Colour in Mathematics

The use of **colour** to add clarity

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Mathematics, Learning and Technology

Autograph – Web Version

From The Complete Mathematics Conference, I really enjoyed Maths Conference #26, July 2021. The highlight for me (apart from very much enjoying my own session presenting with AQA on the New **GCSE Maths tests from Exampro** which been developed for the two-year AQA key stage 4 scheme of work) had to be the first session of the day I attended – a first look at the web version of Autograph.

I have also added this information to a **new page** in the **Autograph** section of my **Use of Technology** series.

A **web version for Autograph** is now available, a game-changer for this sophisticated object-based dynamic geometry system. It has a great deal of functionality already and will be developed even further. I am so pleased to have the option now of sharing pages easily with students. The interface is intuitive. Select the various options to see all the functionality available. Add some points or an equation and experiment! I really like all the style options in Autograph making it possible to create attractive resources. And look at all those colours! That certainly appeals to me with my interest in using colour in Mathematics to add clarity to explanations. Note the colours of the points I have used in my reflection example below. This is just my own first look, I will be learning much more.

Featured Posts

- Rosenshine's Principles in the Mathematics Classroom
- Knowledge Organisers – Mathematics
- The Standards Unit – Mathematics
- Wisweb Applets HTML5
- Arithmagons
- PowerPoint Collection
- Colour in Mathematics

Search …

Algebra – Like terms

$$6a+4b+2a-5b$$

$$= 8a-b$$

The use of colour helps emphasise which sign is associated with each term

$$(2\sqrt{3}+4)(2-\sqrt{3})$$

$$4\sqrt{3} - 2\sqrt{9} + 8 - 4\sqrt{3}$$

$$4\sqrt{3} - 6 + 8 - 4\sqrt{3}$$

$$= 2$$

$$\frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \Rightarrow \frac{2(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$\frac{2\sqrt{3}+2}{3+\sqrt{3}-\sqrt{3}-1} \Rightarrow \frac{2+2\sqrt{3}}{2}$$

Order of Operations

$6 - 5 + 2 - 2$

$8 - 7$

1

$4 + 2 \times 3$

$4 + 6$

10

The use of colour helps emphasise which sign is associated with each term. We can also emphasise operations.

Order of Operations

a) $4 \times 2 + 3$

b) $20 \div 5 - 3$

c) $4 + 2 \times 3$

d) $20 - 15 \div 3$

e) $20 \div 4 + 3 \times 2$

f) $5 \times (3 + 4) - 9$

g) $3 + 2^2$

h) $(3 + 2)^2$

i) $(3 + 4 \times 2)^2$

j) $3 + 4 \times 2^2$

k) $(3 + 1)^2 + 2 \times 5$

l) $(3 + 1 + 2 \times 5)^2$

SMART Board

Graspable Math

Order of Operations

insert | transform | keypad | scrub | draw | erase | arrange | undo | redo | smaller | larger

$$4\times 2+3$$
$$8+3$$
$$11$$

$$4+2\times 3$$
$$4+6$$
$$10$$

$$5\times(3+4)-9$$
$$5\times 7-9$$
$$35-9$$
$$26$$

$$3+2^2$$
$$3+4$$
$$7$$

$$(3+2)^2$$
$$5^2$$
$$25$$

$$(3+1)^2+2\times 5$$
$$4^2+2\times 5$$
$$4^2+10$$
$$16+10$$
$$26$$

$$(3+4\times 2)^2$$
$$(3+8)^2$$
$$11^2$$
$$121$$

$$3+4\times 2^2$$
$$3+4\times 4$$
$$3+16$$
$$19$$

Examples from
Increasingly Difficult Questions - @TAYLORDA01 (weebly.com)
See Order of Operations

Graspable Math

Order of operations - addition and subtraction (Colleen Young)

insert | transform | keypad | scrub | draw | erase | arrange | undo | redo | smaller | larger

Order of Operations Examples

$$6-2+1$$

Addition and subtraction have equal precedence, so these operations could be done in either order.

