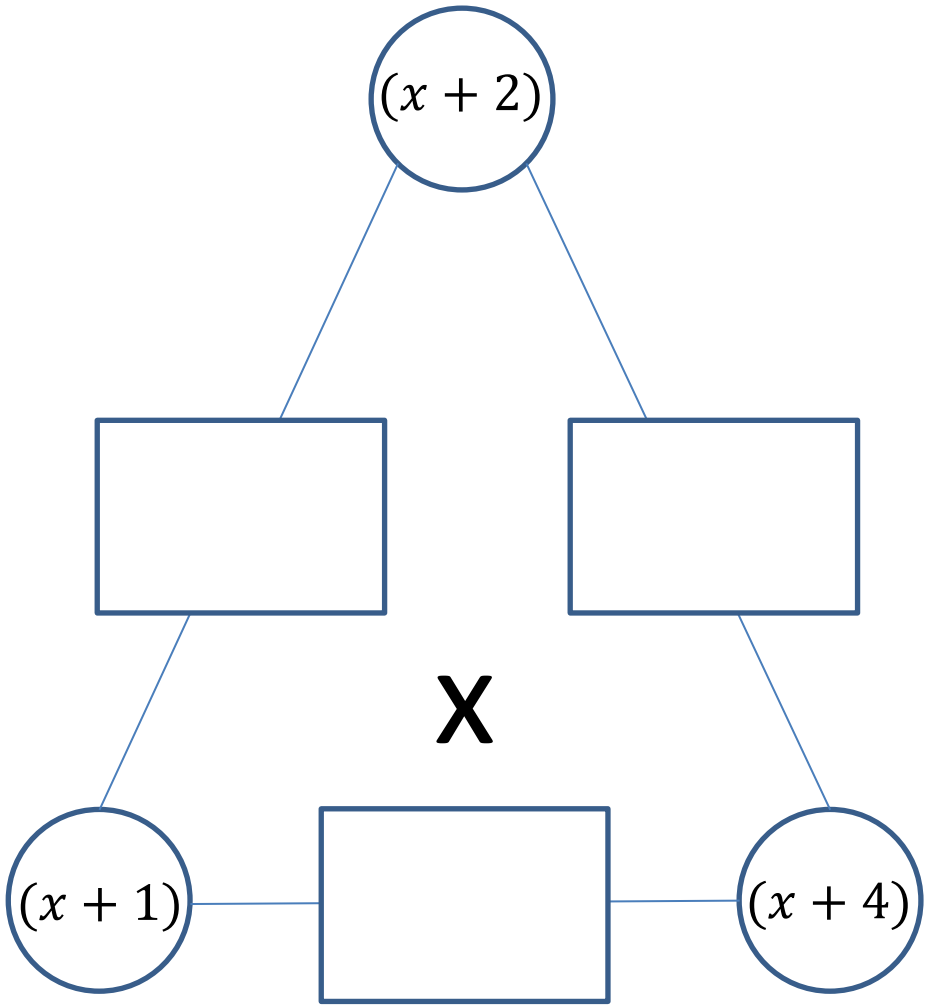
Forwards Arithmagon

- Choose two circles
- Add the two expressions in the circles together and fill your answer in the rectangle in between them
- Continue until you have filled in the other two rectangles

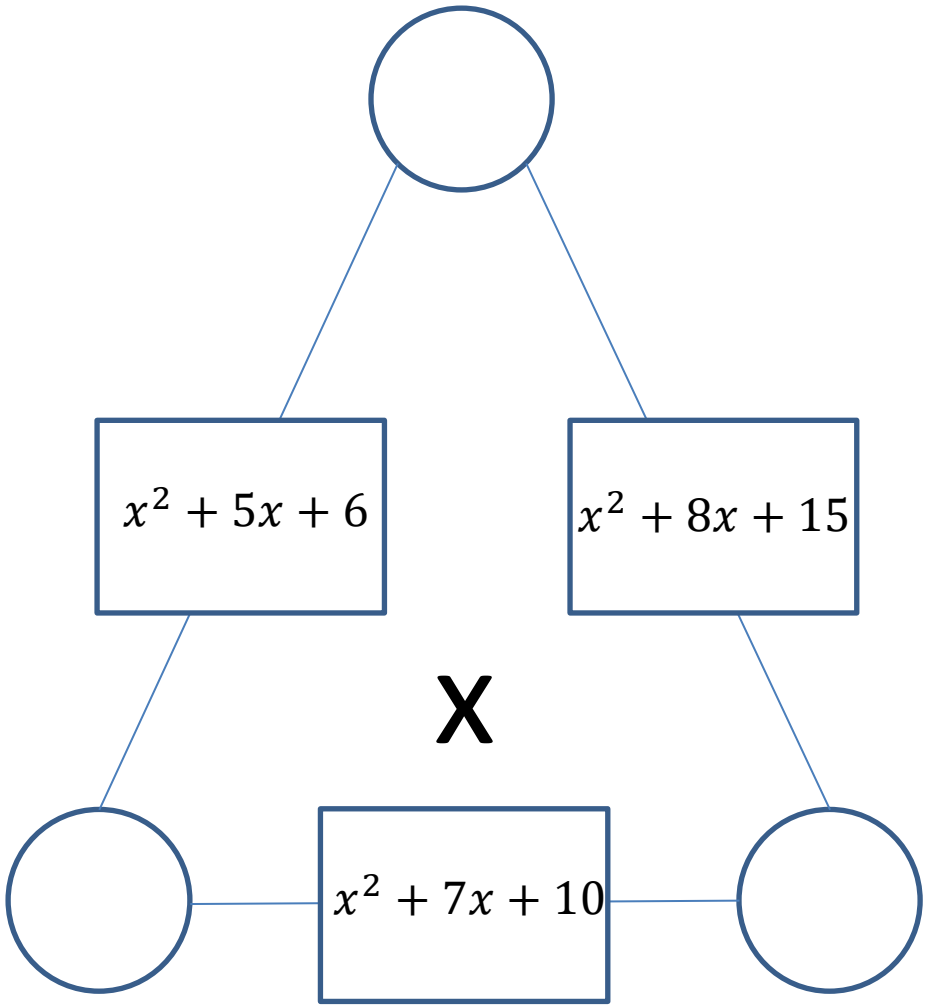
Backwards Arithmagon

- Place an expression in each circle so that each pair of circles adds to give the expression in the rectangle in between them
- Be prepared to describe your strategy!