$6-2+1$

Note that we have $+6$ (conventionally we do not write a positive sign in front of the first term). -2 and $+1$.

$$6-2+1$$
$$4+1$$
$$5$$

Here the subtraction is done first.

$6-2+1$

$4+1$

5

$$6-2+1$$
$$6-1$$
$$5$$

Here the addition is done first.

$6-2+1$

$6-1$

5

Note that we could reorder the numbers.

$$6-2+1$$
$$6+1-2$$
$$7-2$$
$$5$$

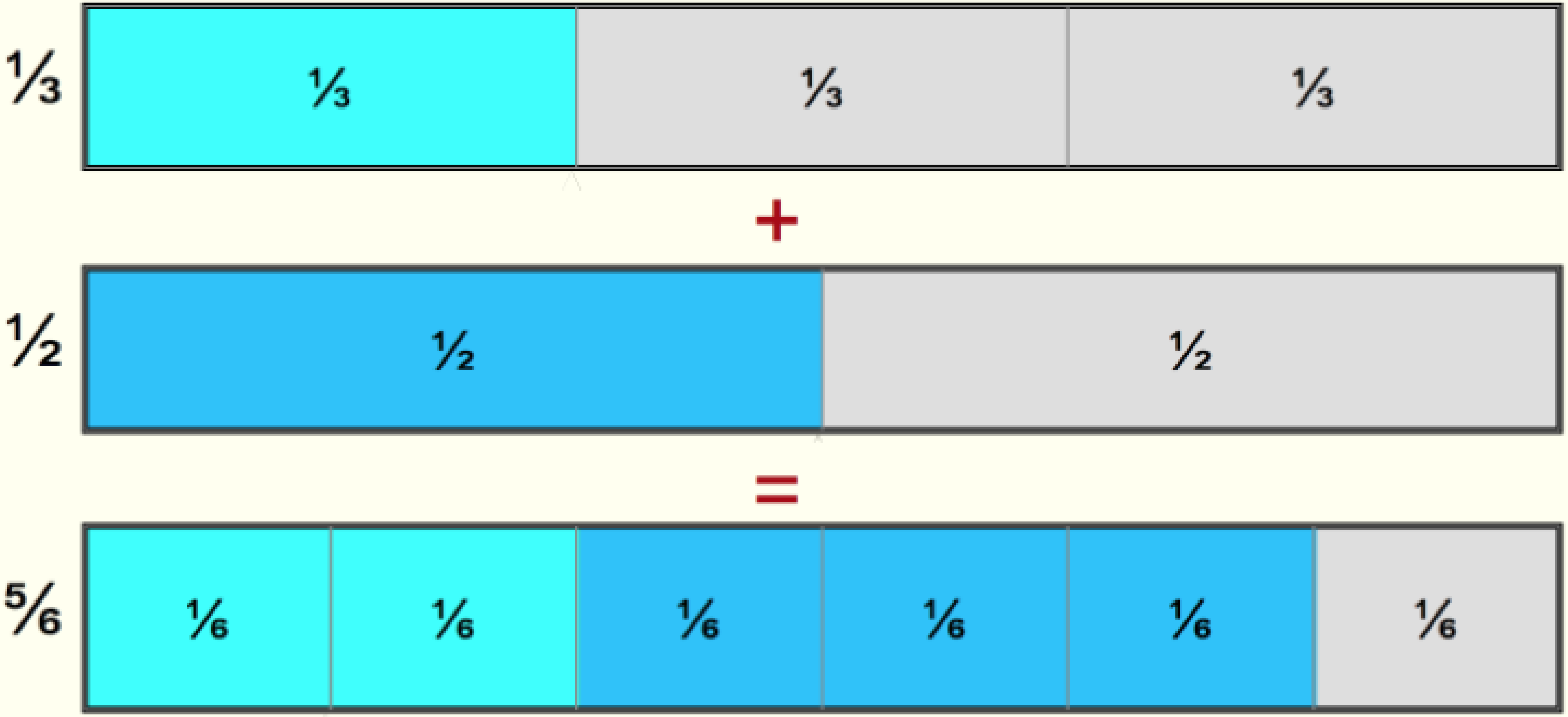
$6-2+1$

is the same calculation as

$6+1-2$

Try some examples yourself:
https://graspablemath.com/canvas/?load=_8622dbc838fbc839