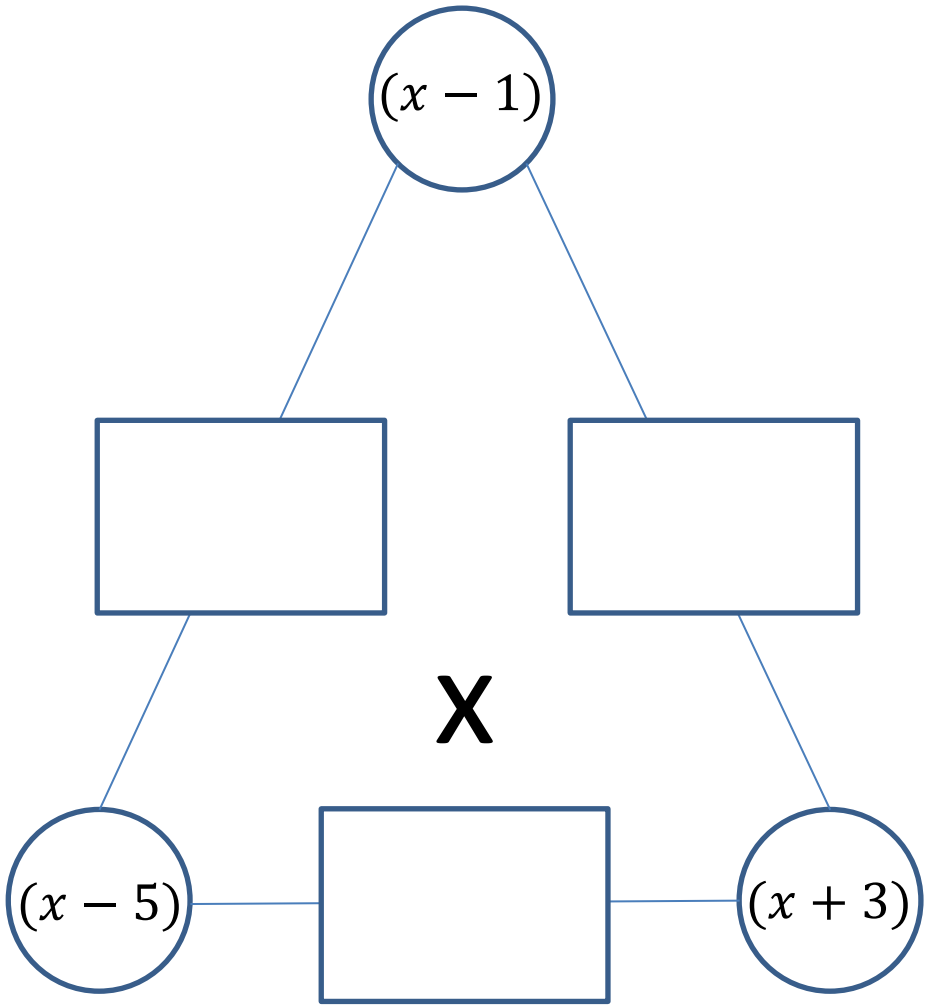
Forwards



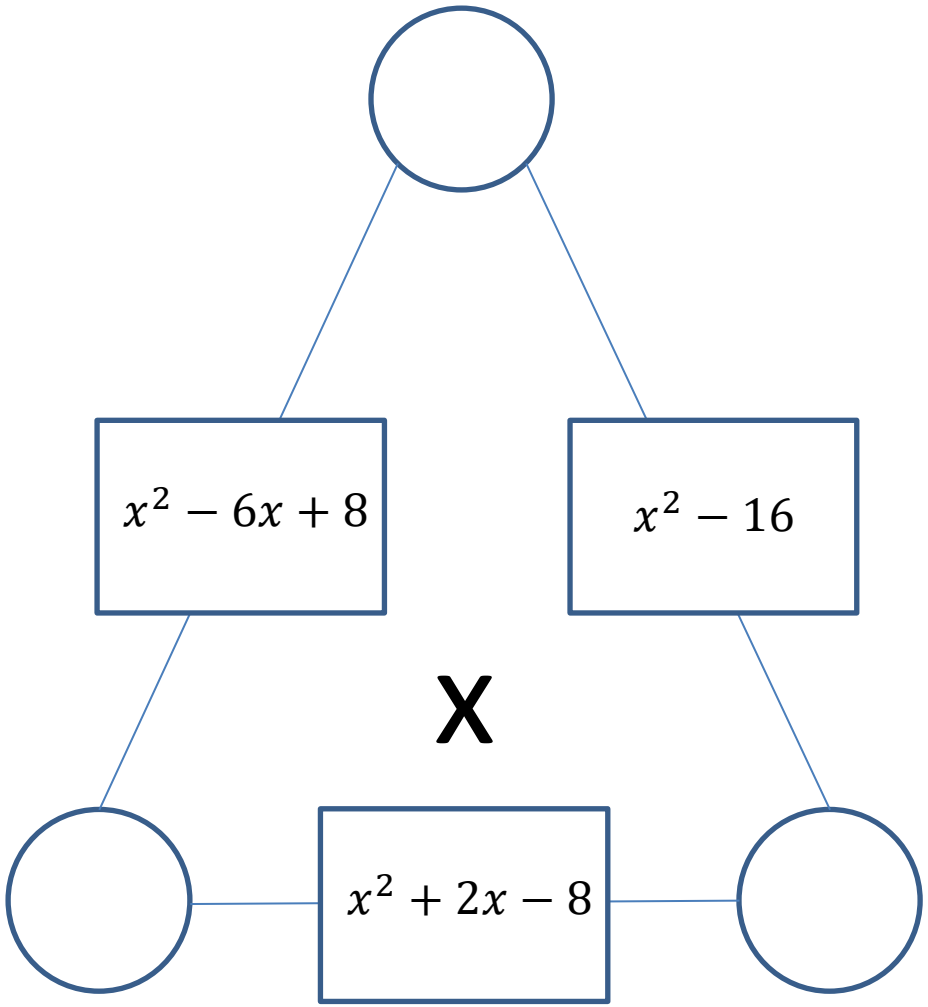
Backwards



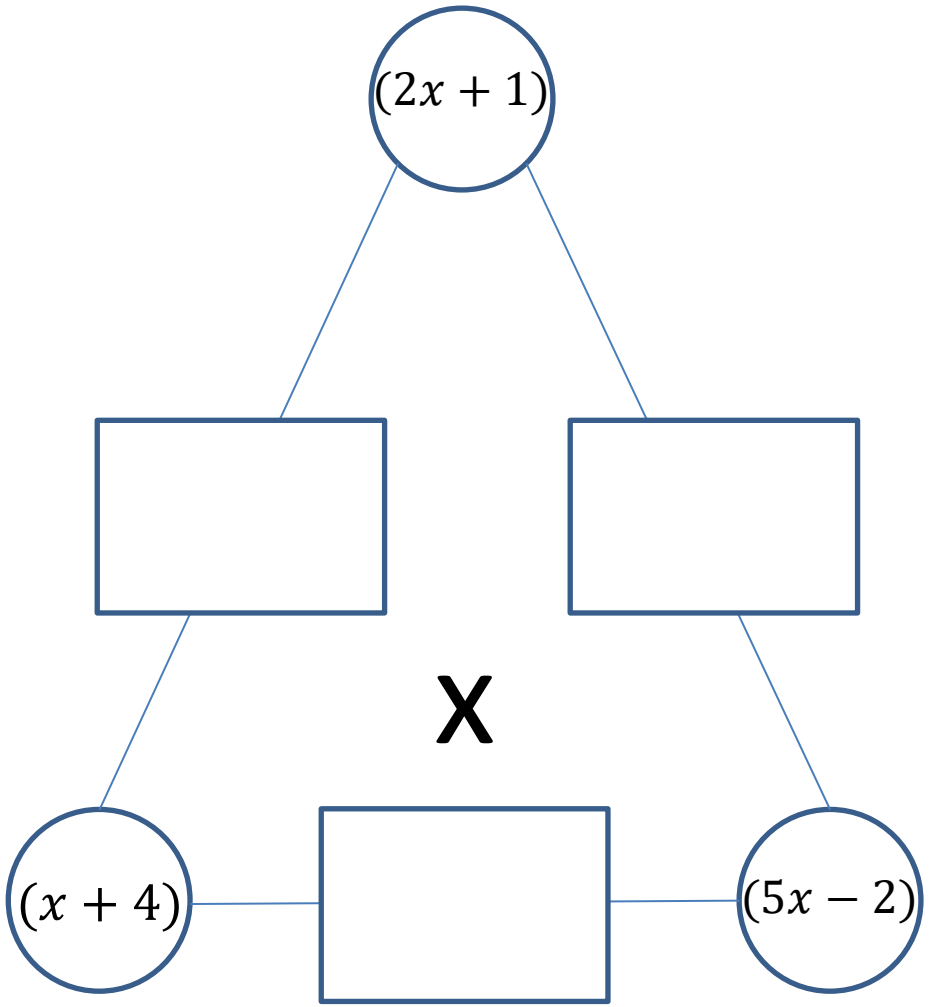
Forwards



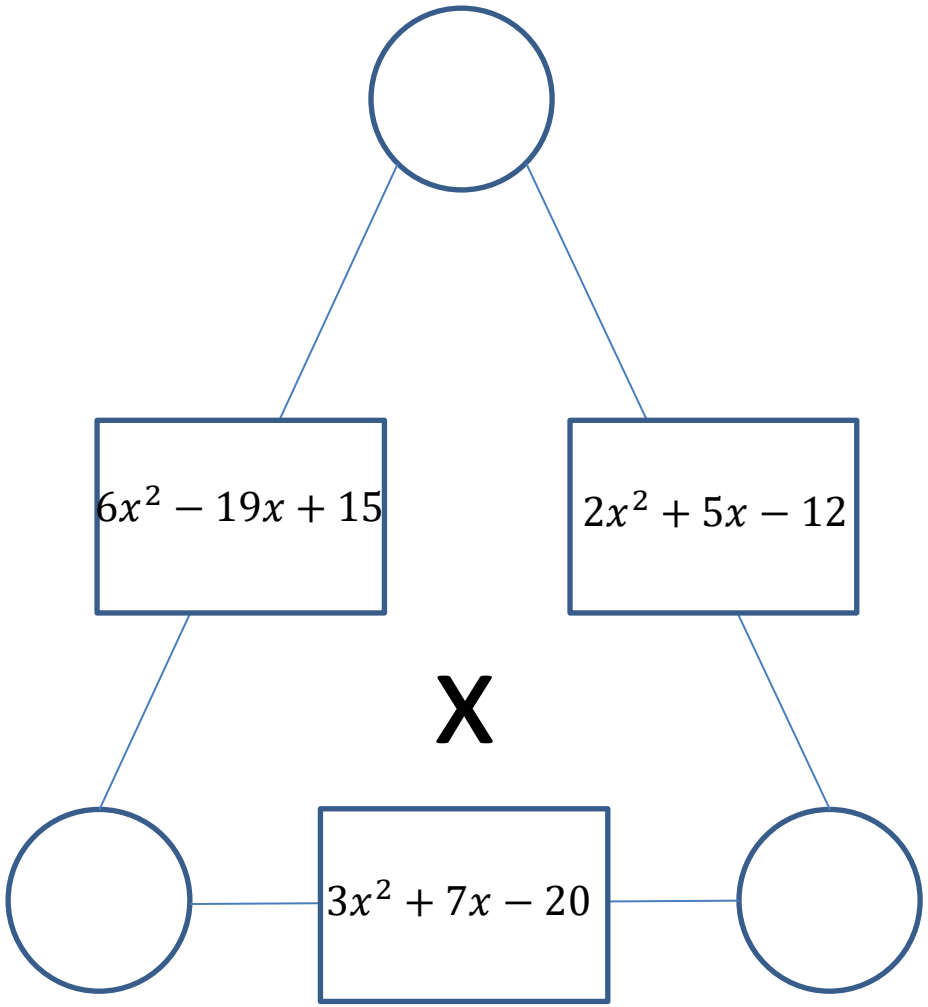
Backwards



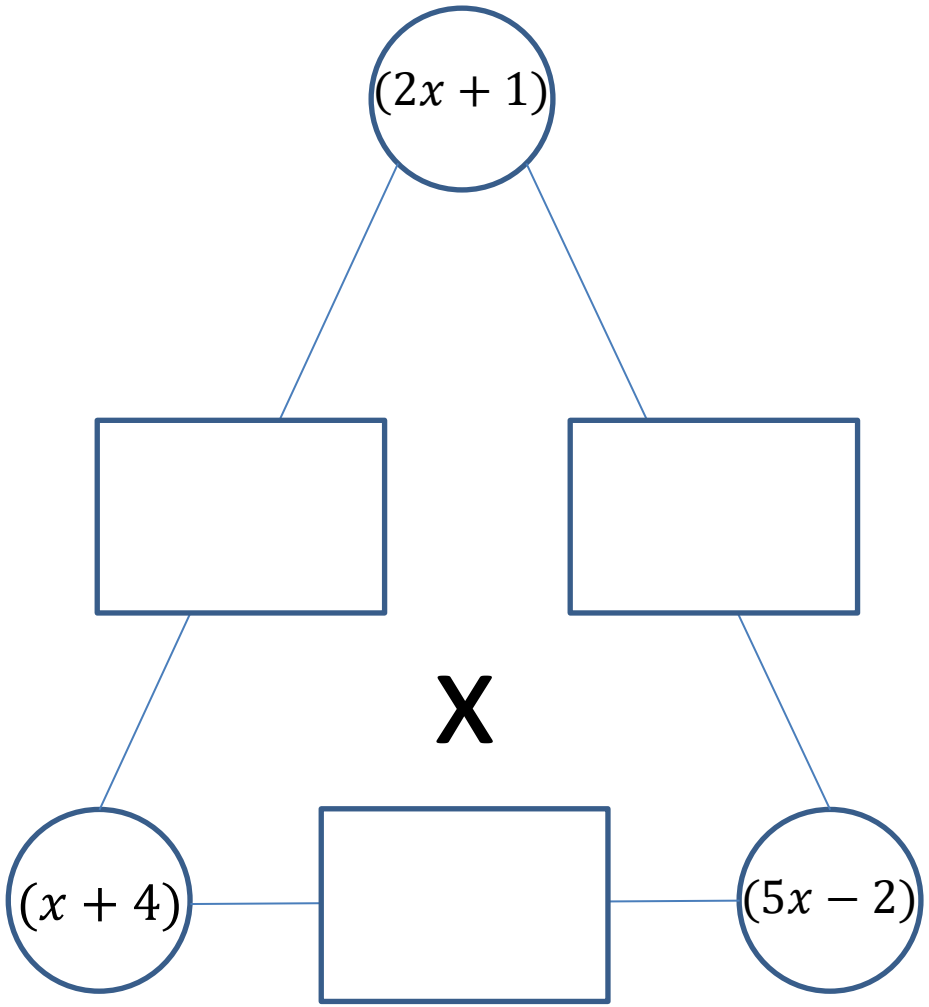
Forwards



Backwards



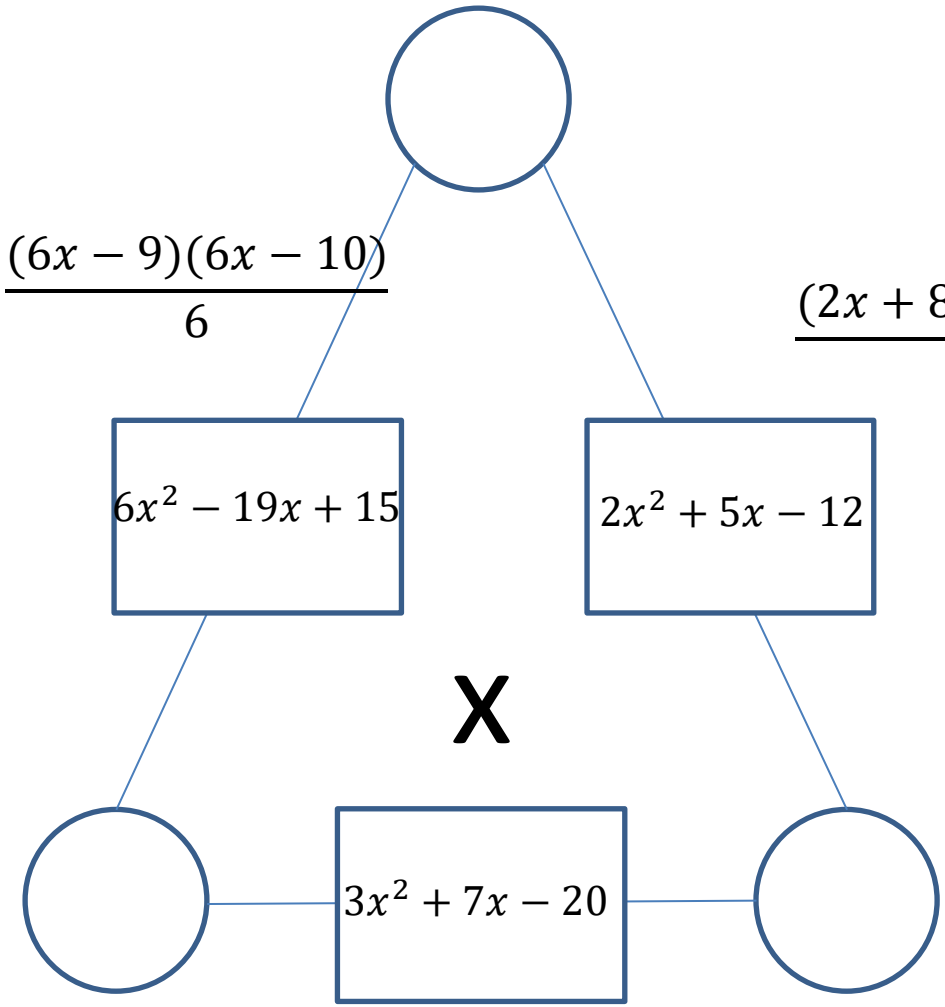
Forwards



Backwards

$\frac{(6x-9)(6x-10)}{6}$

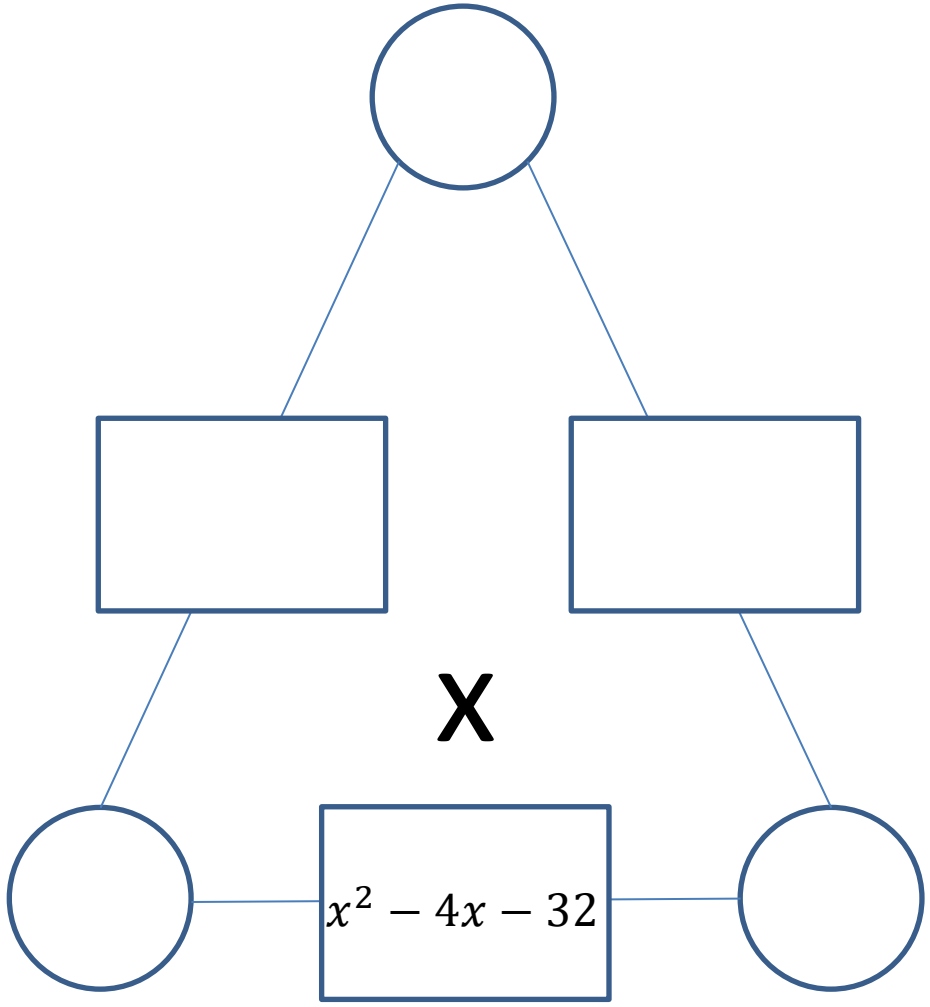
$\frac{(2x+8)(2x-3)}{2}$

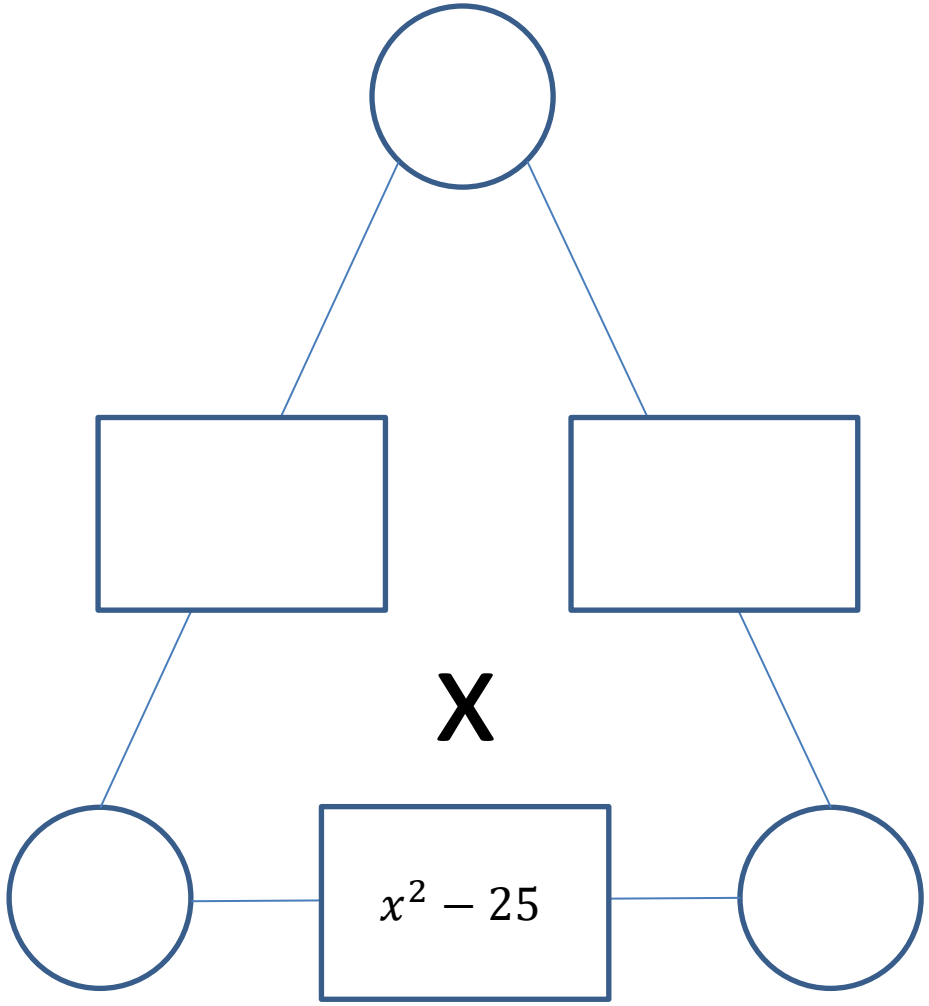


$\frac{(3x+12)(3x-5)}{3}$

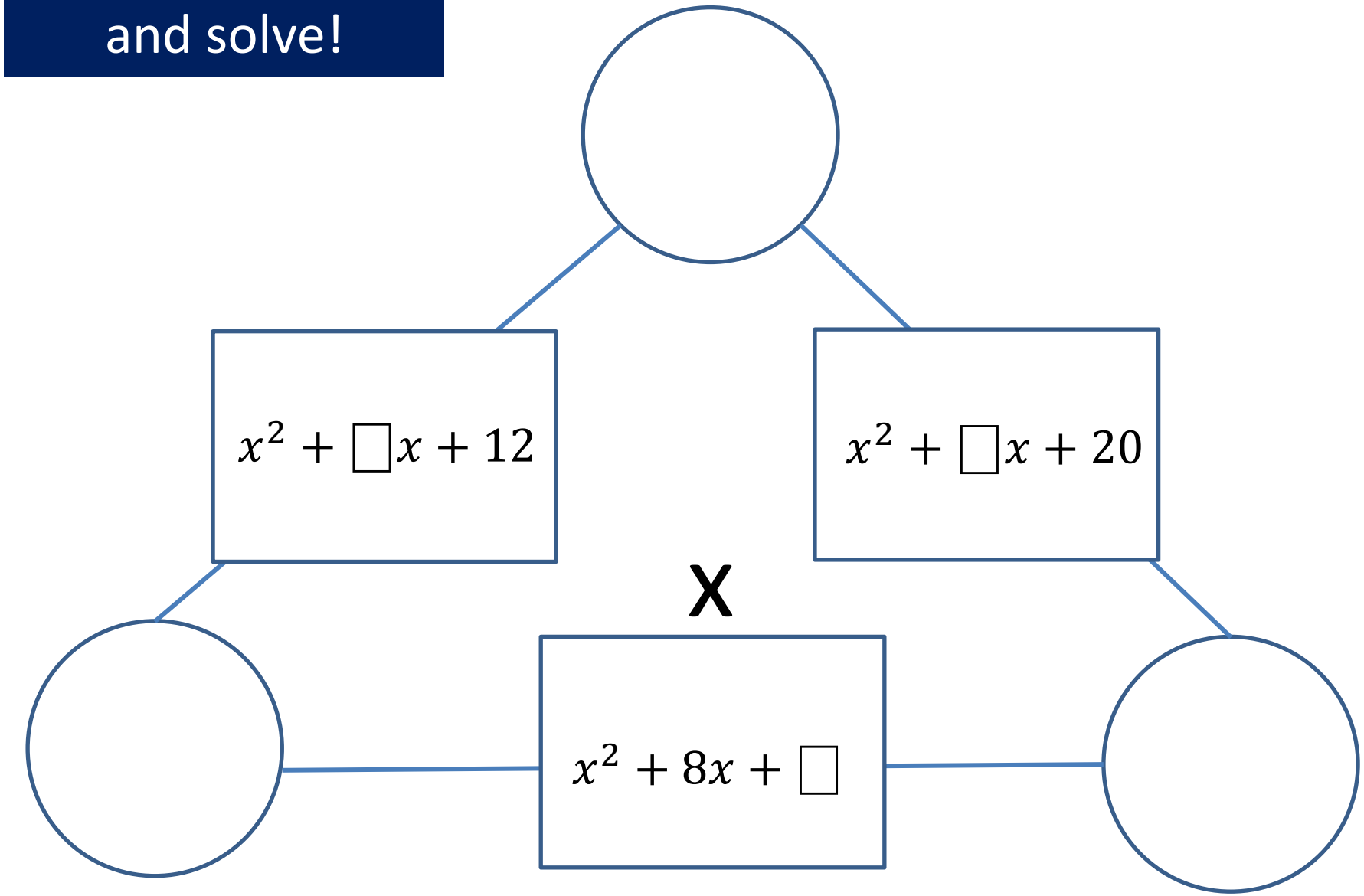
Missing information challenge:

- How many ways can you complete each of the Arithmagons?
- Do you notice anything about the answers that work?





Fill in the blanks and solve!



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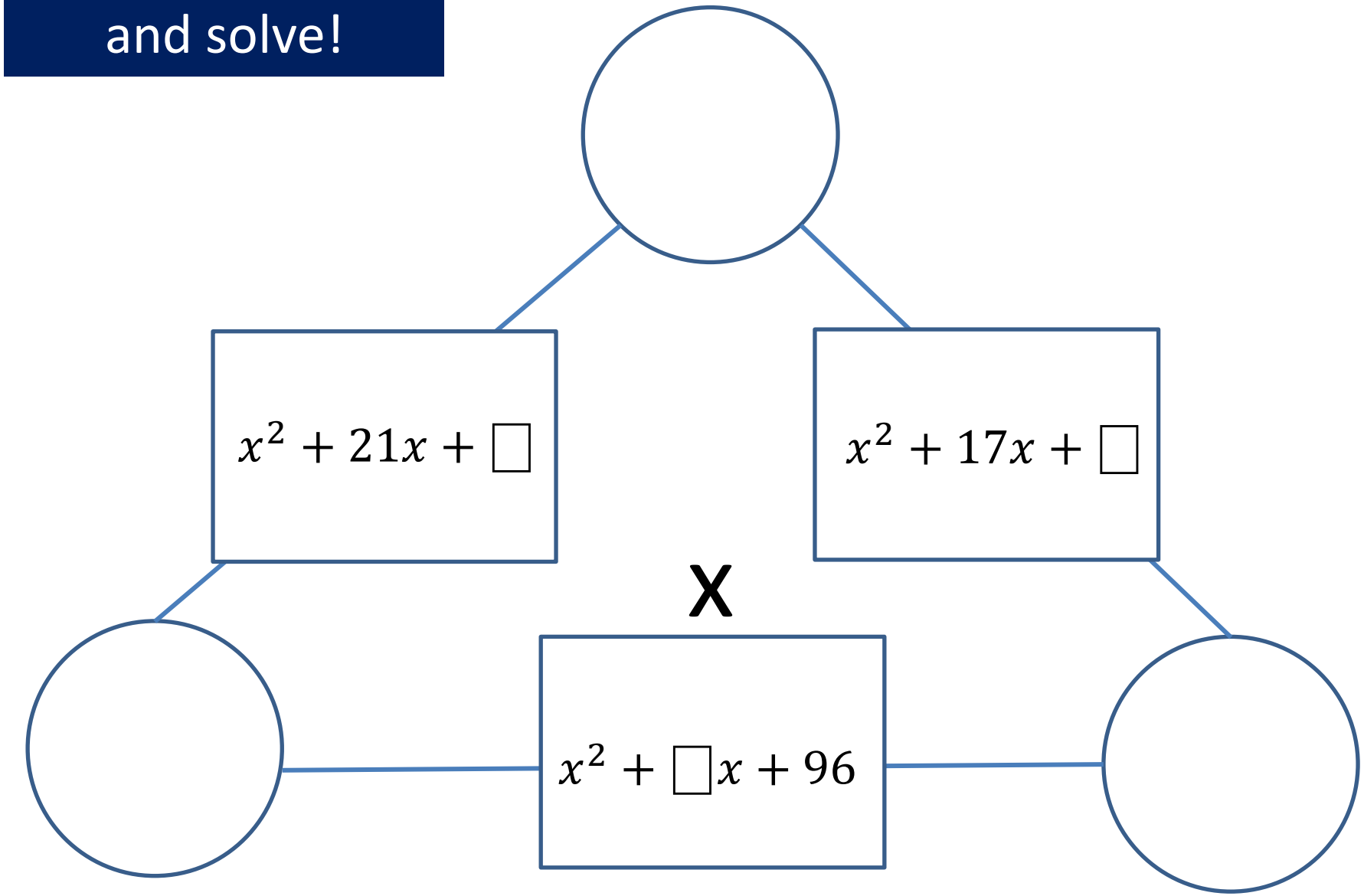


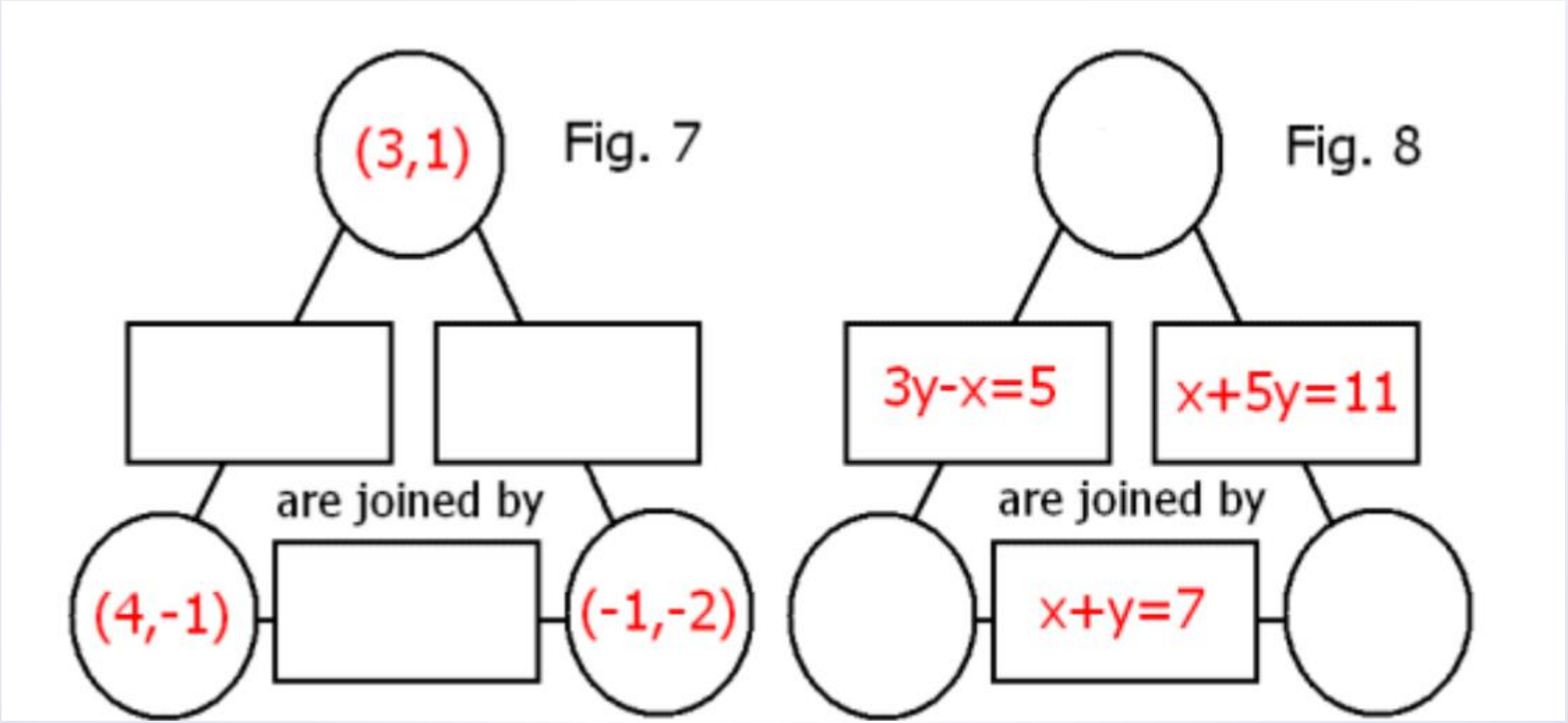
Forwards Arithmagon Challenges

- What maths skills do you need to solve a Forwards Arithmagon?
- Design an easy Forwards Arithmagon and a tricky Forwards Arithmagon
- Fill in two of the circles and one of the rectangles. Can you always complete your Arithmagon? Is there always a unique solution? Does it matter which boxes you fill in?
- Create some different shaped Arithmagons using the blank templates at the end of this PowerPoint

Backwards Arithmagon Challenges

- Describe your strategy to solve the Backwards Arithmagon
- Can you explain why your strategy works?
- Is there a unique solution or several? How do you know?
- Will there always be a solution no matter what 3 values you start with?
- Can you change the values so there are no possible solutions? What conditions are necessary for this to be the case?
- What conditions are necessary to guarantee there will be a single solution?
- How about guaranteeing there will be several solutions?
- How much information can you remove in order to still get a unique solution?
- Make up your own Backwards Arithmagon for each other to solve

Jonny Griffiths - RISPS



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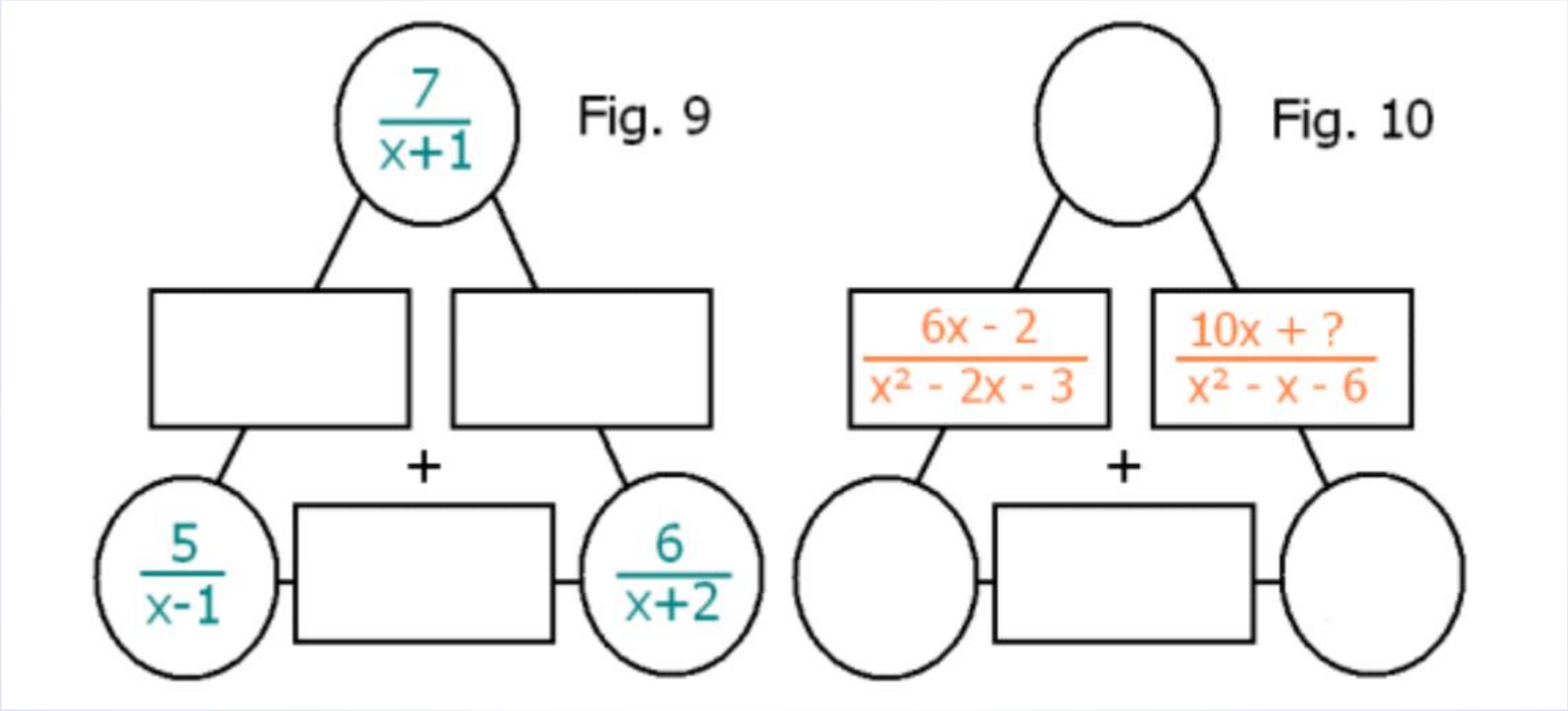

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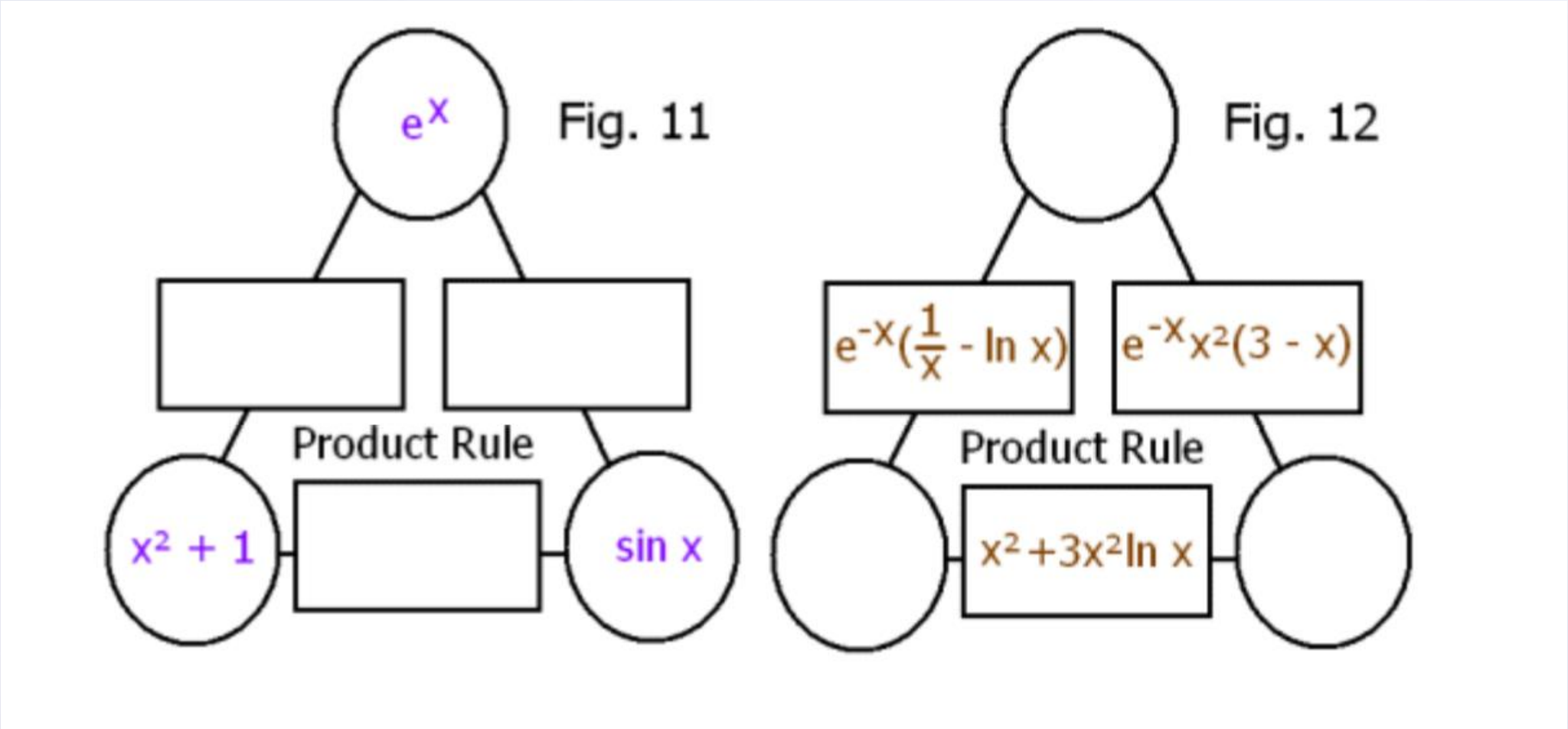
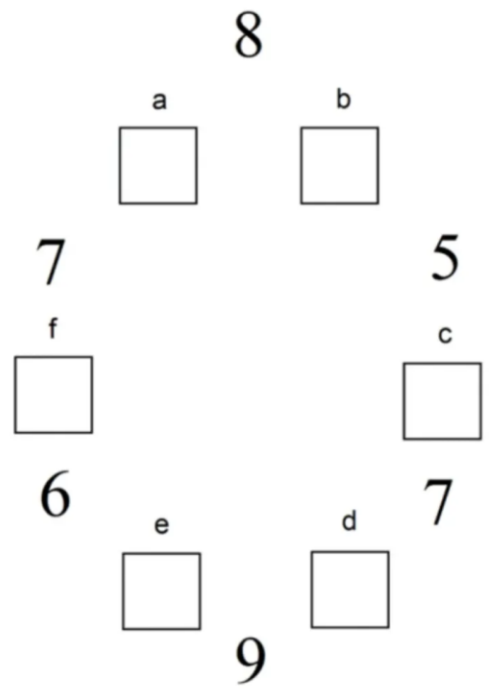


Jonny Griffiths - Digitisers

The missing values a, b, c, d, e and f are the digits 1, 2, 3, 4, 5 and 6 in some order (no repeats!)



Two neighbouring squares add to the numeral between them.

Jonny Griffiths - Digitisers

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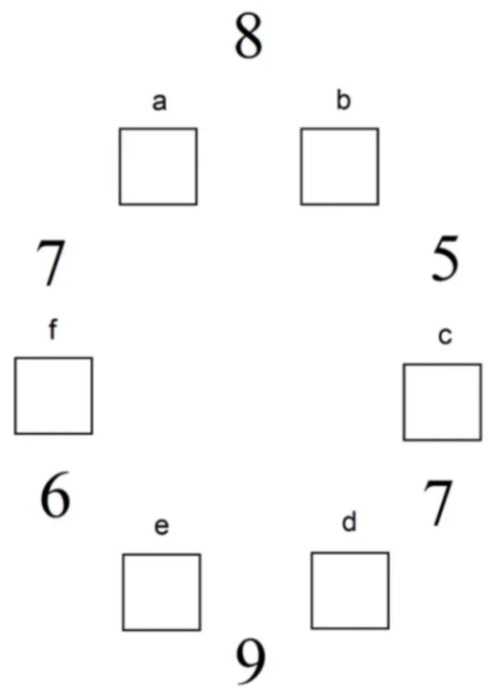


Two neighbouring squares add to the numeral between them.

One possible solution goes like this;

What can b be?

Not 1, since then a = 7.

b could be 2 or 3, but not 4, since then a would be 4.

b can't be 5 or 6, since then c is 0 or -1.

So b is 2 or 3, and c is 3 or 2, and 2 and 3 are taken.

So d is 4 or 5, and e is 5 or 4.

Thus 2, 3, 4 and 5 have all been used,

and f and a must be 1 and 6.

But f can't be 6, for then e would be 0,

so f = 1, e = 5, d = 4, c = 3, b = 2, and a = 6.

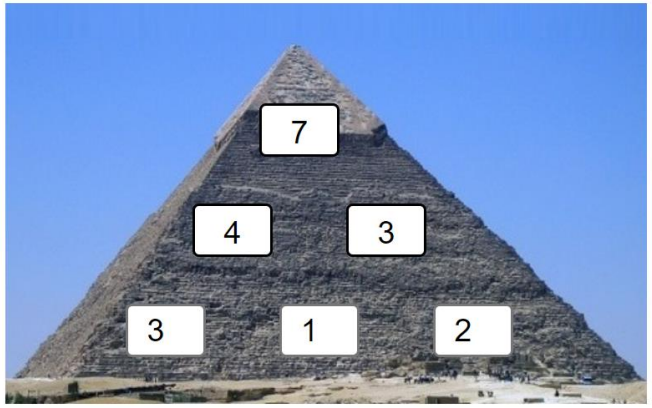
Everything checks out, and the required answer is 623451.

Number Pyramids

Age 11 to 14
Challenge Level ★

In a number pyramid, the numbers on the lower layers determine the numbers above them.

Start by choosing three single-digit numbers and enter them in the bottom row of the interactive number pyramid.



Try entering some different numbers in the bottom row - you don't have to stick to single-digit numbers.
Can you work out how the numbers in the upper layers are generated?

Here are some questions to consider:

If I tell you the numbers on the bottom layer, can you work out the top number without working out the middle layer?

If you change the order of the numbers on the bottom layer, will the top number change?

Given any three numbers for the bottom, how can you work out the largest possible number that could go at the top?

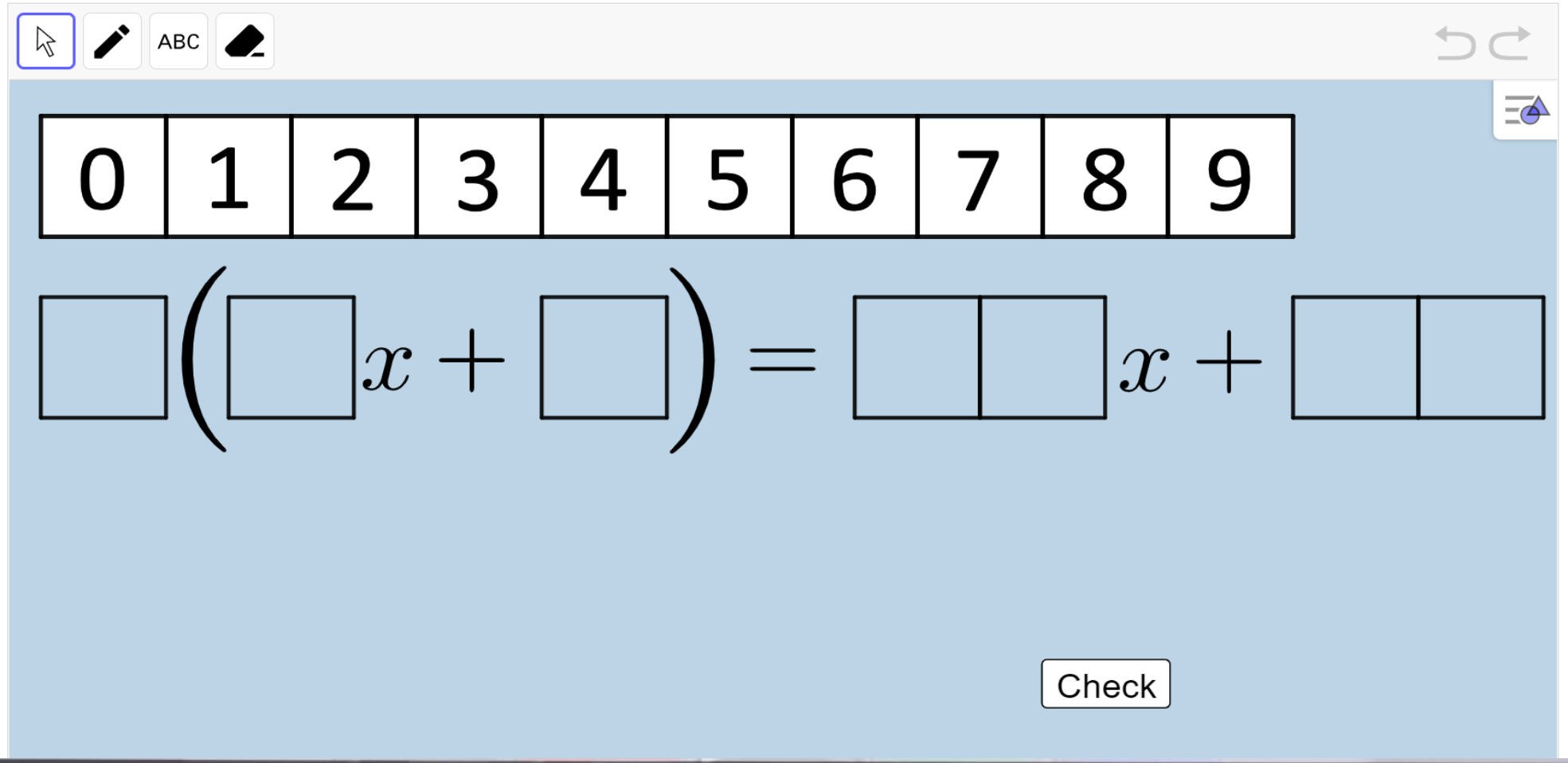
If I give you a target for the top number, can you quickly find three possible numbers for the bottom?

Distributive Property: Open Middle Theme

Author: Tim Brzezinski, Steve Phelps

Topic: Algebra, Equations, Linear Equations

Using digits 0-9 no more than ONE TIME EACH, fill the empty boxes with numbers to create coefficients that make the following statement true.



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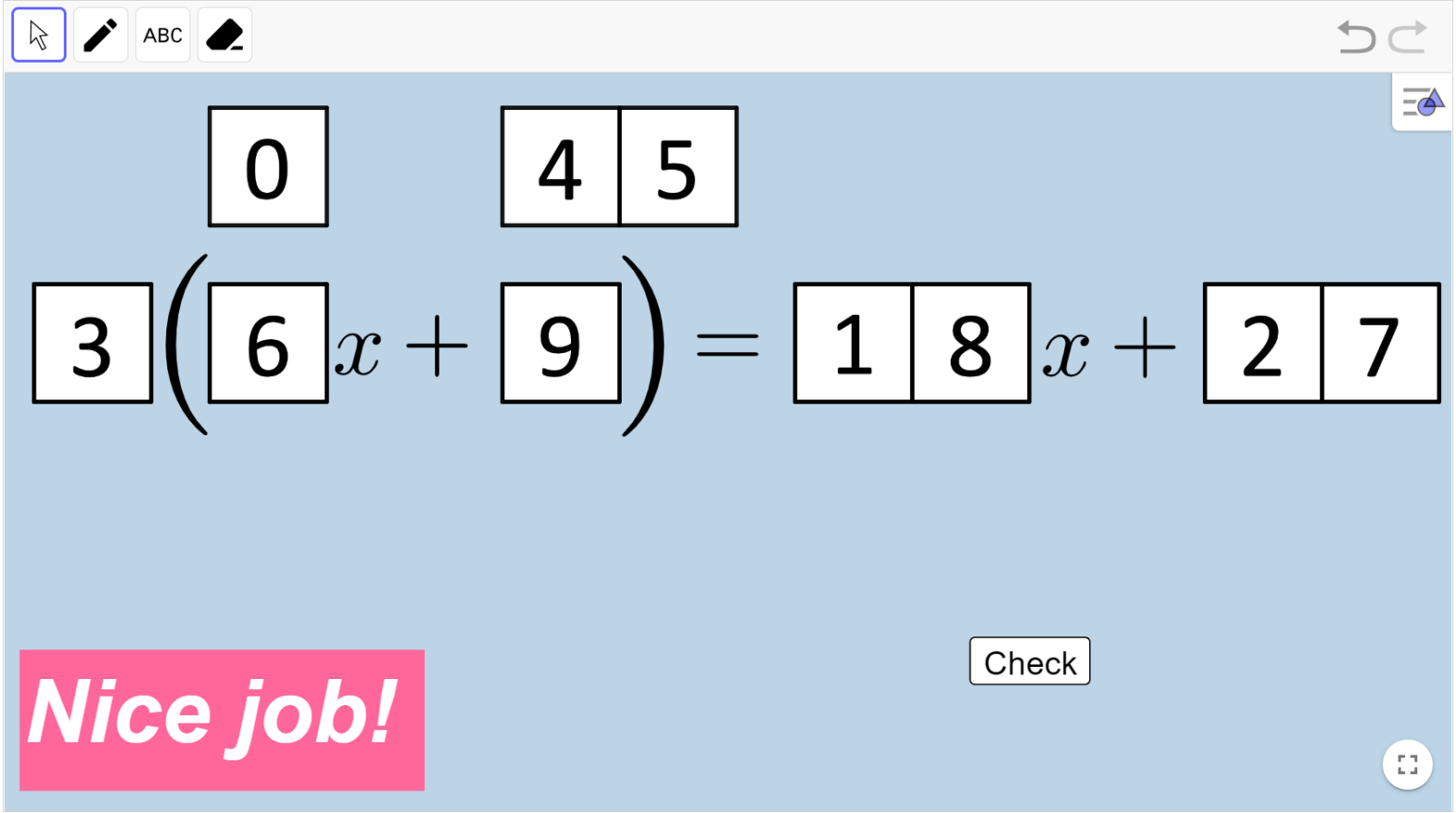


Create another true statement different from the one you made above.

Completion tasks

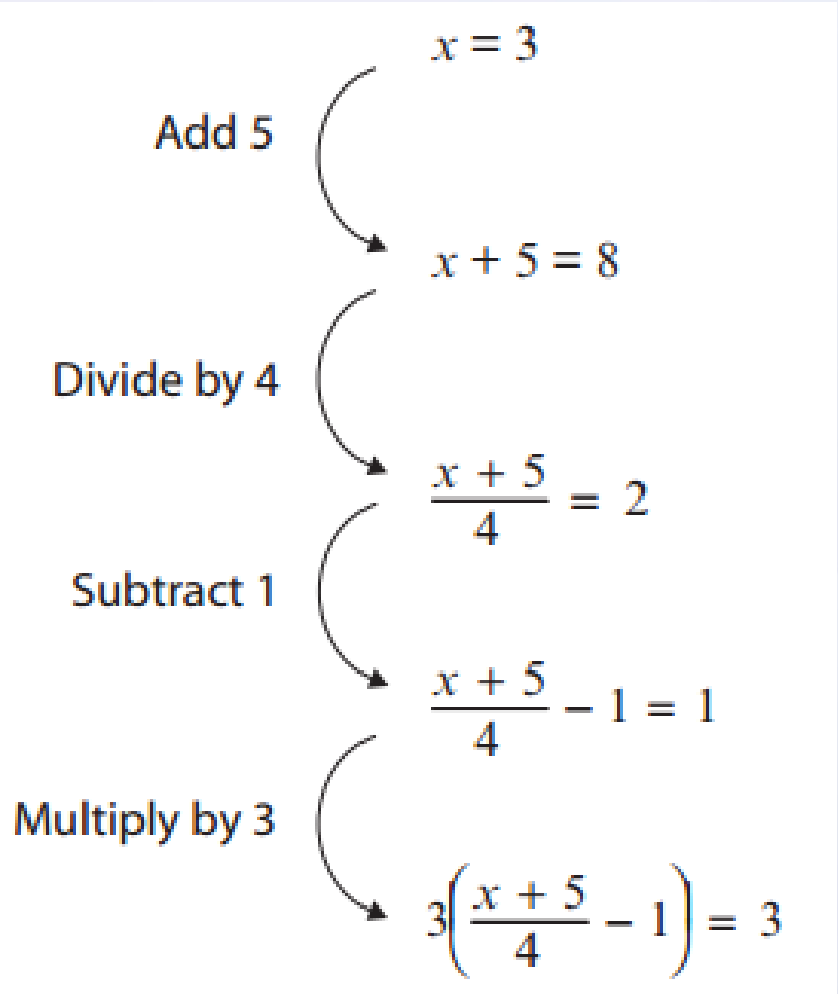
3	(	4	x	+	9	)	=	1	2	x	+	2	7
?	(		x	+	?	)	=	?	?	x	+	?	?
?	(		x	+	?	)	=	?	?	x	+	?	?

Standards Unit – Creating and Solving Equations

1. Build an equation

Write down a letter and its value on the board, e.g. $x = 3$. (This may be done on an OHT of Sheet 1.)

Using learners' suggestions for operations, build up an equation, step by step, using each of the four rules, +, –, ×, ÷ and whole numbers between 1 and 10.