The Mathenæum - Ken Wessen



Algebra – equating coefficients

$2(x + 16) + 4(x - 5)$ simplifies to $a(x + b)$

Work out the values of a and b.

$$2(x+16) + 4(x-5) = a(x+b)$$

$$2x+32 + 4x-20 = ax+ab$$

$$6x+12 = ax+ab$$

$$6=a \quad 12=ab$$

$$12=6b$$

$$a=6 \quad b=2$$

$2x + 32$ or $4x - 20$	M1	Accept $ax + ab$ for M1
$6x + 12$ or $6(x + 2)$	A1	
$a = 6$ and $b = 2$	A1 ft	ft from their $6x + 12$ if M1 earned SC2 $a = 6$ and $b = 12$ SC1 $a = 6$

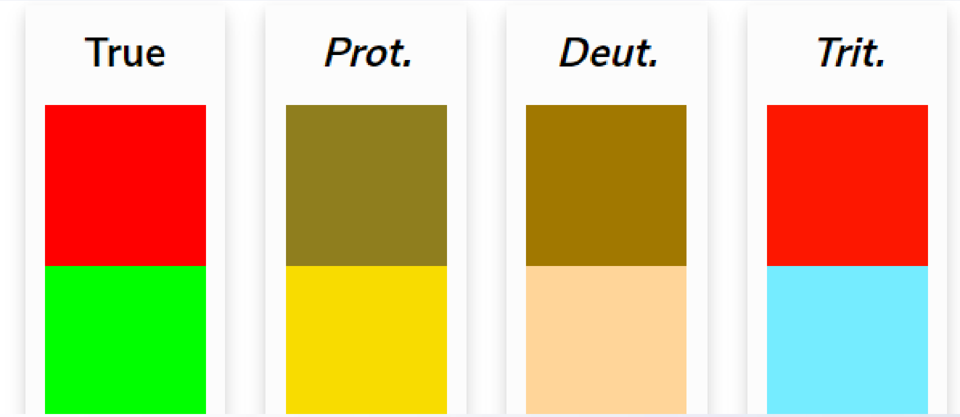
Colour combinations?

$6a+4b+2a-5b$

$= 8a-b$

$6a+4b+2a-5b$

$= 8a-b$



Colour combinations?

$6a+4b+2a-5b$

$= 8a-b$

Colour combinations?

Avoid

red	green
green	brown
green	blue
blue	grey
blue	purple
green	grey
green	black

A Model for **Great Teaching**

<u>Great Teaching</u> <u>Toolkit</u>
<u>Evidence Review June 2020</u>

04 Activating hard thinking

1 Structuring: giving students an appropriate sequence of learning tasks; signalling learning objectives, rationale, overview, key ideas and stages of progress; matching tasks to learners' needs and readiness; scaffolding and supporting to make tasks accessible to all, but gradually removed so that all students succeed at the required level

2 Explaining: presenting and communicating new ideas clearly, with concise, appropriate, engaging explanations; connecting new ideas to what has previously been learnt (and re-activating/checking that prior knowledge); using examples (and non-examples) appropriately to help learners understand and build connections; modelling/demonstrating new skills or procedures with appropriate scaffolding and challenge; using worked/part-worked examples

3 Questioning: using questions and dialogue to promote elaboration and connected, flexible thinking among learners (e.g., 'Why?', 'Compare', etc.); using questions to elicit student thinking; getting responses from all students; using high-quality assessment to evidence learning; interpreting, communicating and responding to assessment evidence appropriately

4 Interacting: responding appropriately to feedback from students about their thinking/knowledge/understanding; giving students actionable feedback to guide their learning

5 Embedding: giving students tasks that embed and reinforce learning; requiring them to practise until learning is fluent and secure; ensuring that once-learnt material is reviewed/revisited to prevent forgetting