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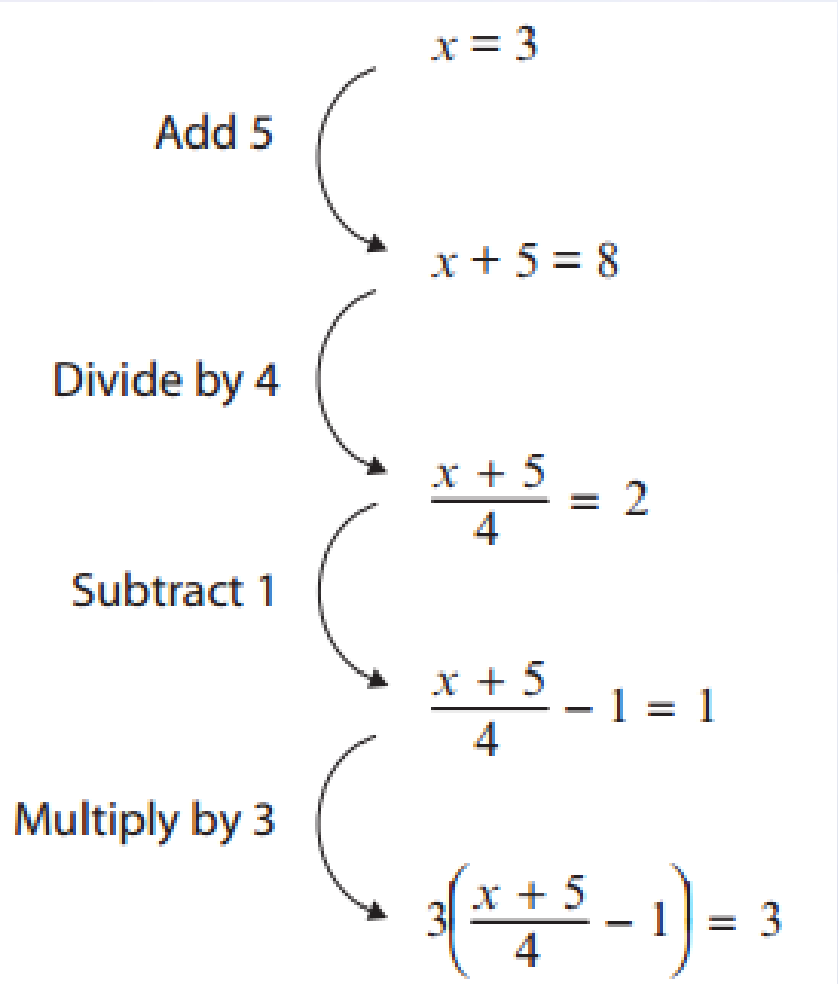
Using learners' suggestions for operations, build up an equation, step by step, using each of the four rules, +, –, ×, ÷ and whole numbers between 1 and 10.

2. Check the equation

Ask the group to check that the original value of x still satisfies the final equation.

$$3\left(\frac{3+5}{4}\right) - 1 = 3\left(\frac{8}{4} - 1\right) = 3(2-1) = 3 \times 1 = 3$$



Standards Unit – Creating and Solving Equations

3. Solve the equation

Hide all the steps except the final equation and ask the group to recall each operation in sequence.

> This equation tells the story of 'a day in the life of x'.
>
> What happened to it first? How can you tell by looking only at the equation?
>
> What then?
>
> What then?
>
> What was the last thing that happened?

In this way, show that the final equation tells the story of the operations used.

$$3\left(\frac{x+5}{4}-1\right)=3$$

Standards Unit – Creating and Solving Equations

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>
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$$3\left(\frac{x+5}{4}-1\right)=3$$

Standards Unit – Creating and Solving Equations

Suppose you had started with this equation and you wanted to find the value of x.

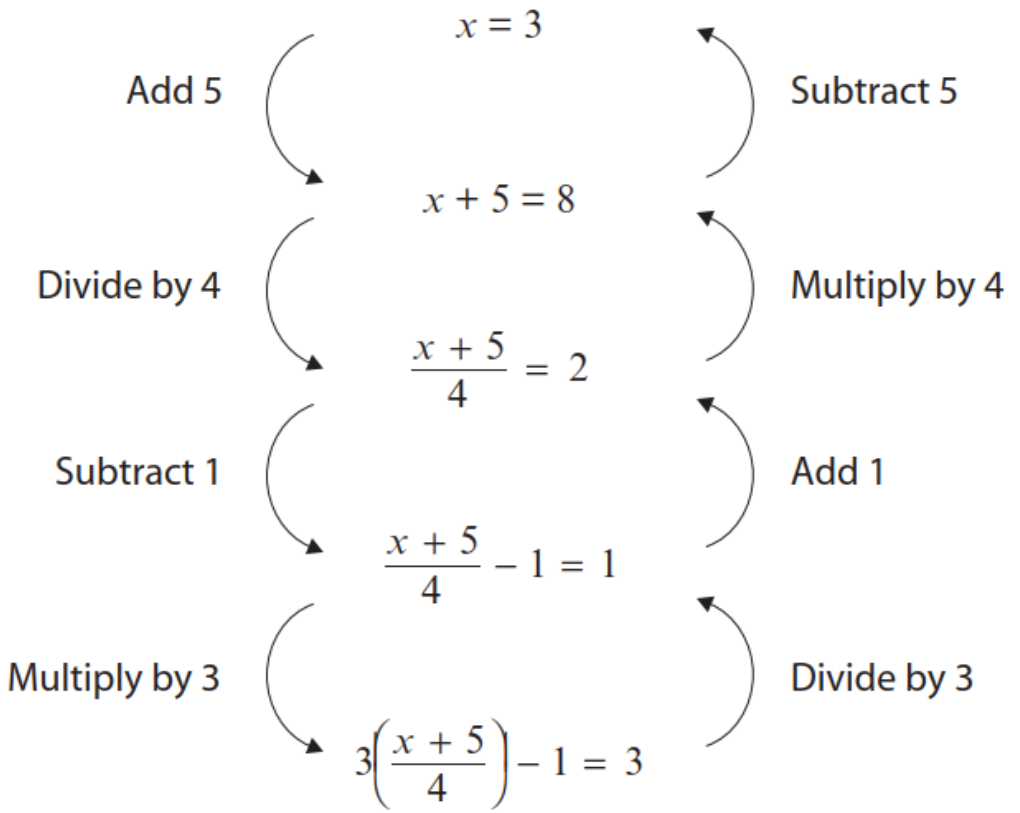
How could you do this?

How can you undo what we have just done?

You take your socks and boots off in the reverse order to the order you put them on.

It's the same here.

Gradually get the group to unpick each step in reverse order. As they do this, uncover the preceding equations one by one and write the corresponding operation to the right of each equation (with upward arrows):



Standards Unit – Creating and Solving Equations

4. Create your own equation

Ask learners to create two equations of their own in a similar way. Sheet 1 – *Creating equations* provides a structure for this. After creating each equation, learners should check that it works by substituting the answer back into it.

Learners who struggle may be asked to restrict themselves to fewer steps and operations to start with.

When learners are satisfied that their equations work (and maybe when they have checked them with you), ask them to write the equations on Sheet 2 – *Solving equations*.

"These resources are designed to develop mathematical thinking. We have attempted to do this through a number of different activity types. These types are not there to simply provide variety (though they do); they are devised to develop different ways of thinking."

4.4 **Creating problems**
Learners devise their own problems or problem variants for other learners to solve. This offers them the opportunity to be creative and 'own' problems. While others attempt to solve them, they take on the role of teacher and explainer. The 'doing' and 'undoing' processes of mathematics are vividly exemplified.

Worded Expressions

Expressions: 8 Max Variables: 2 Mixed Letters?

Hide Sentences | Show Expressions | Jumble | New | Print | Background:

1) 11 minus a number. a)

2) A number minus 6 all multiplied by 6. b)

3) 5 times a number added to another number all multiplied by 8. c)

4) A square number divided by another square number. d)

5) 6 divided by a number. e)

6) A number multiplied by 3 then all squared. f)

7) A number multiplied by 4 then all squared. g)

8) A number times by itself, then times by itself again. h)

Mathsbot – Jonathan Hall

Worded Expressions

Expressions: 8 Max Variables: 2 Mixed Letters?

Show Sentences | Hide Expressions | Jumble | New | Print | Background:

1)	a)	$11 - t$
2)	b)	$6(b - 6)$
3)	c)	$8(5c + w)$
4)	d)	$\frac{x^2}{h^2}$
5)	e)	$\frac{6}{k}$
6)	f)	$(3n)^2$
7)	g)	$(4e)^2$
8)	h)	k^3

Forming Expressions - Mathsbot

Forming Expressions

n = 10

New | Show All | Hide All | Random

Variable	1st Operation	2nd Operation	3rd Operation	Expression	When $n = 10$
n	$\div 10$	$\times 3$	$+ 5$		
n	$\div 10$	$+ 5$	$\times 3$		
n	$\times 3$	$\div 10$	$+ 5$		
n	$\times 3$	$+ 5$	$\div 10$		
n	$+ 5$	$\div 10$	$\times 3$		
n	$+ 5$	$\times 3$	$\div 10$		

Forming Expressions - Mathsbot

Forming Expressions

n = 10

New | Show All | Hide All | Random

Variable	1st Operation	2nd Operation	3rd Operation	Expression	When $n = 10$
n	$\div 10$	$\times 3$	$+ 5$	$\frac{3n}{10} + 5$	8
n	$\div 10$	$+ 5$	$\times 3$	$3(\frac{n}{10} + 5)$	18
n	$\times 3$	$\div 10$	$+ 5$	$\frac{3n}{10} + 5$	8
n	$\times 3$	$+ 5$	$\div 10$	$\frac{3n+5}{10}$	3.5
n	$+ 5$	$\div 10$	$\times 3$	$3(\frac{n+5}{10})$	4.5
n	$+ 5$	$\times 3$	$\div 10$	$\frac{3(n+5)}{10}$	4.5

Forming Expressions - Mathsbot

Variable	1st Operation	2nd Operation	3rd Operation	Expression	When $n = 10$
n	$\div 10$	$\times 3$	$+ 5$	$\frac{3n}{10} + 5$	8
n	$\div 10$	$+ 5$	$\times 3$	$3(\frac{n}{10} + 5)$	18
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n	$\times 3$	$+ 5$	$\div 10$	$\frac{3n+5}{10}$	3.5
n	$+ 5$	$\div 10$	$\times 3$	$3(\frac{n+5}{10})$	4.5
n	$+ 5$	$\times 3$	$\div 10$	$\frac{3(n+5)}{10}$	4.5

Variable	1st Operation	2nd Operation	3rd Operation	Expression	When $n = 10$
n	$\div 10$	$\times 3$	$+ 5$	$\frac{3n}{10} + 5$	8
n	$\div 10$	$+ 5$	$\times 3$		18
n	$\times 3$	$\div 10$	$+ 5$		8
n				$\frac{3n+5}{10}$	3.5
n				$3(\frac{n+5}{10})$	4.5
n				$\frac{3(n+5)}{10}$	4.5

Variation Grids

A Collection of Structured Variation Grids, Tables, and Charts

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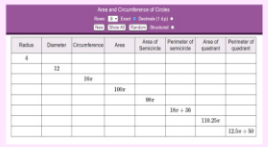
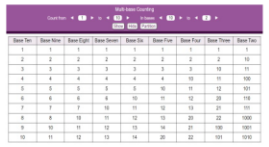
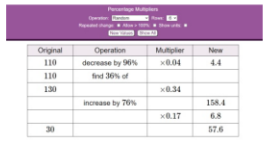
Grids



Area and Circumference of Circles

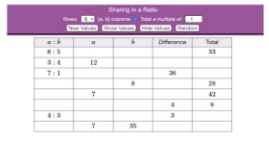


Multi-base Counting

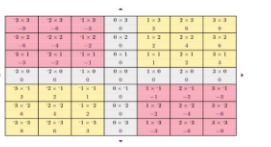


Original	Operation	Multiplier	New
110	decrease by 96%	×0.04	4.4
110	find 36% of		
130		×0.34	
	increase by 76%		158.4
		×0.17	6.8
30			57.6

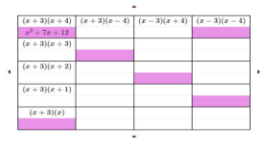
Percentage Multipliers



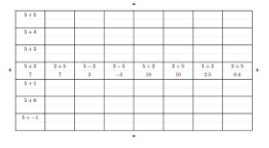
Sharing in a Ratio



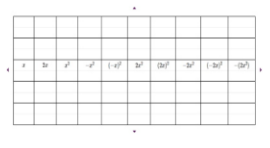
SVG: Arithmetic



SVG: Binomials

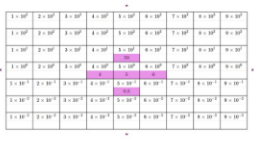


SVG: Commutative

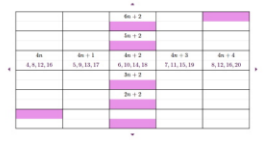


SVG: Evaluating Expressions

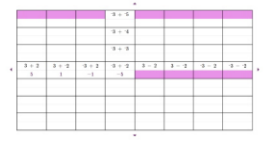
SVG: Fibonacci Sequences



SVG: Gattegno Tens Chart



SVG: Linear Sequences



SVG: Subtraction

CREATE A SYSTEM OF TWO EQUATIONS

Directions: Using the digits 1 to 30, at most one time each, fill in the boxes to create a system of two linear equations where (3, 2) is the solution to the system.

$$\begin{cases} \square x + \square y = \square \\ \square x + \square y = \square \end{cases}$$

Hint

Answer

Source: Daniel Luevanos

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Create a System of Two E... Save

desmos

Open Middle Problem by Daniel Luevanos

http://www.openmiddle.com/create-a-system-of-two-equations/

$ax + by = c$

$a = 7$ 1 30

$b = 4$ 1 13

$c = 3a + 2b$

$c = 29$

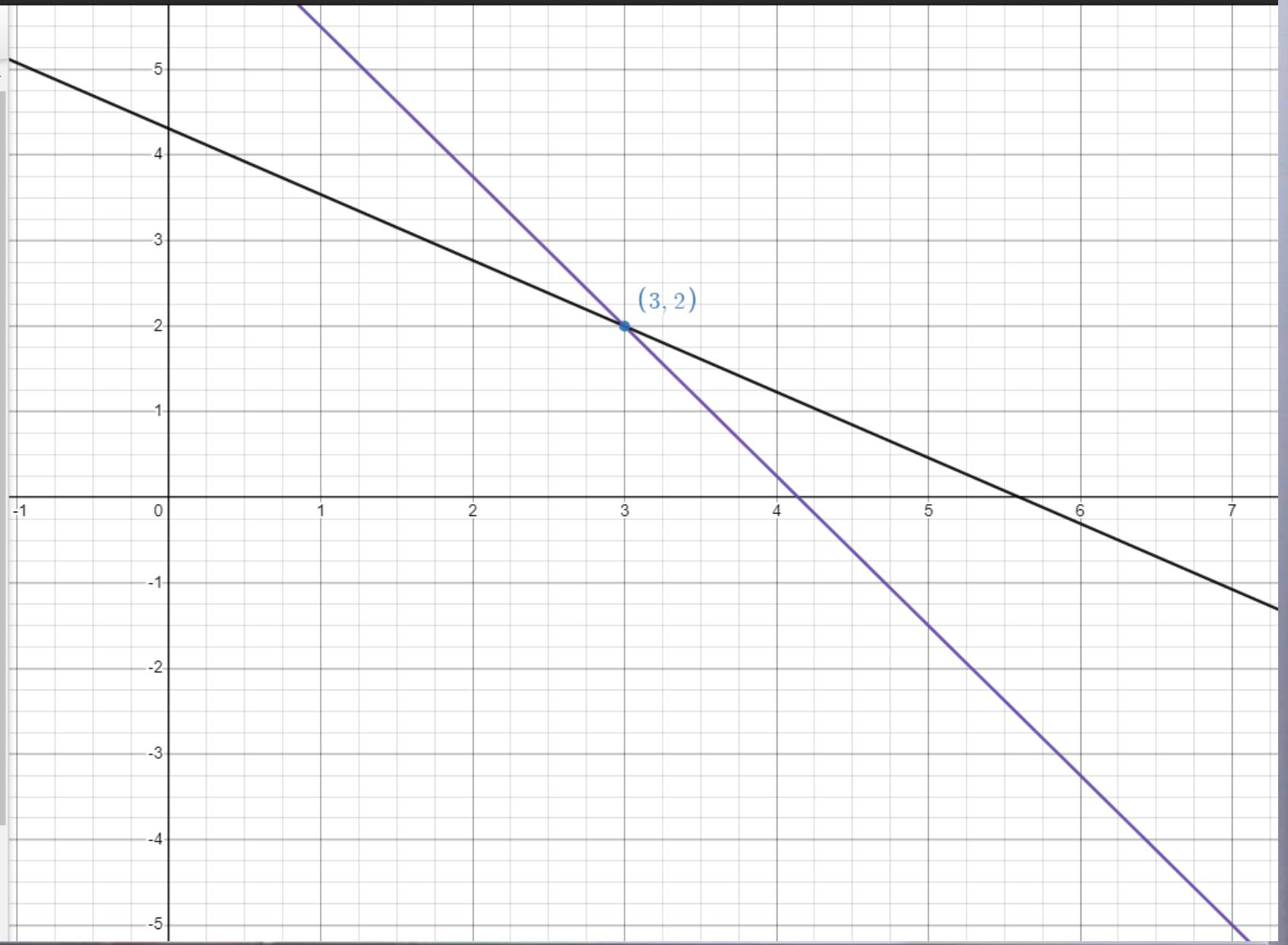
$dx + fy = g$

$d = 10$ 1 30

$f = 13$ 1 13

$g = 3d + 2f$

$g = 56$



Fill in the blanks

There is also a good deal of evidence that the use of worked examples can be helpful in introducing new ideas (Booth et al., 2017; Sweller et al., 2019).

Particularly effective are 'completion problems' where students are given partial solutions and required to complete them.

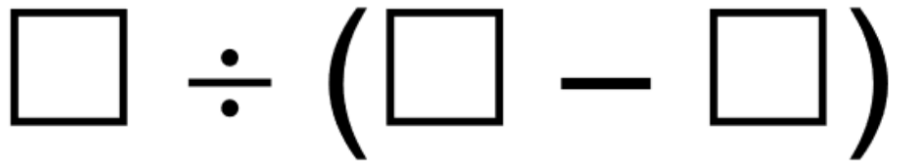
These can help students to focus on the examples but also manage the difficulty level while retaining authentic tasks.

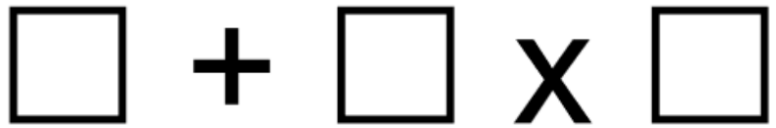
2	Explaining: presenting and communicating new ideas clearly, with concise, appropriate, engaging explanations; connecting new ideas to what has previously been learnt (and re-activating/checking that prior knowledge); using examples (and non-examples) appropriately to help learners understand and build connections; modelling/demonstrating new skills or procedures with appropriate scaffolding and challenge; using worked/part-worked examples

Great Teaching Toolkit – Evidence Review, June 2020 Rob Coe, C.J. Rauch, Stuart Kime, Dan Singleton

ORDER OF OPERATIONS 5

Directions: Using the digits 0 to 9 at most one time each, place a digit in each box so that each expression is simplified to a different odd number.





Hint

How do we know which digits are easier to place?

How can we tell which operations might affect getting a whole number result for our expression?

From the Open Middle Starter Pack

Why We Love It

The conversations students have as they develop conceptual understanding are phenomenal!
So much great learning happens around discussing misconceptions that come out.

Hints

- How do we know which digits are easier to place?
- How can we tell which operations might affect getting a whole number result for our expression?

Answer

There are many solutions to this problem including:
$5 \div (8 - 7) = 5$
$9 + 0 \times 6 = 9$
$3 - 4 / 2 \div 1 = 1$

Problem Source

https://www.openmiddle.com/order-of-operations-5/

Problem Creator

Molly Rawding

DEFINITE INTEGRAL 3

Source: Robert Kaplinsky

Directions: Using the digits 1 to 9 at most one time each, place a digit in each box to make a solution that is as close to 100 as possible.

$$\int_{\square}^{\square} x^{\square}\,dx$$

Answer

I only know how to figure this out via brute force and would love to learn if anyone has a conceptual approach to this.

I created this spreadsheet and tried all 9 potential exponent values. The bounds and exponent that result in a value as close to 100 as possible are:

upper bound: 8
lower bound: 6
exponent: 2
value: 98.666…

and a close second place is:

upper bound: 2
lower bound: 1
exponent: 9
value: 102.30

Why We Love It

Students may know how to solve a definite integral without having deep understanding of how the bounds and exponents work. This problem forces students to develop and use their conceptual understanding to efficiently find closer and closer solutions.

Hints

- How do the upper and lower bounds determine whether the solution is negative or positive?
- How does the exponent affect the solution's value?

Answers

The solution whose value is 98.666 and is as close to 100 as possible comes from $\int_2^8 x^2\,dx$

Problem Source

https://www.openmiddle.com/definite-integral-3/

Problem Creator

Robert Kaplinsky

MULTIPLYING BINOMIALS

Directions: Fill in the boxes with any numbers that make the equation true.

$$(\square x - 3)(\square x + \square) = 12x^2 - \square x - 15$$

Hint

Answer

Source: Dane Ehlert

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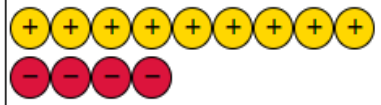
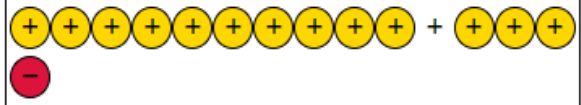
Depth of Knowledge Matrix – Fifth Grade Math

Robert Kaplinsky

Topic	Evaluating Expressions	Rounding Decimals	Multi-Digit Multiplication	Multiplying Decimals
CCSS Stand.	• 5.OA.1	• 5. NBT.4	• 5.NBT.5	• 5.NBT.7
DOK 1 Example	Evaluate the expression. $56 \div (8 - 1)$	Round the decimal to the nearest tenth. 7.163	Find the product. 37×45	Solve. $3.4 \times 2.5 =$
DOK 2 Example	Using the digits 0 through 9, at most one time each, place a digit in each box to create two true statements: one where the value on each side of the equal sign is greater than 30 and one where it's less than 30. You may reuse all the digits for each equation. $\square\square \div (\square - \square) = \square + \square \times \square$	Using the digits 0 to 9 at most one time each, place a digit in each box to create two different decimals that are equivalent when rounded to the nearest tenth. $\square.\square\square\square$ $\square.\square\square\square$	Using the digits 0 to 9 at most one time each, place a digit in each box to create a true equation. $\square\square \times \square\square = \square\square\square$	Using the digits 1 to 9 at most one time each, fill in the boxes to make a true number sentence. $\square.\square \times 3.2 = \square.\square$
DOK 3 Example	Using the digits 0 through 9, at most one time each, place a digit in each box to create the greatest possible value.	Using the digits 0 to 9 at most one time each, place a digit in each box to create two different decimals that	Using the digits 0 to 9 at most one time each, place a digit in each box to create a true equation with the greatest	Using the digits 1 to 9 at most one time each, fill in the boxes so that the product is as close to 50 as

Directed Number

Rows: 8 | New Values | Show All | Hide All | Random

Subtraction	We say:	Equivalent addition	We say:	Model	Answer
$9 - 4$	*"nine minus four"*	$9 + {}^{-}4$	*"nine plus negative four"*		5
	"ten minus two"				
		$^{-}4 + 4$			
			"negative three plus six plus negative six"		
					
$3 - 6 - 9$					-12
	"negative nine minus negative ten minus negative two minus negative nine"				
		$1 + {}^{-}5 + 10 + 9$			

Fill In The Blanks...