6 Activating: helping students to plan, regulate and monitor their own learning; progressing appropriately from structured to more independent learning as students develop knowledge and expertise

01 Understanding the content

1 Having deep and fluent knowledge and flexible understanding of the content you are teaching

2 Knowledge of the requirements of curriculum sequencing and dependencies in relation to the content and ideas you are teaching

3 Knowledge of relevant curriculum tasks, assessments and activities, their diagnostic and didactic potential; being able to generate varied explanations and multiple representations/analogies/examples for the ideas you are teaching

4 Knowledge of common student strategies, misconceptions and sticking points in relation to the content you are teaching

17 Principles of Effective Instruction

1 Begin a lesson with a short review of previous learning.

2 Present new material in small steps with student practice after each step.

3 Limit the amount of material students receive at one time.

4 Give clear and detailed instructions and explanations.

5 Ask a large number of questions and check for understanding.

6 Provide a high level of active practice for all students.

7 Guide students as they begin to practice.

8 Think aloud and model steps.

9 Provide models of worked-out problems.

10 Ask students to explain what they have learned.

11 Check the responses of all students.

12 Provide systematic feedback and corrections.

13 Use more time to provide explanations.

14 Provide many examples.

15 Reteach material when necessary.

16 Prepare students for independent practice.

17 Monitor students when they begin independent practice.

Algebra – expanding brackets

$$(2x + 3)(5x + 2)(4x + 1)$$

$$(10x^2 + 15x + 4x + 6)(4x + 1)$$

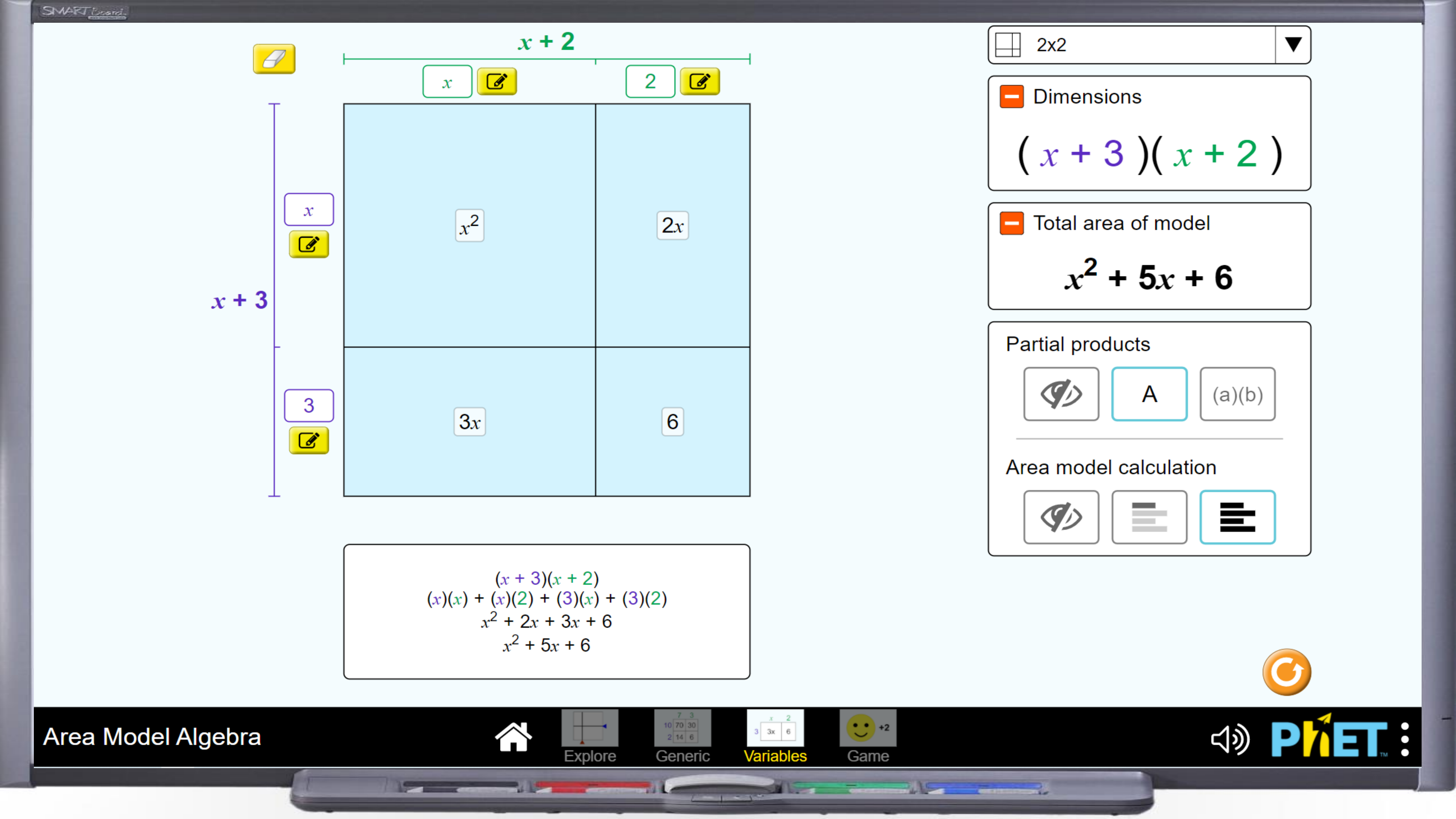
$$(10x^2 + 15x + 4x + 6)(4x + 1)$$

$$(10x^2 + 19x + 6)(4x + 1)$$

$$(40x^3 + 76x^2 + 24x + 10x^2 + 19x + 6)$$

$$(40x^3 + 76x^2 + 24x + 10x^2 + 19x + 6)$$

$$(40x^3 + 86x^2 + 43x + 6)$$



x + 2
x
2
x + 3
x
3
x^2
2x
3x
6
(x + 3)(x + 2)
(x)(x) + (x)(2) + (3)(x) + (3)(2)
x^2 + 2x + 3x + 6
x^2 + 5x + 6
2x2
Dimensions
(x + 3)(x + 2)
Total area of model
x^2 + 5x + 6
Partial products
A
(a)(b)
Area model calculation
Area Model Algebra
Explore
Generic
Variables
Game
PhET