Dividing Fractions

Division	Equivalent Multiplication	Unsimplified Answer	Simplified Answer (where possible)
$\frac{2}{3} \div 6$	$\frac{2}{3} \times \frac{1}{6}$	$\frac{2}{18}$	$\frac{\square}{\square}$
$\frac{2}{5} \div 4$	$\frac{2}{5} \times \frac{1}{4}$	$\frac{\square}{\square}$	$\frac{\square}{\square}$
$\frac{5}{8} \div 10$	$\frac{\square}{\square} \times \frac{\square}{\square}$	$\frac{\square}{\square}$	$\frac{\square}{\square}$

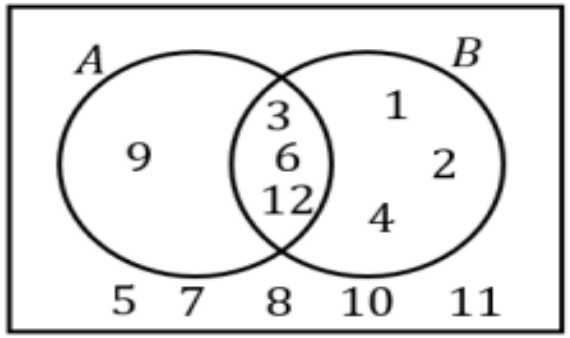
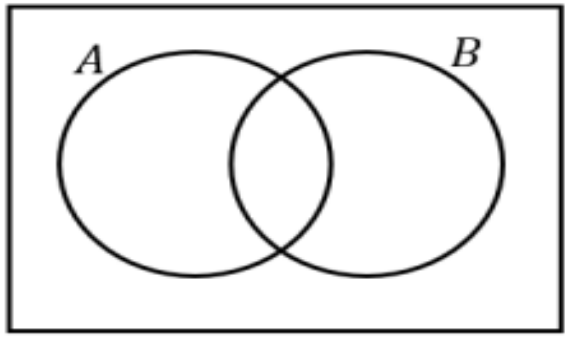
$\frac{11}{16} \div \frac{3}{4}$	$\frac{\square}{\square} \times \frac{\square}{\square}$	$\frac{\square}{\square}$	$\frac{\square}{\square}$
$\frac{\square}{\square} \div \frac{\square}{\square}$	$\frac{2}{9} \times \frac{6}{5}$	$\frac{\square}{\square}$	$\frac{\square}{\square}$
$\frac{\square}{\square} \div \frac{\square}{\square}$	$\frac{3}{8} \times \frac{\square}{\square}$	$\frac{12}{24}$	$\frac{\square}{\square}$
$\frac{\square}{\square} \div \frac{2}{5}$	$\frac{\square}{\square} \times \frac{\square}{\square}$	$\frac{15}{20}$	$\frac{\square}{\square}$
$\frac{\square}{\square} \div \frac{\square}{\square}$	$\frac{5}{12} \times \frac{\square}{\square}$	$\frac{10}{12}$	$\frac{\square}{\square}$
$\frac{\square}{\square} \div \square$	$\frac{\square}{\square} \times \frac{1}{3}$	$\frac{\square}{\square}$	$\frac{3}{10}$

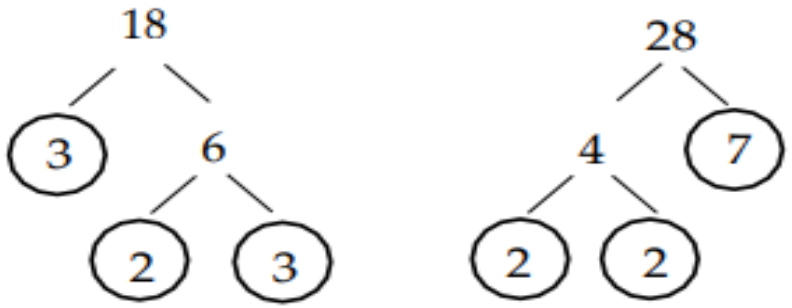
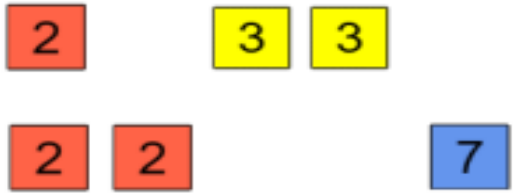
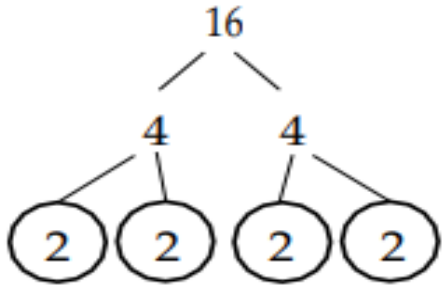
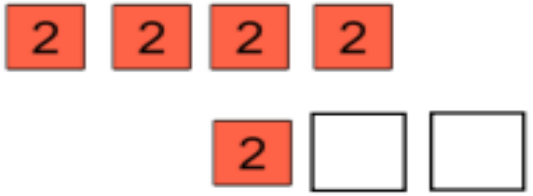
Dr Austin Maths

Fill In The Blanks...

Venn Diagrams and Set Notation

In all questions $\xi = \{Integers\ from\ 1\ to\ 12\ inclusive\}$

Sets	Venn Diagram	$A \cup B$	$A \cap B$	$A' \cap B$	$(A \cup B)'$
$A = \{multiples\ of\ 3\}$ $B = \{factors\ of\ 12\}$			$\{3, 6, 12\}$		$\{5, 7, 8, 10, 11\}$
$A = \{prime\ numbers\}$ $B = \{odd\ numbers\}$					

Prime Factorisation	Prime Factor Tiles	LCM	HCF
		2 x 2 x 3 x 3 x 7 = 252	2
			3
		2 x 2 x 2 x 2 x ? x ? =	

Fill In The Blanks

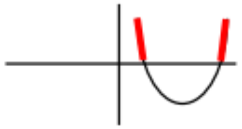
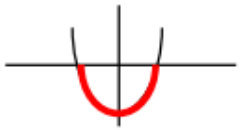
ACCESS MATHS

Can you fill in the multiplication square?

x	4x	b	y^2	z		
a	4xa		ay^2			
b						
3x						
2y						$8yx^2$
		4ab				
	-8xz				-4zx	

Quadratic Inequalities – Fill in the blanks!

ACCESS MATHS

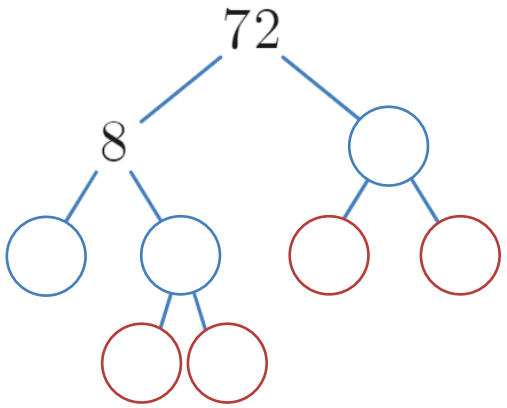
Equation	Factorisation	Roots	Sketch	Solve >0	Sketch	Solve < 0
$x^2 + 4x + 3$						
$x^2 - x - 20$						
		$x=8$ $x=-3$				
				$x<5$ and $x>7$		
						$-7<x<7$
$2x^2 - 11x + 9$						
$2x^2 + 3x - 2$						

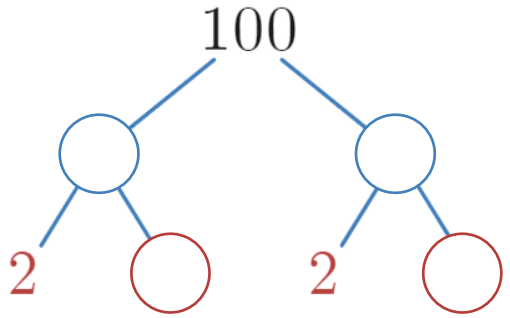
Malcolm Swan on Practice

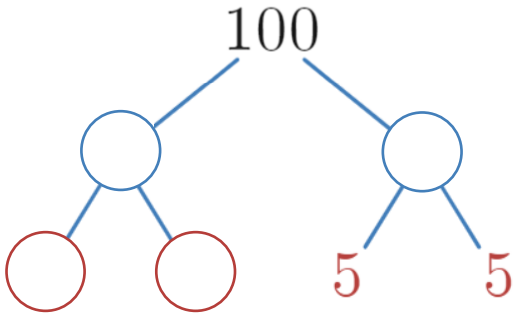
“If practice is just repeating the same procedure with different numbers, chosen randomly, then it has no purpose. Some appear to think that such practice is like training a muscle, where repeated exercise builds up some kind of inner mental strength and speed. In fact it usually results in boredom. Variation theory tells us that by systematically changing significant aspects of a task, keeping the rest fixed, we can focus the students’ attention on those aspects and conceptual change can result. But the emphasis in making such variations is not to develop speed but to develop an awareness of pattern, leading to conjecture, generalisation, explanation and deeper understanding.”

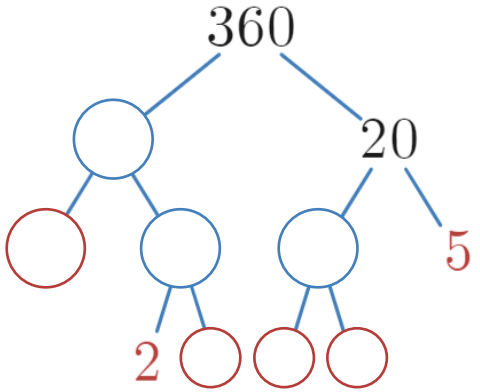
Professor Malcolm Swan. (NAMA, 2015)

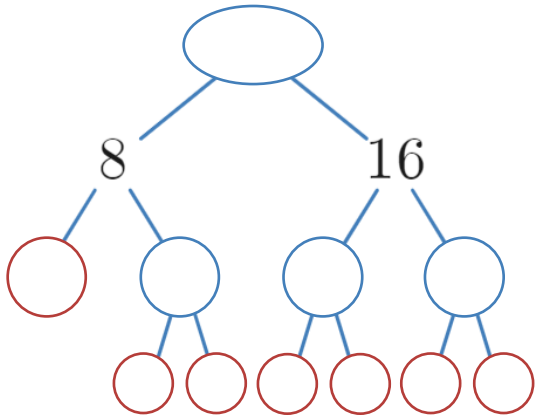
Factor Trees

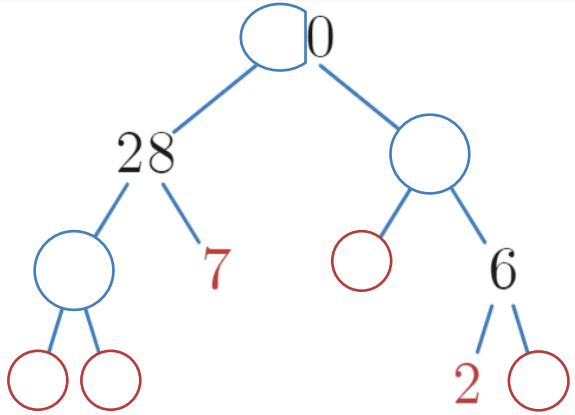


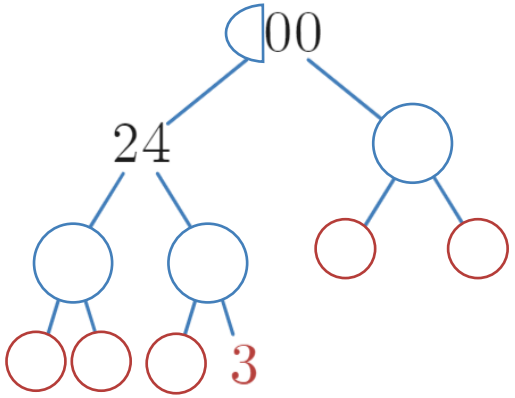


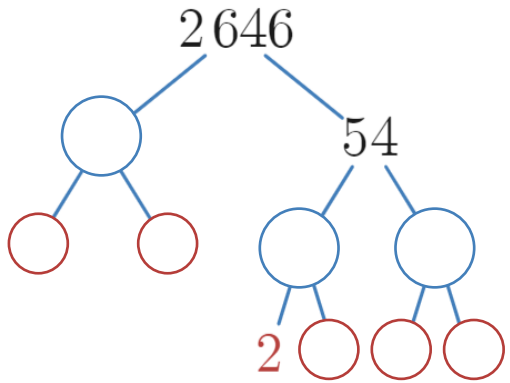


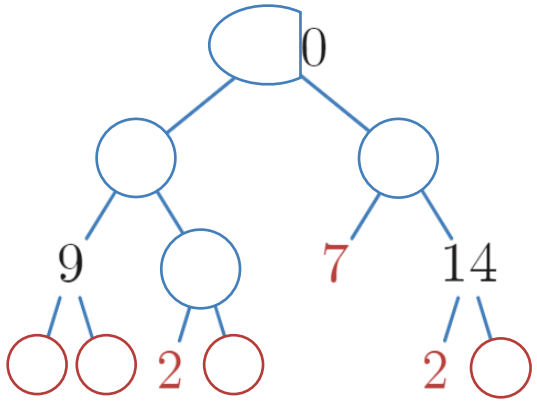


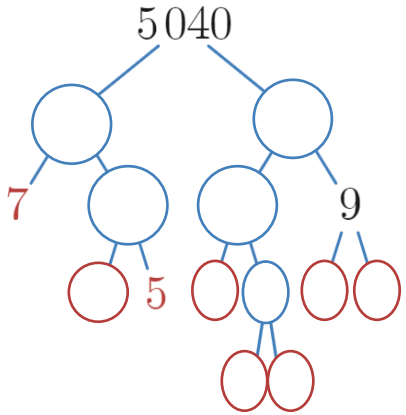


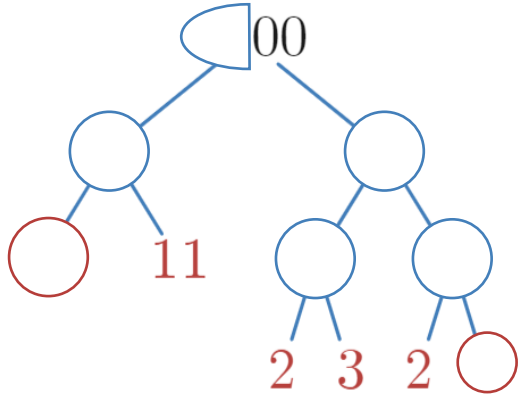


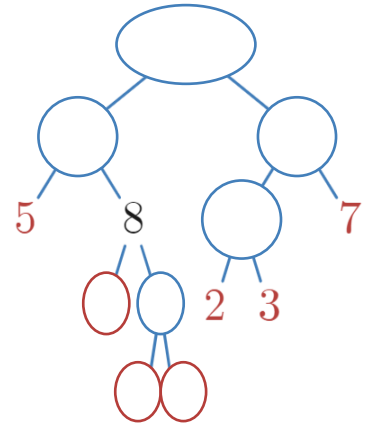












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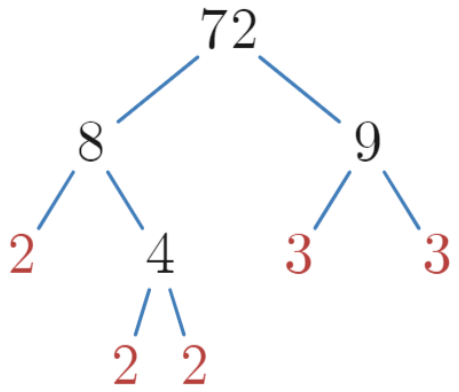

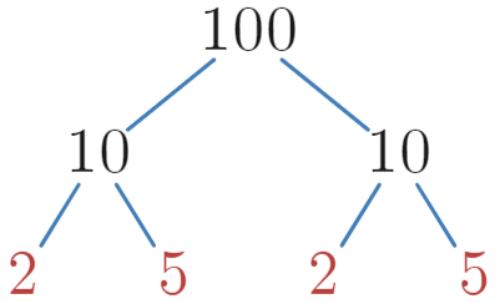

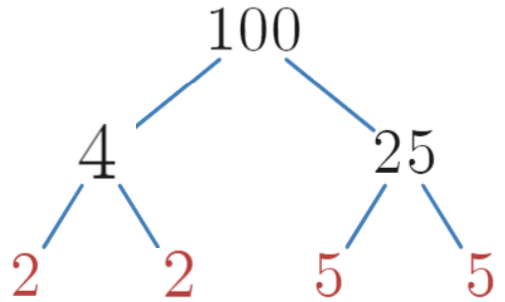

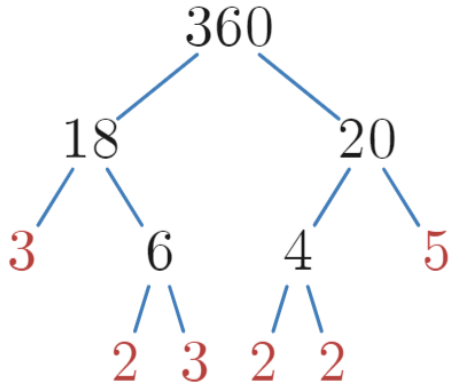

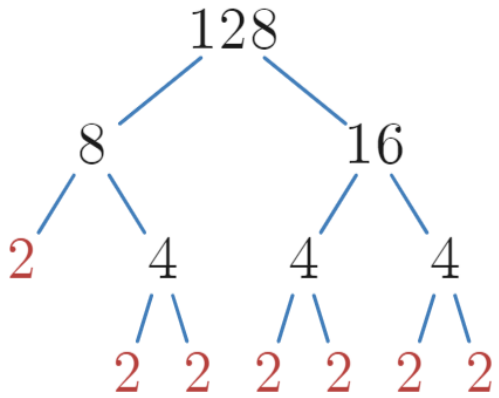

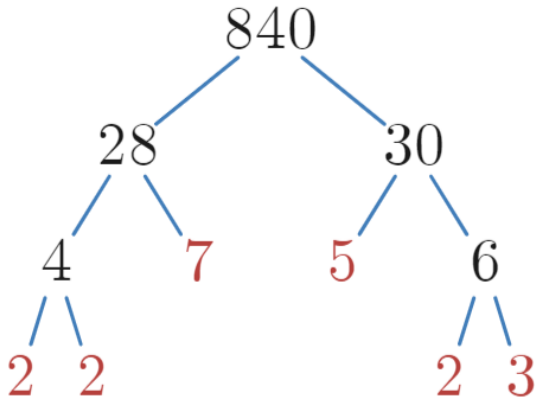

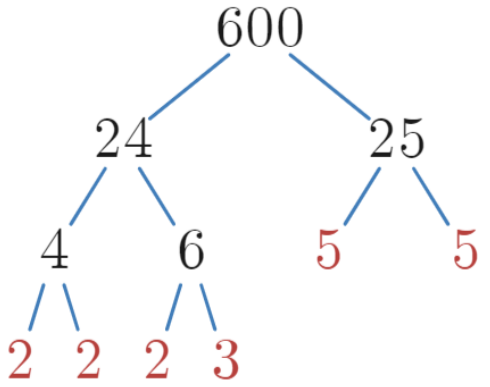

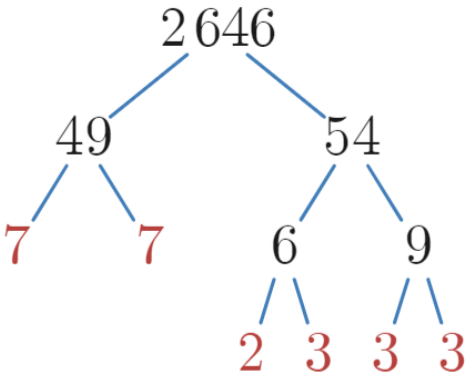

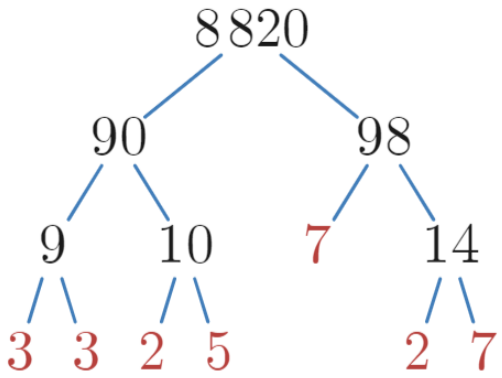

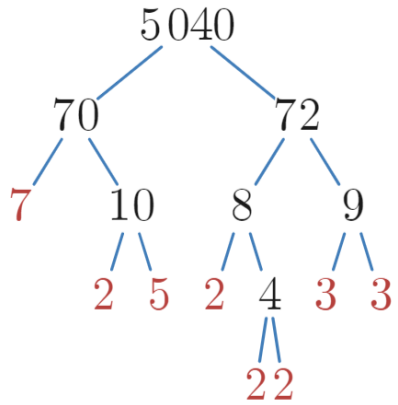

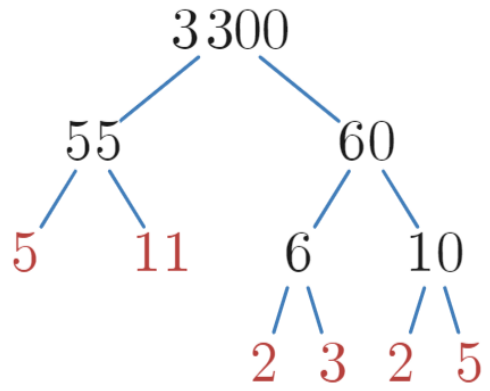

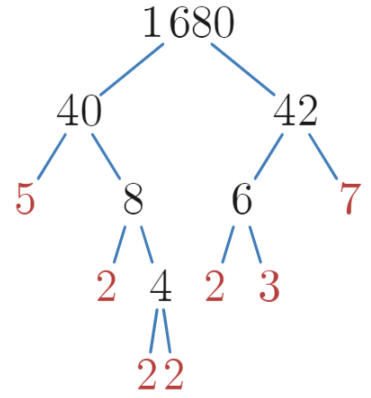

Factor Trees 2

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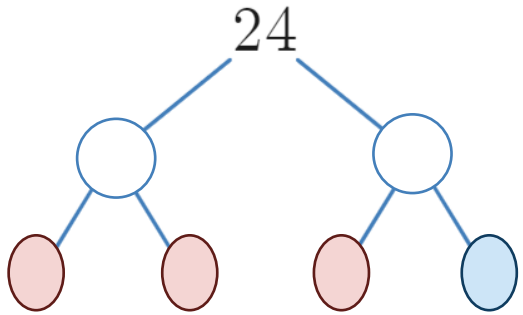

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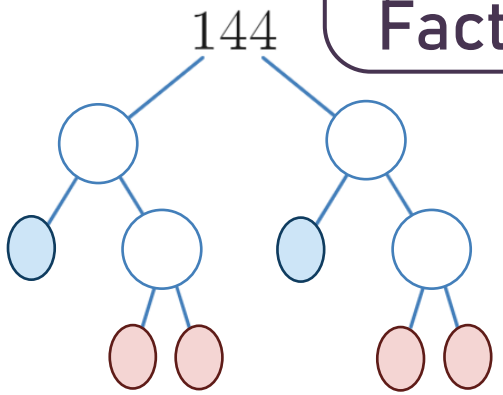

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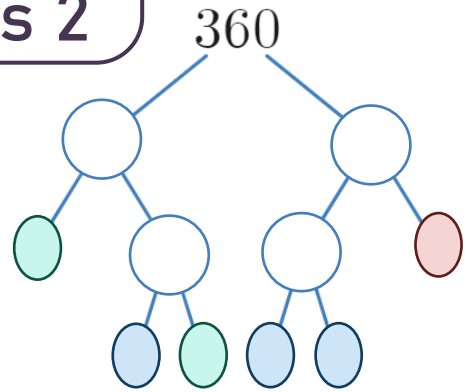

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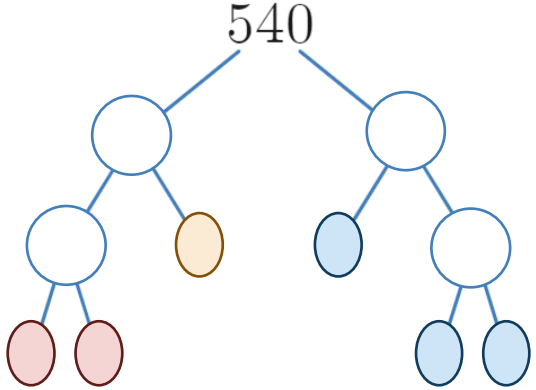

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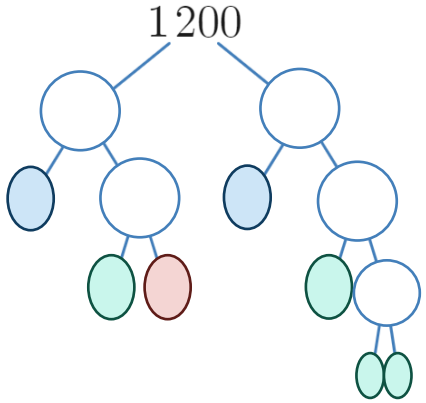

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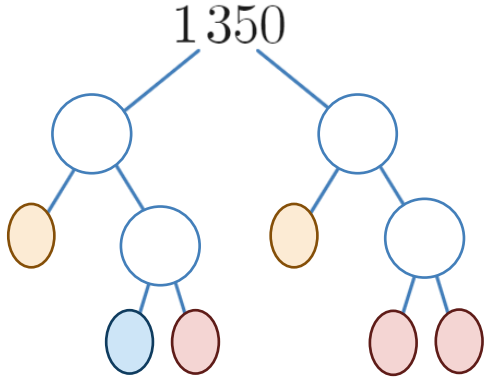

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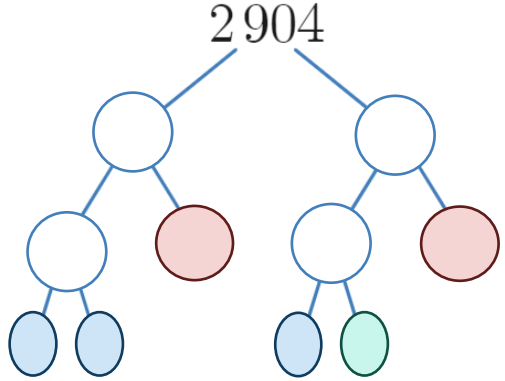

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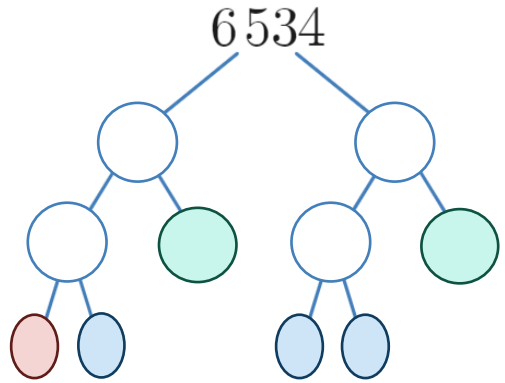

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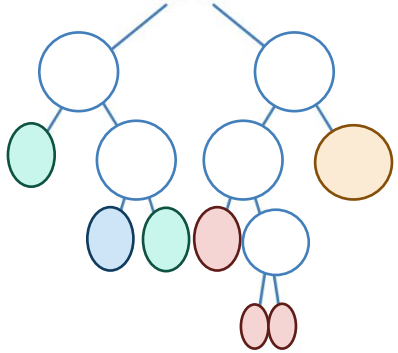

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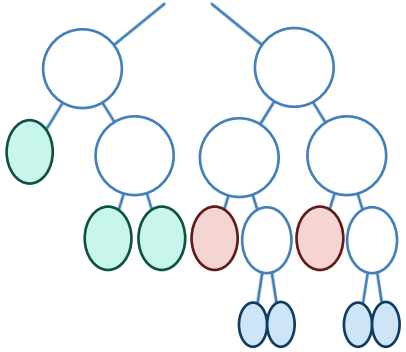

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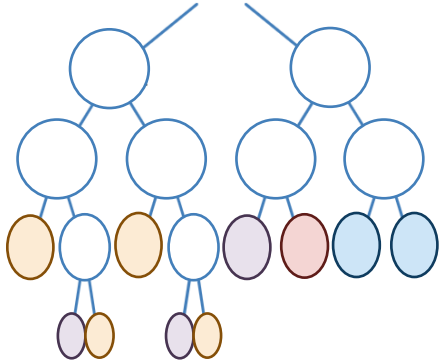

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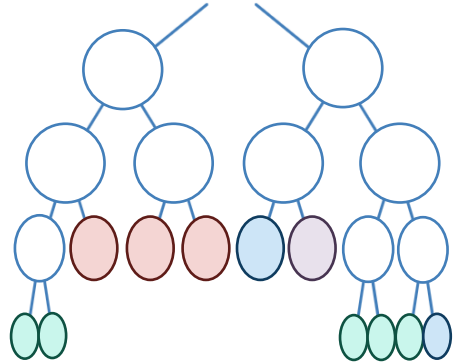
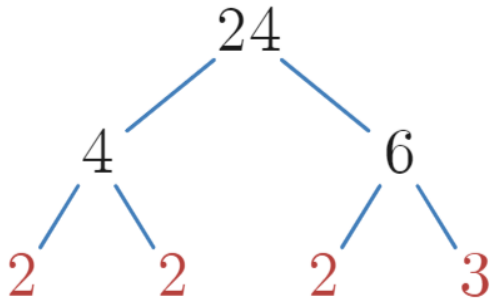
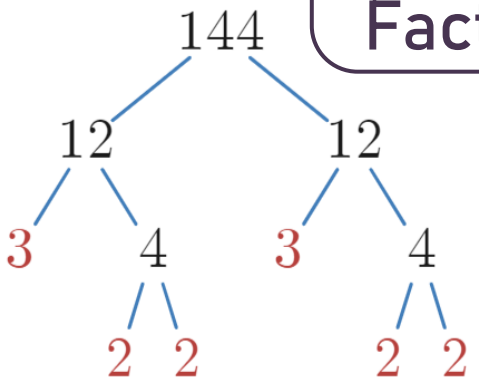
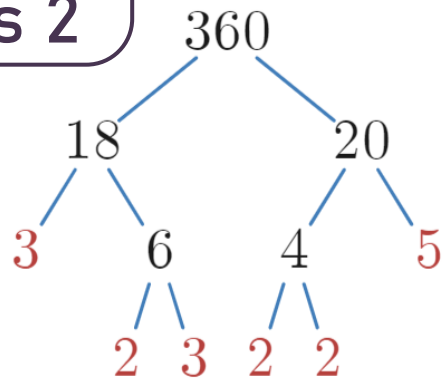
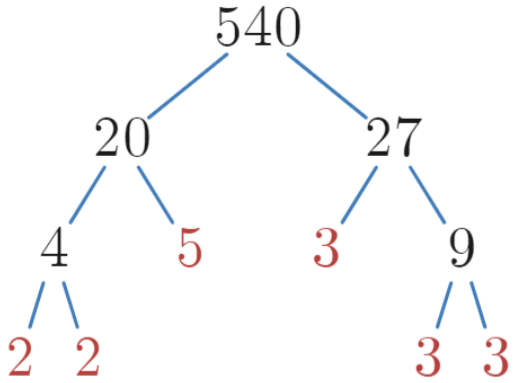


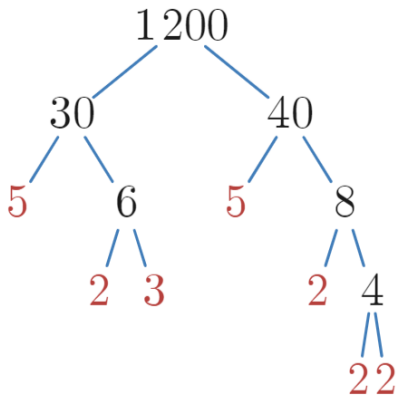
Factor Trees 2

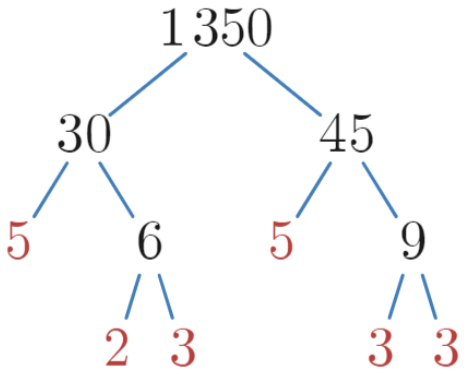


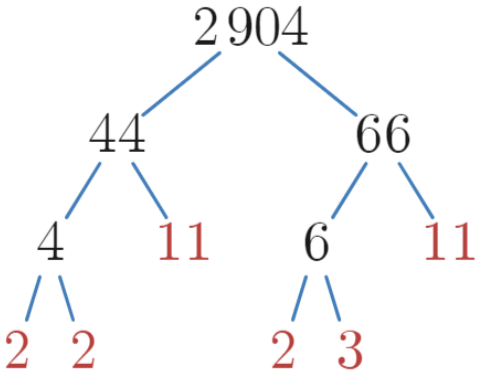


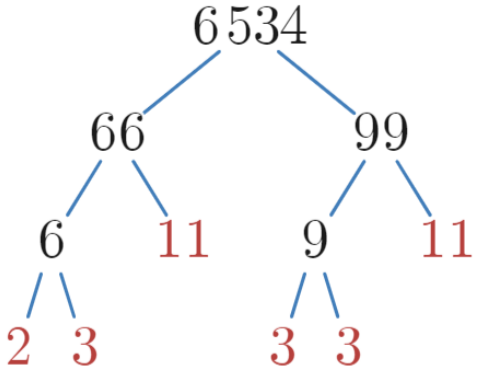


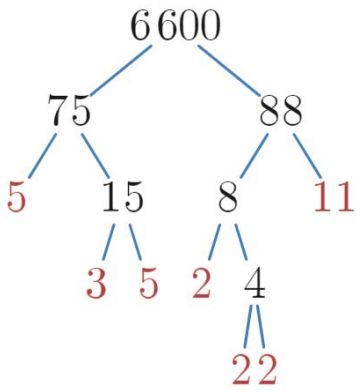


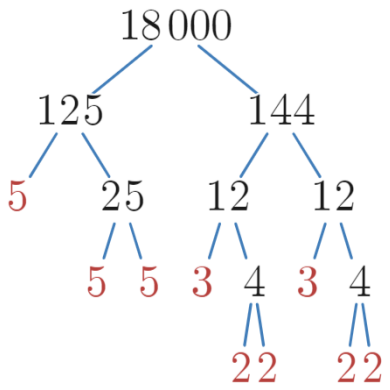


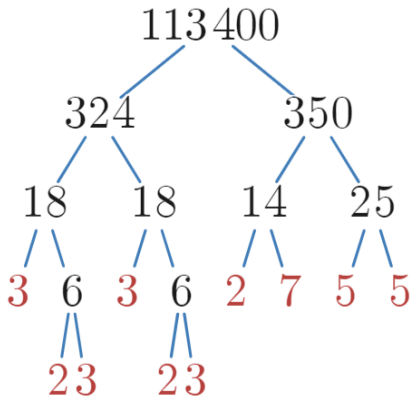


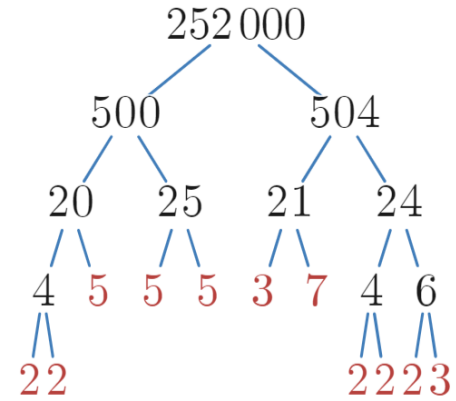












Year 12 Pure – Coordinate Geometry.

Rob Southern

Fill in the missing information in the table.

Point A	Point B	**Midpoint** of the line segment AB	**Length** of the line segment AB	**Gradient** of the line segment AB	**Equation** of the line through A and B.
$(1,3)$	$(5,11)$				
$(-3,2)$	$(5,-6)$				
$\left(\frac{-7}{3},\frac{-22}{3}\right)$	$\left(\frac{11}{3},\frac{-4}{3}\right)$				
	$(-7,11)$	$\left(-11,\frac{7}{2}\right)$			
	$(-2,-4)$		$2\sqrt{5}$		$x+2y+10=0$
		$(4,1)$	20	$\frac{3}{4}$	
$(4,1)$			$4\sqrt{13}$		$2x+3y-11=0$

Coordinate Geometry Table

Fill in the empty cells of this table with information about lines and coordinates.

Enclose coordinates in brackets. Use the / symbol to show a fraction and the root button to insert the square root sign if required. Express mixed numbers as improper fractions.

Point A	Point B	Midpoint of the line segment AB	Length of the line segment AB	Gradient of the line segment AB	Equation of the line through A and B
$(1, 3)$	$(5, 11)$				
$(-3, 2)$	$(5, -6)$				
$(\frac{-7}{3}, \frac{-22}{3})$	$(\frac{11}{3}, \frac{-4}{3})$				

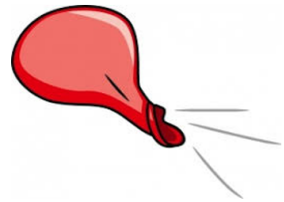
I have the sequence below:

_____, _____, _____, _____

Find the nth term of the sequence.

Expand the brackets:

_____(_____)

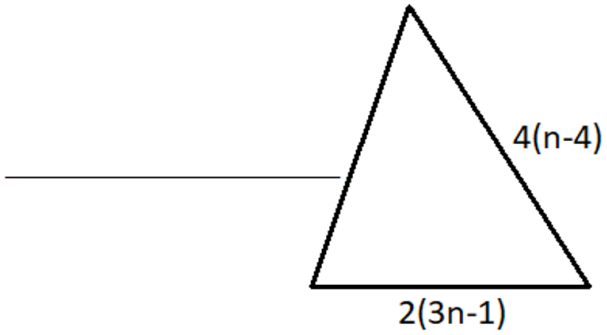


The answer is $12n - 8$

I have a rectangle of length $2n + 1$ and width _____.

What is the perimeter of the rectangle?

Find the perimeter of the triangle:



I have the sequence below:

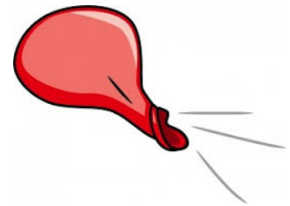
4, **16**, **28**, **40**

Find the nth term of the sequence.

Expand the brackets:

$$\frac{4}{2}(3n-2)$$

$$6n-4$$



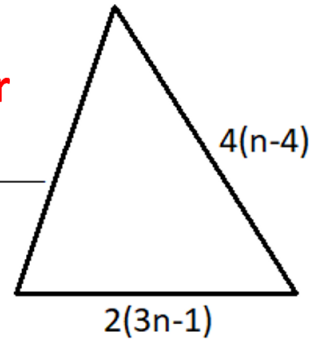
The answer is $12n - 8$

I have a rectangle of length $2n + 1$ and width **$4n - 5$**.

What is the perimeter of the rectangle?

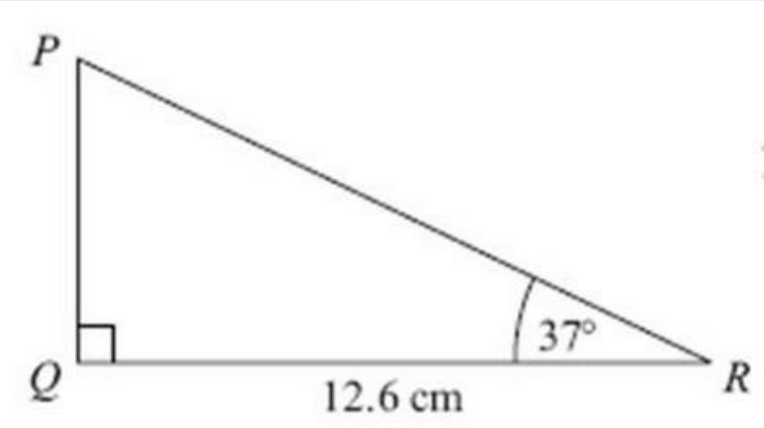
Find the perimeter of the triangle:

$2n + 10$ or $2(n + 5)$



Geometry – here's the diagram..

For a variation on here's the answer, what's the question?



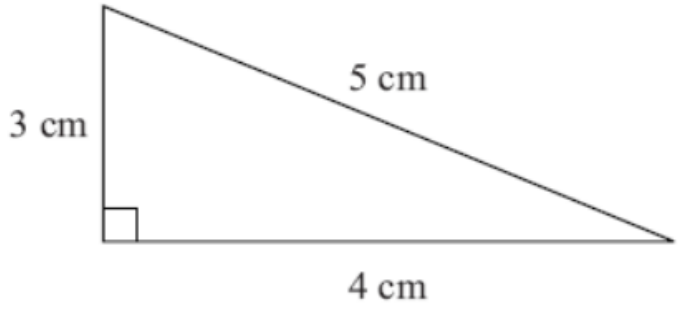
Try, here's the diagram what's the question?