Algebra – Factorisation

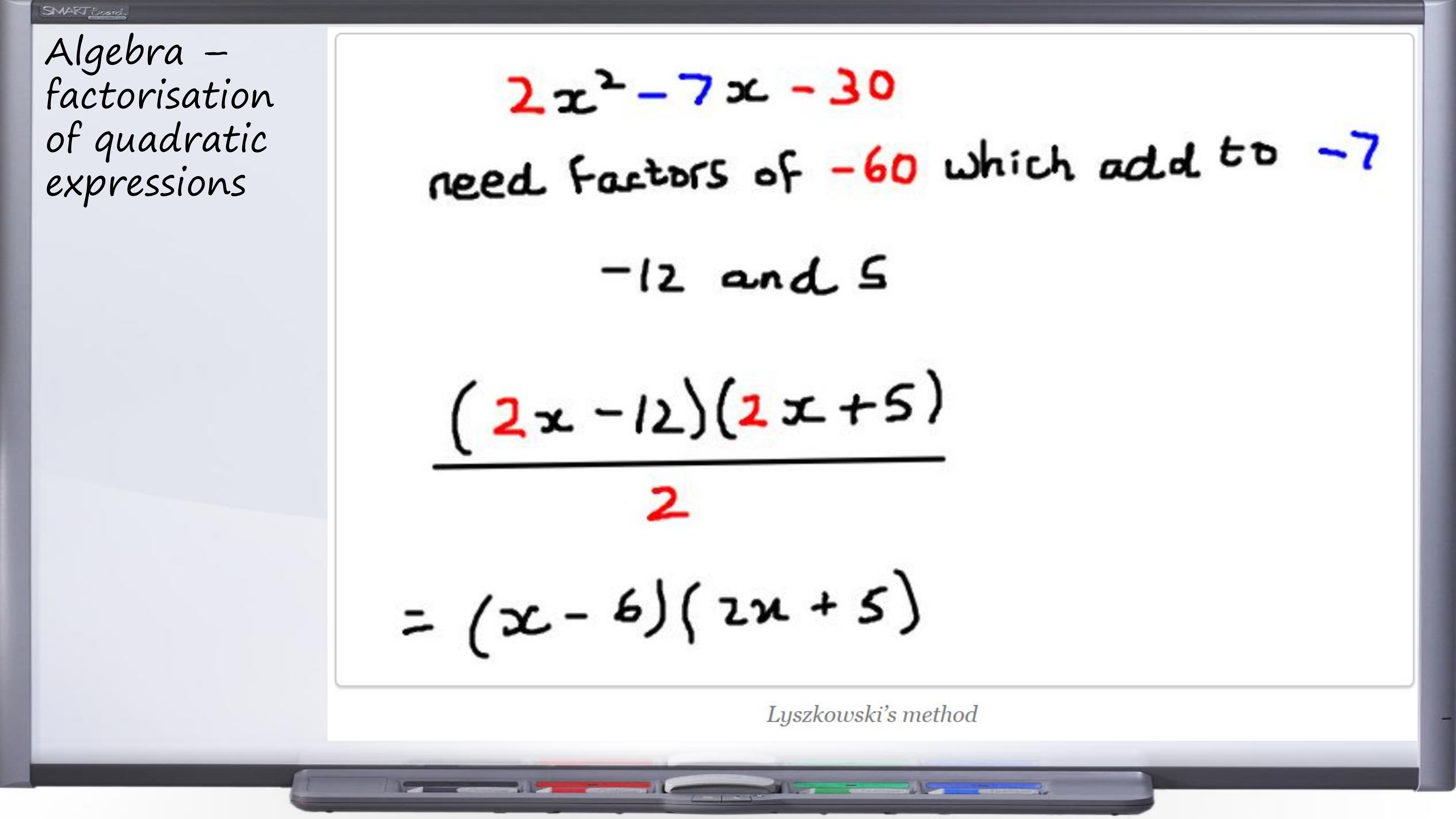
$$5x^2y + 15xy^2$$

$$5xxy + 5 \times 3xyy$$

$$5xy(x + 3y)$$

Students often find this method helpful

Algebra – factorisation of quadratic expressions



Lyszkowski's method

Algebra – Composite Functions

$f(x) = 2x^2$, $g(x) = x+3$

(a) Write down fg(4)

(b) Write down gf(4)

Solution

(a) fg(4) is f(g(4)) so first we must work out g(4)

$g(4) = 4 + 3 = 7$

So $f(g(4)) = f(7) = 2 \times 7^2 = 2 \times 49 = 98$

Algebra – Composite Functions

$f(x) = 2x^2$, $g(x) = x+3$

(a) Write down fg(4)

(b) Write down gf(4)