In any triangle ABC

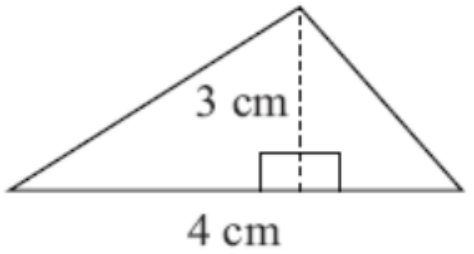
Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

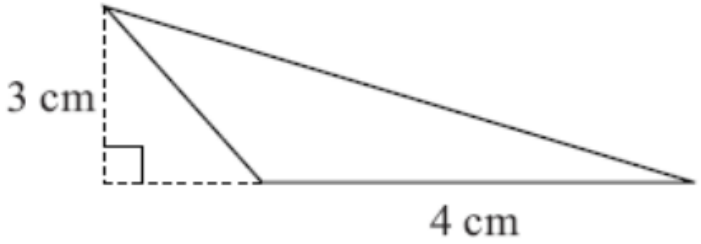
Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$



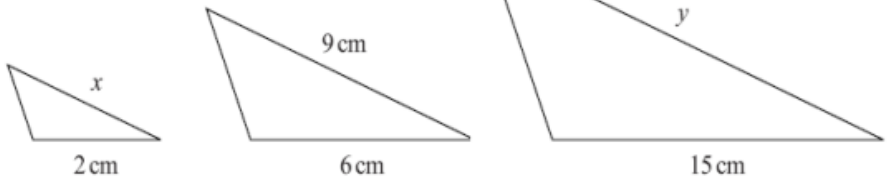
B

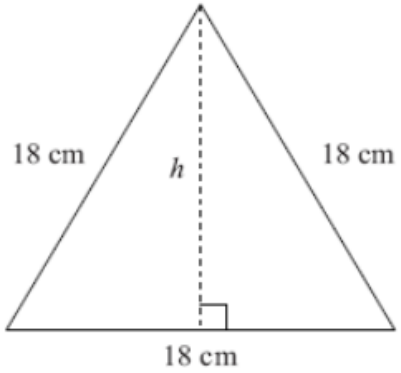


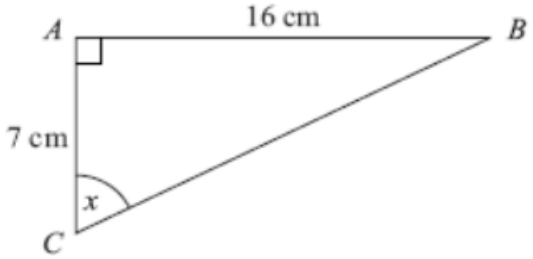
C

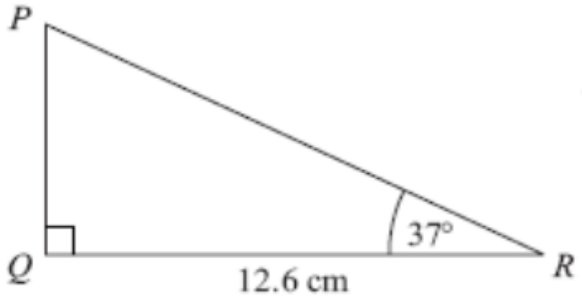


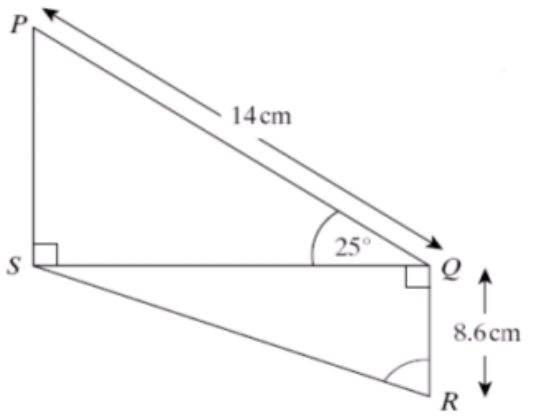
Not drawn accurately



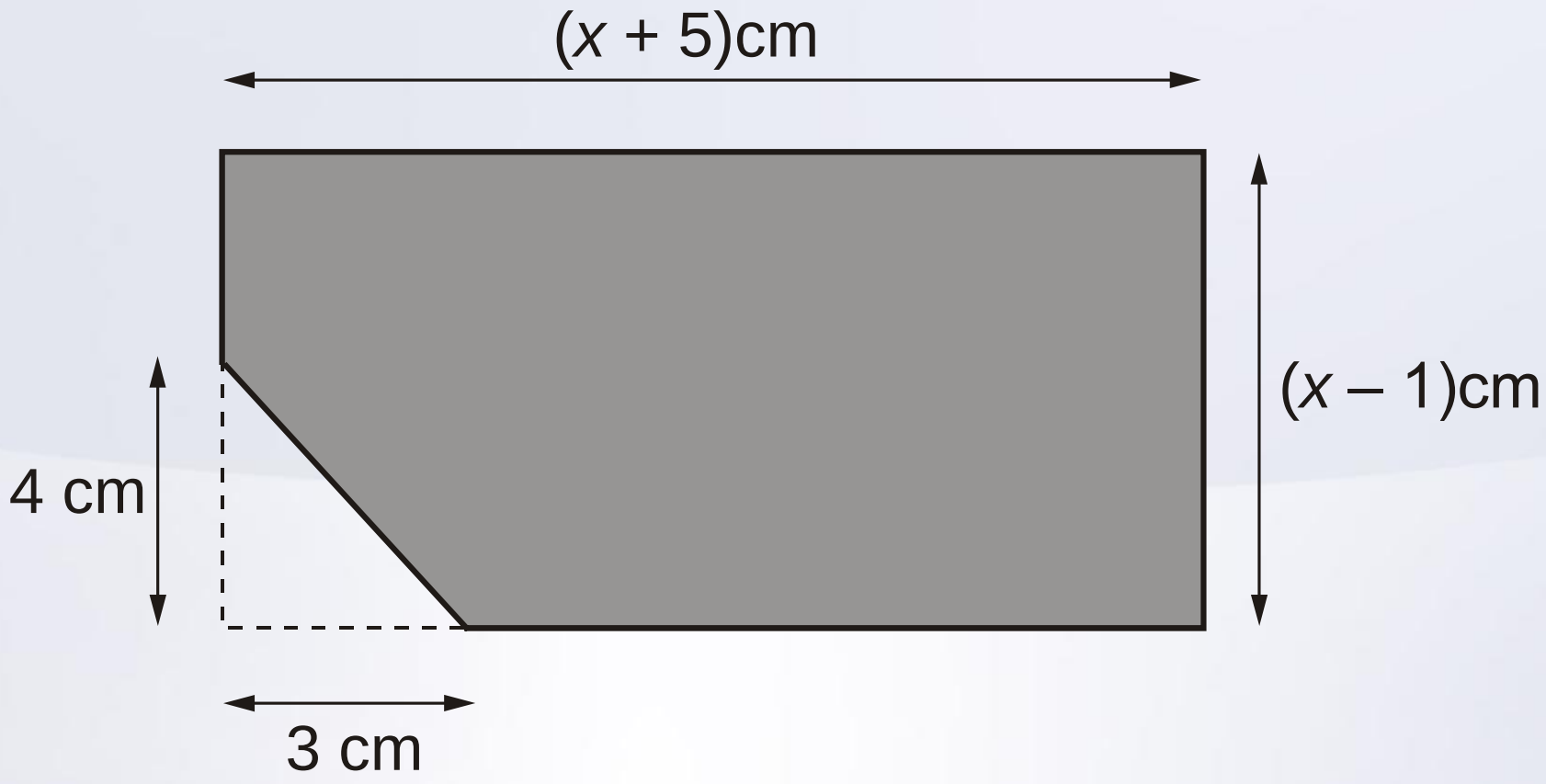






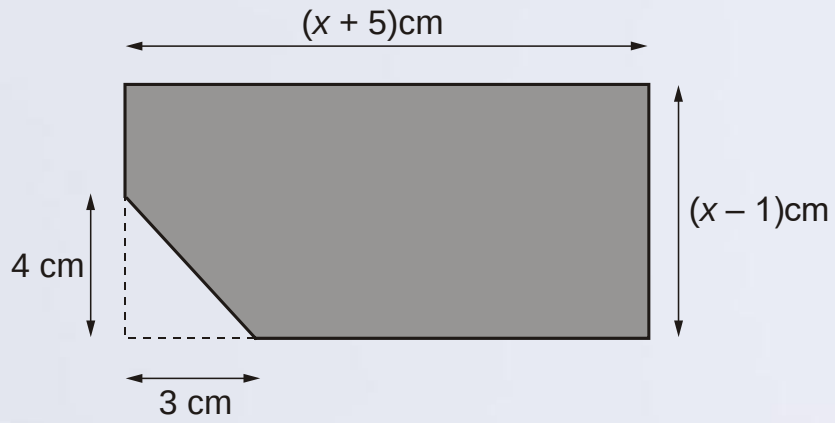


Problem 1



Question 1

A rectangle has length $(x + 5)$ cm and width $(x - 1)$ cm.
A corner is removed from the rectangle as shown.



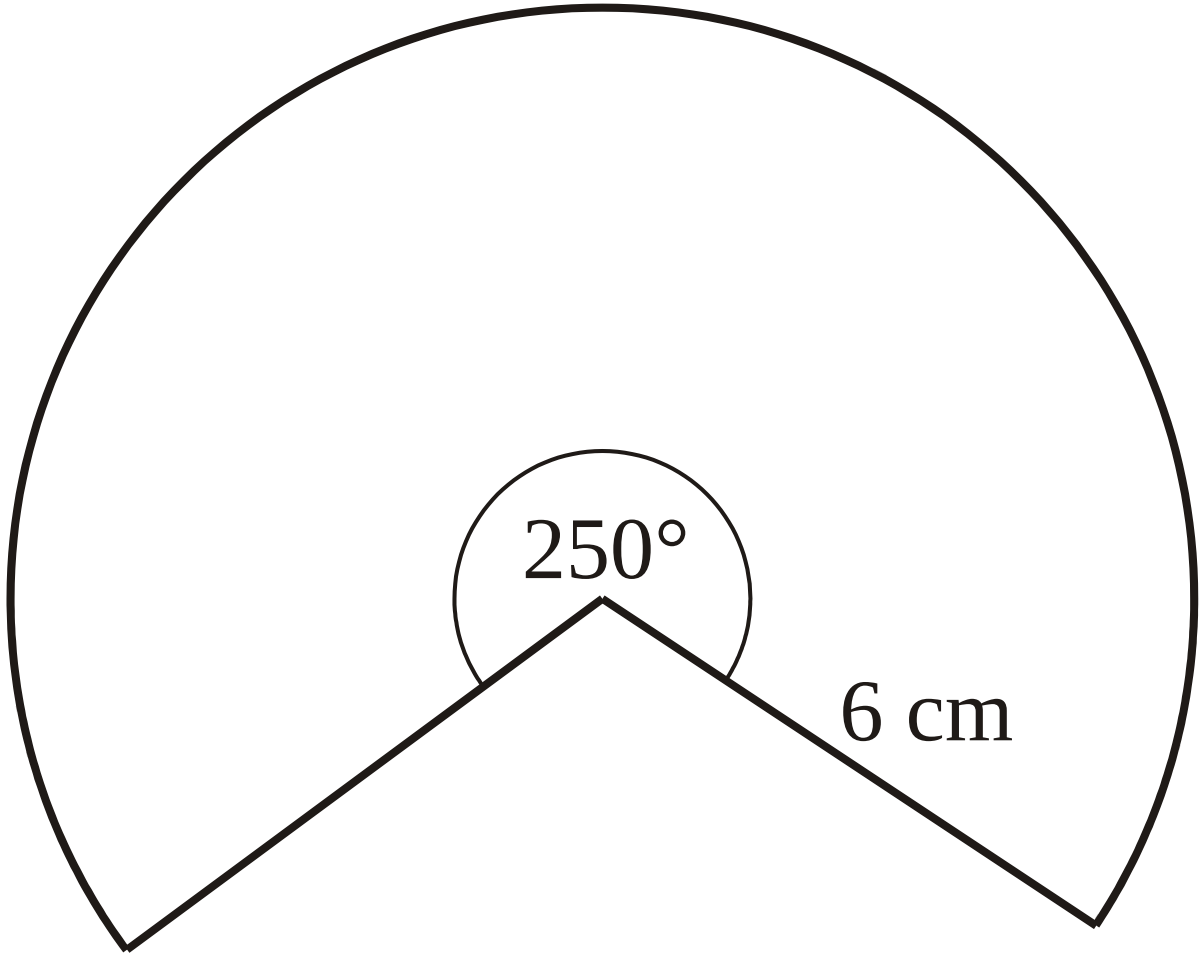
(a) Show that the shaded area is given by $x^2 + 4x - 11$.

(b) The shaded area is 59 cm^2.

(i) Show that $x^2 + 4x - 70 = 0$.

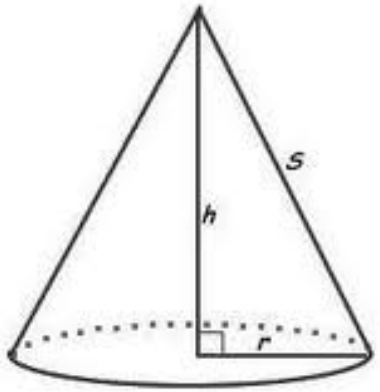
(ii) Calculate the value of x.

Problem 2

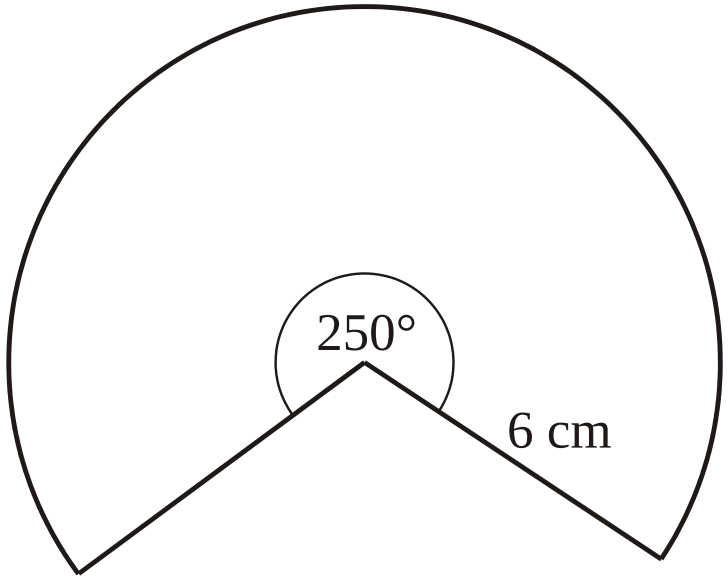


Question 2

The diagram shows the net of the curved surface of a cone.



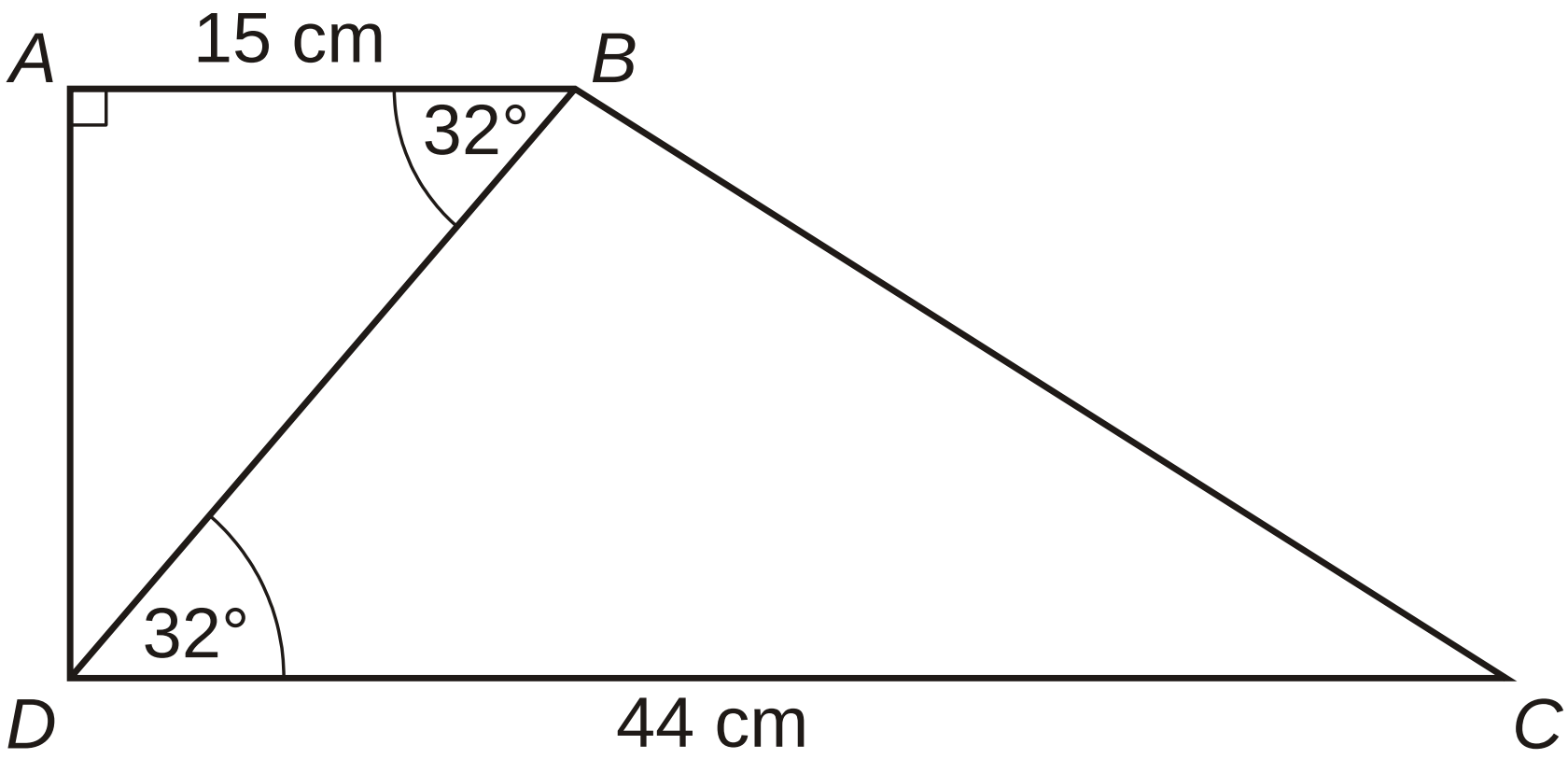
Volume of cone = $\frac{1}{3}\pi r^2 h$



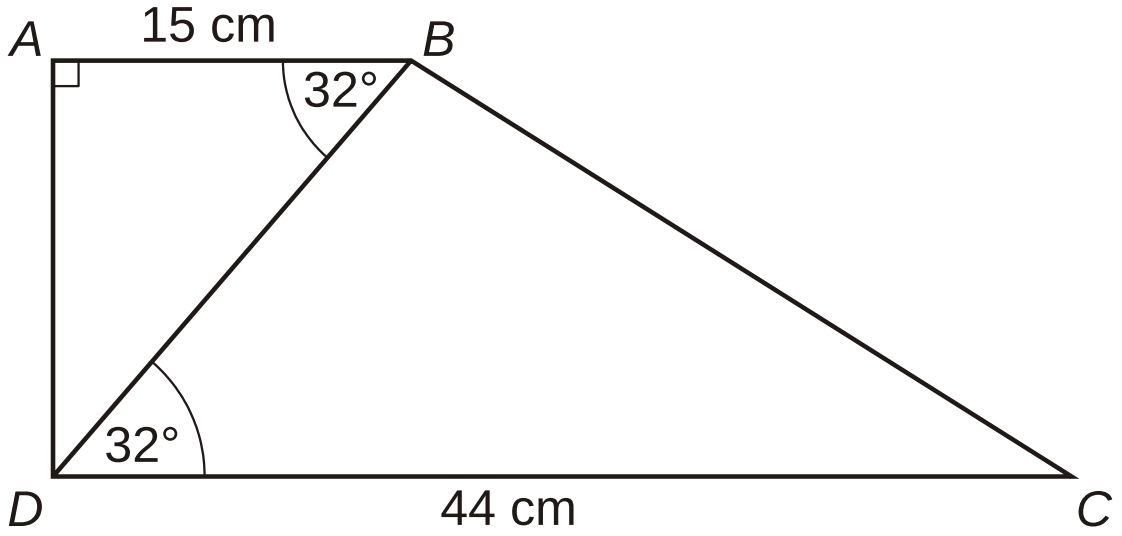
Not to scale

Work out the volume of the cone.

Problem 7



Question 7



ABCD is a trapezium.
Angle *BAD* = 90°.
Angle *BDC* = angle *ABD* = 32°
AB= 15cm and *DC*= 44cm.
Calculate the length of *BC*
Give your answer to a suitable degree of accuracy.

Algebra Snippets..

For a variation on here's the answer, what's the question?

Try, here's the diagram what's the question?

Or try Algebra Snippets

$$c = \frac{10(t-d)}{d}$$

What might the question be?

My brother is 5 years older than me.
The sum of our ages is 111 years.

w, x and y are three positive integers.

w is 20% of y.

x is one-sixth of y.

y is less than 100.

$4(x + 5) = 28$

$y^2 + 8y$

$x^4 \times x^4$

$x^4 \div x^4$

7 11 15 ....

$7x - 5 < 3x - 1$

$10x^2 - 15xy$

$3cd^5 \times c^2d$

$a(3x + 2) + b(4x - 5) \equiv 34x - 31$

$\sqrt{75} \times \sqrt{3}$

$\frac{\sqrt{18}}{\sqrt{2}}$

$y^2 = x + 3$

$y = x - 3$

The nth term of a sequence is $100 - 3n$.

$a = 10$, $b = 2$ and $c = -6$

$\frac{ab - c}{c + 4}$

$2x + 5$ $3x - 8$ $4x - 21$

p is a number.
12 is the highest common factor of 24 and p.

n is an integer and $-2.5 \leqslant n < 1$

In a sale the price of a computer is decreased by 30%.

The price in the sale is £560.

$\frac{10c^2 + 8cd}{(2ab^4)^3}$

$c = \frac{10(t - d)}{d}$

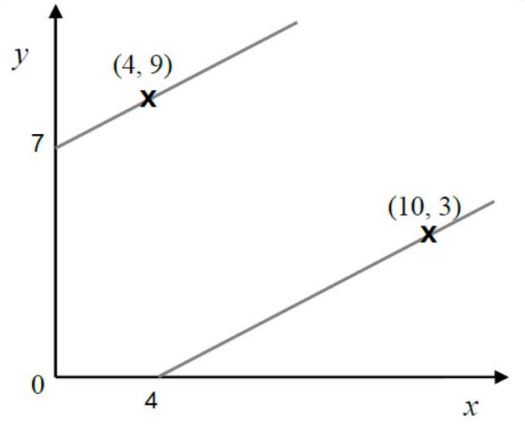
$\frac{3^0}{27^{\frac{1}{3}}}$ $4^{\frac{5}{2}}$

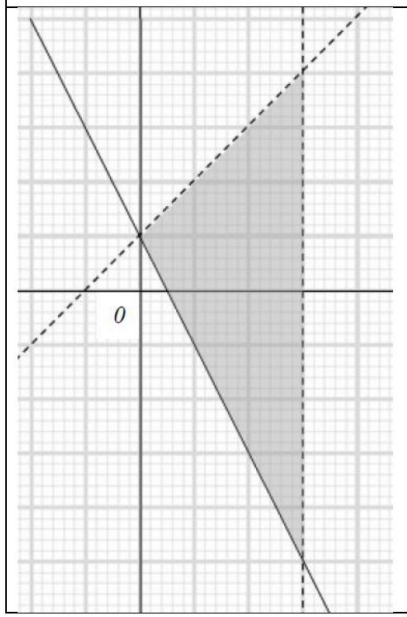
$y = 1 - 2x$

$y = x^2 - 3x - 5$

$r = \sqrt{y^2 - x^2}$

$y = 5\sqrt{2}$ and $x = \sqrt{6}$





Building blocks

Ab-surd!

Each line is a set of equivalent fractions. Fill in the blanks in the fractions to make each line complete, including the multiplier used to get from one fraction to the next.

(1) $\frac{1}{\sqrt{2}}\left(\times \_\_\_\_\right) = \frac{\sqrt{2}}{\_\_\_\_}\left(\times \_\_\_\_\right) = \frac{\sqrt{6}}{\_\_\_\_}\left(\times \_\_\_\_\right) = \frac{\_\_\_\_}{6}$

(2) $\frac{2}{5\sqrt{3}}\left(\times \_\_\_\_\right) = \frac{\_\_\_\_}{15}\left(\times \_\_\_\_\right) = \frac{2\sqrt{6}}{\_\_\_\_}\left(\times \_\_\_\_\right) = \frac{\_\_\_\_}{60}$

(3) $\frac{5}{2+\sqrt{2}}\left(\times \_\_\_\_\right) = \frac{10-5\sqrt{2}}{\_\_\_\_}\left(\times \_\_\_\_\right) = \frac{\_\_\_\_}{20+10\sqrt{2}}$

(4) $\frac{2-\sqrt{3}}{4}\left(\times \_\_\_\_\right) = \frac{\_\_\_\_}{8+4\sqrt{3}}\left(\times \_\_\_\_\right) = \frac{\_\_\_\_}{16}$

Thinking about Numbers

Building blocks

Ab-surd!

Teacher Notes

Printable version

Why use this resource?

This resource asks students to rationalise and de-rationalise a set of equivalent fractions. This offers plenty of practice at manipulating surds, alongside a chance to notice what multipliers are leading to rational denominators. This allows students the opportunity to draw conclusions about how to rationalise a fraction.

Possible approach

This task could be used as a way to introduce rationalising denominators or to review a student's understanding.The problem could be completed individually before giving students the opportunity to feedback to small groups or the whole class about the method they approached it with, what they have noticed, and any conjectures they have made.

Key questions

- How would you rationalise fractions in the following form: $\frac{a}{\sqrt{b}}$, $\frac{a}{b\sqrt{c}}$ and $\frac{a}{b+\sqrt{c}}$?
- Is there more than one way to rationalise a fraction?

Possible support

If students are having difficulty in moving from one fraction to the next, encourage them to use other fractions in the row to find the numerator (or denominator) and then work backwards to find the multiplier.

They can also check their work by simplifying the fractions at the end of the row and seeing whether they match one of the earlier fractions.

Rich example

Divide it up ☆

Use a multiplication grid to evaluate 23×17 and complete the complementary number statements on the right-hand side.

	10	7
20		
3		

$23 \times 17 = \ldots$

$\frac{\ldots}{17} = 23$

$\frac{\ldots}{23} = 17$

Use a multiplication grid to expand $(x + 4)(2x - 3)$ and complete the complementary statements on the right-hand side.

	x	$+4$
$2x$		
-3		

$(x + 4)(2x - 3) = \ldots$

$\frac{\ldots}{x+4} = 2x - 3$

$\frac{\ldots}{2x-3} = x + 4$

Polynomials & Rational Functions

Rich example

Divide it up

Printable/supporting materials Fullscreen mode Teacher notes

Warm-up | Problem | Things you might have noticed

Use the same thinking to complete the multiplication grid and statements below.

	x	-2
…	$4x^2$	
…		-12

$$(x-2)(\ldots) = 4x^2 - 2x - 12$$

$$\frac{4x^2 - 2x - 12}{x-2} = \ldots$$

$$\frac{4x^2 - 2x - 12}{\ldots} = x - 2$$

Next

Rich example

Divide it up

Printable/supporting materials Fullscreen mode Teacher notes

Warm-up | Problem | Things you might have noticed

What do you notice about the two problems below?

$$\frac{6x^3 + 13x^2 + 9x + 2}{3x + 2} = \ldots$$

	$\ldots x^2$	$\ldots x$	$\ldots$
$3x$			
$+2$			

How can you use the multiplication grid on the right-hand side to find the result of dividing $6x^3 + 13x^2 + 9x + 2$ by $3x + 2$?

Key questions

- What type of answer are you expecting when you carry out this division / multiplication?
- Do you expect there to be a remainder?
- In what order could you fill in the missing values in the multiplication grid? Do you have any choice?
- Which values did you find easiest / more difficult to complete?

Rich example

Divide it up

What do you notice about the two problems below?

Key questions

- What type of answer are you expecting when you carry out this division / multiplication?
- Do you expect there to be a remainder?
- In what order could you fill in the missing values in the multiplication grid? Do you have any choice?
- Which values did you find easiest / more difficult to complete?

✱ How can you use the multiplication grid on the right-hand side to find the result of dividing $6x^3 + 13x^2 + 9x + 2$ by $3x + 2$?

What stays the same and what changes, if the division on the left-hand side is $\frac{6x^3 + 13x^2 + 9x + 5}{3x + 2} = \ldots$ instead?

Can you use a similar approach to divide $4x^4 + 3x^3 + 2x + 1$ by $x^2 + x + 2$? Are you convinced that taking this approach gives you the same result as other methods?

Rich example

Divide it up

What do you notice about the two problems below?

$$\frac{6x^3 + 13x^2 + 9x + 2}{3x + 2} = \ldots$$

	$\ldots x^2$	$\ldots x$	$\ldots$
$3x$			
$+2$			

- How can you use the multiplication grid on the right-hand side to find the result of dividing $6x^3 + 13x^2 + 9x + 2$ by $3x + 2$?

First let's think about the fraction on the left-hand side.

What type of result do you expect if this division is carried out?

The multiplication grid on the right-hand side also features the expression $3x + 2$ but is set up to have a quadratic expression along the top.

Rich example

Divide it up

What type of polynomial do you expect to find inside the multiplication grid?

If we think about the division and the multiplication grid simultaneously, we can begin to fill in some of the missing values. In the grid below we suggest the order in which you might choose to complete values.

	$(2)x^2$	$(5)x$	(8)
$3x$	(1)	(4)	(7)
$+2$	(3)	(6)	(9)

- Why might we complete the value in the top left box of the grid first?
- In each case try to justify why that value could be found next and try to fill it in.
- Which values do you find more tricky to complete? Why?

Rich example

Divide it up

Completed grid

	$2x^2$	$3x$	1
$3x$	$6x^3$	$9x^2$	$3x$
$+2$	$4x^2$	$6x$	2

Can you now write down any number statements that this grid might represent?

How do we now bring all of our thinking together to complete the division $\frac{6x^3 + 13x^2 + 9x + 2}{3x + 2} = \ldots?$

Polynomials & Rational Functions

Rich example

Divide it up

Teacher Notes

Printable version

Why use this resource?

This resource is designed to help students to make links between multiplication and division of polynomials using multiplication grids.

The warm-up is carefully designed to draw on students' prior knowledge, possibly from as far back as their Primary education. They will be reminded of how the grid method for multiplication can be used in a variety of situations and that working forwards and backwards through a problem can be helpful.

Preparation

It is helpful if students are familiar with using the grid method for multiplication of two numbers.

Possible approach

You might like to begin by giving students an opportunity to work on the warm-up problems individually for a short time before introducing the main problem. The three warm-up problems are closed and build on each other so it is important that students do consider each one before moving to the next (even if they think they look easy!).

When students are invited to think about the main problem it could be helpful to allow them to see all three of the questions posed at once. This will encourage them to make links between the multiplication grid and a variety of situations as well as allowing them to work at their own pace.

Key questions

- What type of answer are you expecting when you carry out this division / multiplication?
- Do you expect there to be a remainder?
- In what order could you fill in the missing values in the multiplication grid? Do you have any choice?
- Which values did you find easiest / more difficult to complete?

Many ways problem

Two-way algebra ☆

Each column and row has a property which some equation or inequality may or may not have. If an example has the properties of the corresponding row and column, then it can appear in that cell.

We have omitted some headings, and some entries in cells. Can you complete the table?

		Solutions lie between 1 and 6		All values of x are solutions
	$x^2 + 4 = 0$	$5x - 3 = 9 - x$		
Inequalities			$8 - x < 2 - 3x$	
	$y = 3x$ and $\frac{y}{3} - 1 = x$		$y = 3x + 5$ and $y = -x - 3$	

- Did you have any choice about the row and column headings?
- Can you simplify any of your examples?
- Do all your examples require some 'solving' to check they fit the attributes of the cell? If not, can you make it so that they all do?
- If we required all the cells to contain quadratics, would it still be possible to fill all the cells?

Two-way algebra

Why use this resource?

Two-way algebra encourages students to appreciate the different types of solutions that various equations and inequalities can give. **Students might have to work backwards from a solution to construct their answers, so there is an opportunity to think carefully about the structure of equations and inequalities.** It may also help to develop the thinking of students who have common misconceptions, such as, that a pair of simultaneous equations will always have one solution, or that a quadratic will always have two (real) solutions.

Many ways problem

Two-way algebra ☆

Here is one possible solution.

	No real solutions	Solutions lie between 1 and 6	Negative solutions only	All values of x are solutions
Equations in x	$x^2 + 4 = 0$	$5x - 3 = 9 - x$	$2x - 5 = -12$	$(x-1)^2 - 1 = x^2 - 2x$
Inequalities	$x^2 < 0$	$5 < 2x - 3 < 15$	$8 - x < 2 - 3x$	$x^2 \geq 0$
Simultaneous equations in x and y	$y = 3x$ and $\frac{y}{3} - 1 = x$	$3y = x$ and $y = x - 2$	$y = 3x + 5$ and $y = -x - 3$	$y = 2x$ and $\frac{y}{2} = x$

Did you have any choice about the row and column headings?

We have put the first row as 'Equations in x', so examples all have a single variable only. How is this different from just writing 'Equations'? Could simultaneous equations appear here if this was your row heading?

Can you simplify any of your examples?

Instead of giving the equation $5x - 3 = 9 - x$, which has a solution between 1 and 6, we could have written $x = 2$. This is still an equation, albeit a very simple one, that has the properties required. Is it possible to give all the examples in this form, so you can immediately see whether they have the attributes of their cell?

Do all the examples require some 'solving' to check they have the attributes of the cell? If not, can you make it so that they all do?

If you gave an answer such as $x = \sqrt{-4}$ for an equation that has no real solutions, you can easily see that it matches the properties of the cell. However, you could have written it as $x^2 + 4 = 0$, so that it requires some manipulation to check it is correct.

If we required all the cells to contain quadratics, would it still be possible to fill all the cells?

It might be best to try to sketch what the answers might look like graphically to help you decide if they are all possible.

Two-way algebra

Key questions

Alongside the questions in the resource, you might ask your students to reflect on:

- Which cells were the easiest to fill in?
- Which cells were the most challenging to fill in?

Possible support

Students may have difficulty working with simultaneous equations with no solutions. Encouraging them to think of the graphs of the equations may help them to understand their answers.

The final column will perhaps be the most unfamiliar idea for the students. They could be asked to leave this out initially, so it can be discussed and completed as a class.

Possible extension

The final question, asks students to decide whether it is possible to fill all the cells with quadratics. This could be done by students sketching graphs to show which they think are possible, or they could try and re-do the table and come up with new examples only using quadratics. It might be a good task to return to at a later date, perhaps as a starting point for teaching quadratic inequalities.

Many ways problem

Can you find... cubic edition ☆

Problem | Explore | Things you might have tried

Can you find a cubic curve that...

(a) ... passes through the x-axis at $x = 1$ and $x = -1$?

(b) ... passes through the origin and touches the x-axis at $x = -3$?

(c) ... touches the x-axis at $x = 2$ and crosses the y-axis at 12?

(d) ... crosses the y-axis at -6 and has three integer roots?

(e) ... crosses the y-axis at $y = 5$ and touches the x-axis at $x = 1$?

Are any of the curves described above unique?

Many ways problem

Can you find... cubic edition

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Problem Explore Things you might have tried

In this section we have provided some tools to help you explore cubic curves. You should try to get as far as you can with the problem *before* using the tools. For example,

- If you have found examples of cubic curves that satisfy the criteria, can you generalise your examples?
- If you haven't managed to come up with an equation for an example, have you tried to sketch one or more examples of cubic curves that satisfy the criteria?
- If you have been able to express the criteria in algebra, have you thought about what else you need to know to find an example that meets these criteria?

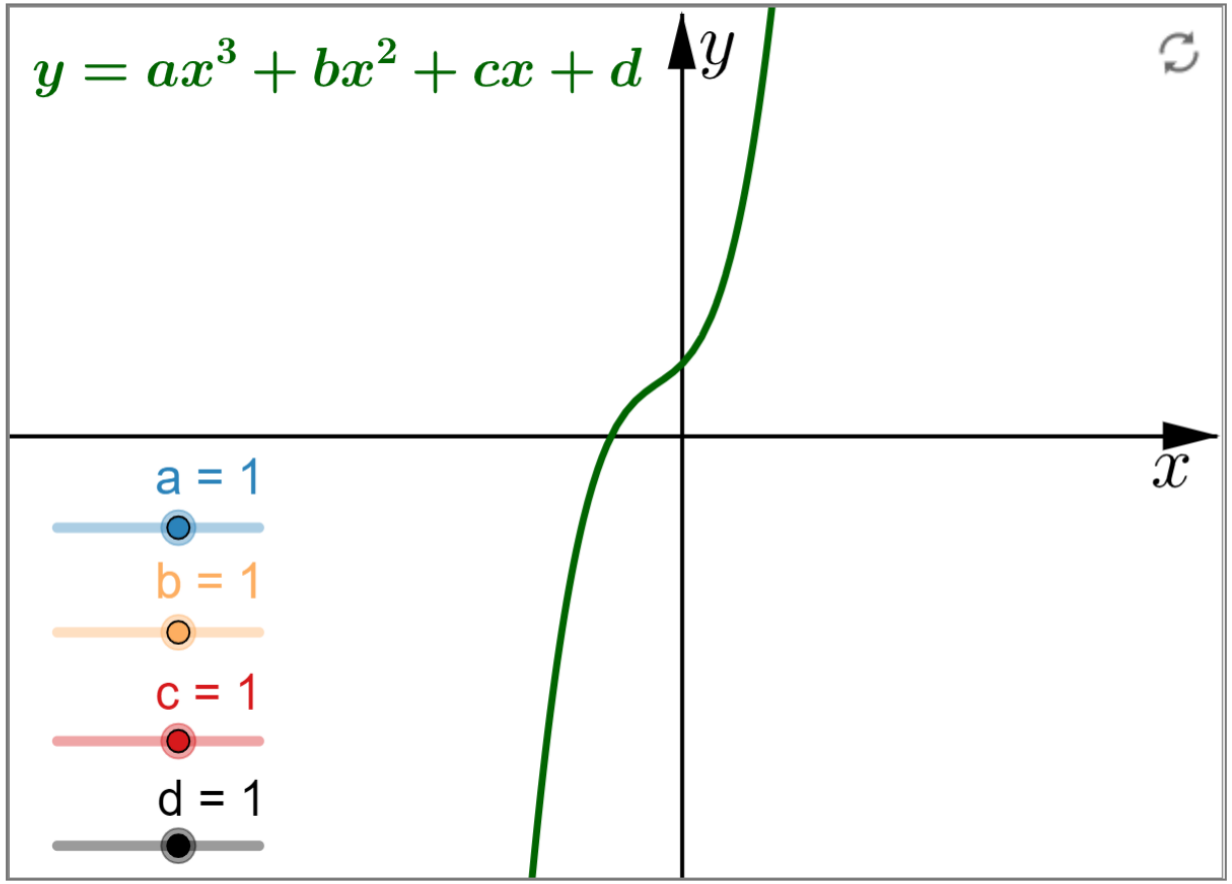
Thinking about cubics

- If you know $f(x)$ is a cubic function of x, what can you say algebraically about $f(x)$?
- What do you know geometrically about the graph of $f(x)$? Sketch some different shapes that can cubic graphs be.
- How many pieces of information do we need to define a cubic curve? In what form might this information be given?

Exploring coefficients and graphs

One of the standard ways to write a cubic in one variable is $ax^3 + bx^2 + cx + d$. Use the sliders for a, b, c and d to explore what happens when you vary the coefficients.

- What different shapes the can the graph of $y = ax^3 + bx^2 + cx + d$ be?
- How many roots can a cubic have?



Download GeoGebra file

Polynomials & Rational Functions

Many ways problem

Can you find... cubic edition

Teacher Notes

Printable version

Why use this resource?

This resource invites students to identify cubic curves given certain properties. It develops the link between the geometrical and algebraic properties of cubics, as well as looking at the general form for cubic equations.

Possible approaches

If this is used before students are familiar at working with cubic equations, this could be done as a sketching exercise. For each question students could see how many different possible sketches can they find for each curve.

When students are more aware of the links between graphs and equations, we have seen that they often write down possible equations without thinking about the sketch. They should be encouraged to sketch the graphs, which makes it easier to recognise different possibilities, and to generalise their equations.

Key questions

- How many different possible sketches are there for each question?
- What is the general form for a cubic equation when it has three/one/repeated roots?
- Are any of the curves described unique?

Possible support

We have provided the Explore page which could be used by students who are finding the problems challenging. It has GeoGebra applets and questions that ask students to make links between the geometric properties of cubics and their equations. For example, it looks at what impact the coefficients have when the cubics are written in different forms.

Review question

Can we find the three inequalities that define this region?

Ref: R8408

In Fig. 1 (not drawn to scale) the curve has equation $x^2 + 4y^2 = 16$. The straight line has a gradient of -2 and passes through the intersection of the curve with the positive y-axis.

Write down the three inequalities which are sufficient to define the shaded area (including the three lines bounding it).

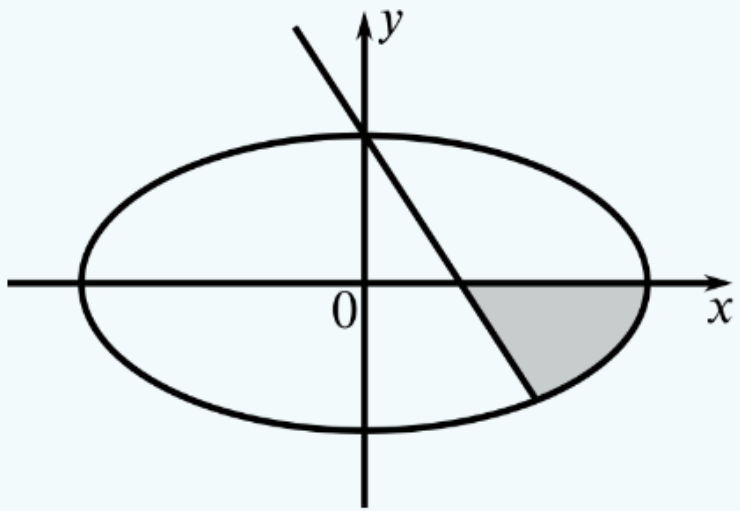


Fig. 1

Statistics - Problems

AQA

Mean Set

A set of five numbers has:

A mode of 12

A median of 11

A mean of 10

What could the numbers be?

Mean Street

Three numbers have a mean of 23

Two of the numbers have a mean of 12

Two of the numbers have a mean of 30

What are the three numbers?

Statistics – Problems

AQA

Mean Set

A set of five numbers has:

A mode of 12

A median of 11

A mean of 10

What could the numbers be?

Mean Street

Three numbers have a mean of 23

Two of the numbers have a mean of 12

Two of the numbers have a mean of 30

What are the three numbers?

Mean Set	5, 10, 11, 12, 12; other possibilities based upon (in ascending order) a, b, 11, 12, 12 where $a + b = 15$ and $a \neq b$
Mean Street	9, 15, 45 (in any order)

Learn it both ways!

Learn everything from right to left as well as left to right!

If you know your laws of indices you should recognise that $x^{10} = x^1 \times x^9$

For every derivative you know, you also know an integral!

.....and so on...and on....and on!

Laws of logs

$$\log x + \log y = \log xy$$

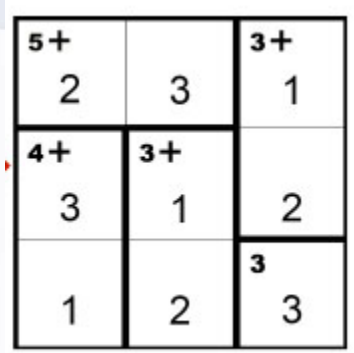
$$\log x - \log y = \log \frac{x}{y}$$

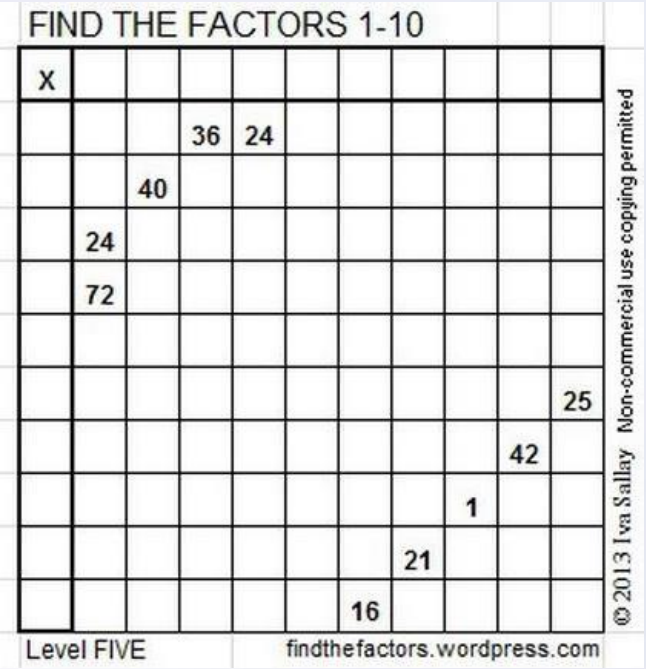
$$n\log x = \log x^n$$

Learn everything from right to left as well as left to right!

Puzzles

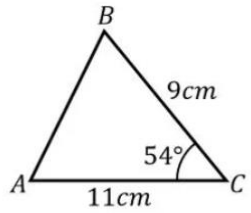
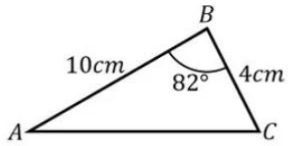
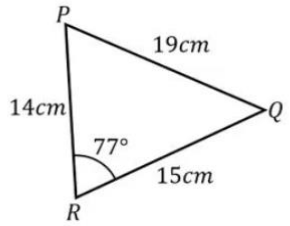
- Sudoku, see these Nrich resources
- Ken Ken
- Find the Factors





OCT 10 2021

Fill in the blanks …

Shape	Calculation	Answer
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad °$	$Area = \qquad cm^2$
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad °$	$Area = \qquad cm^2$
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad °$	$Area = \qquad cm^2$

Trigonometry (Area) – Fill In The Blanks, Andy Lutwyche

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