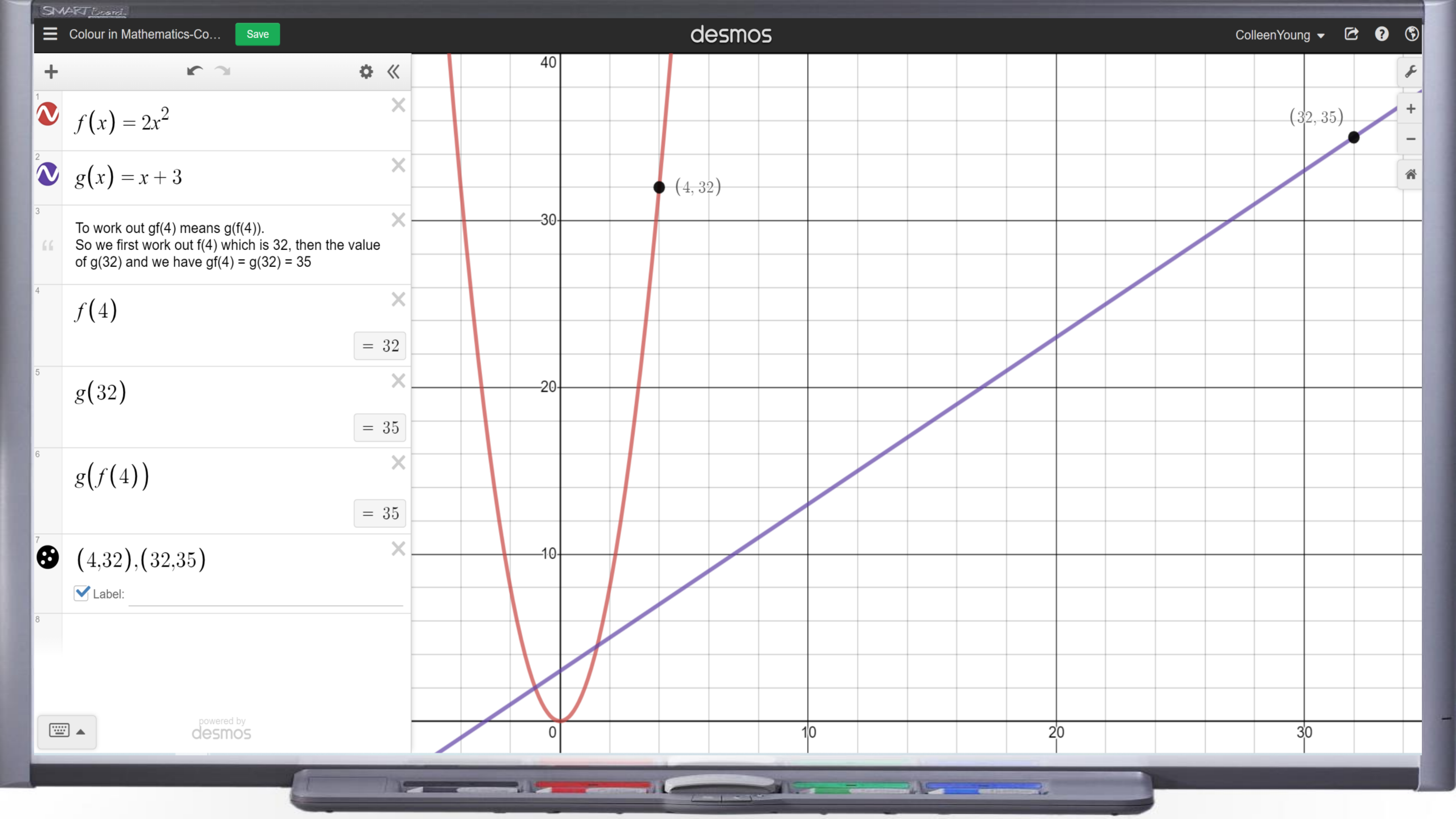
Solution

(b) gf(4) is g(f(4)) so first we must work out f(4)

$f(4) = 2 \times 4^2 = 2 \times 16 = 32$

So $g(f(4)) = g(32) = 32 + 3 = 35$

Note that we can check values of functions on Desmos …



Colour in Mathematics-Co...
Save
desmos
ColleenYoung
$f(x) = 2x^2$
$g(x) = x + 3$
To work out gf(4) means g(f(4)).
So we first work out f(4) which is 32, then the value of g(32) and we have gf(4) = g(32) = 35
$f(4)$
$= 32$
$g(32)$
$= 35$
$g(f(4))$
$= 35$
$(4,32),(32,35)$
Label:
powered by desmos
(4, 32)
(32, 35)
40
30
20
10
0
10
20
30

Algebra – Composite Functions

$f(x) = 2x^2$, $g(x) = x+3$

(c) Write down fg(x)

(d) Write down gf(x)

Solution

Looking at the algebra now to find f(g(x), we need f(g(x)).

$f(g(x)) = f(x+3) = 2(x+3)^2$.

$2(x+3)^2 = 2(x^2+6x+9) = 2x^2 + 12x + 18$

$fg(x) = 2x^2 + 12x + 18$

Inequalities

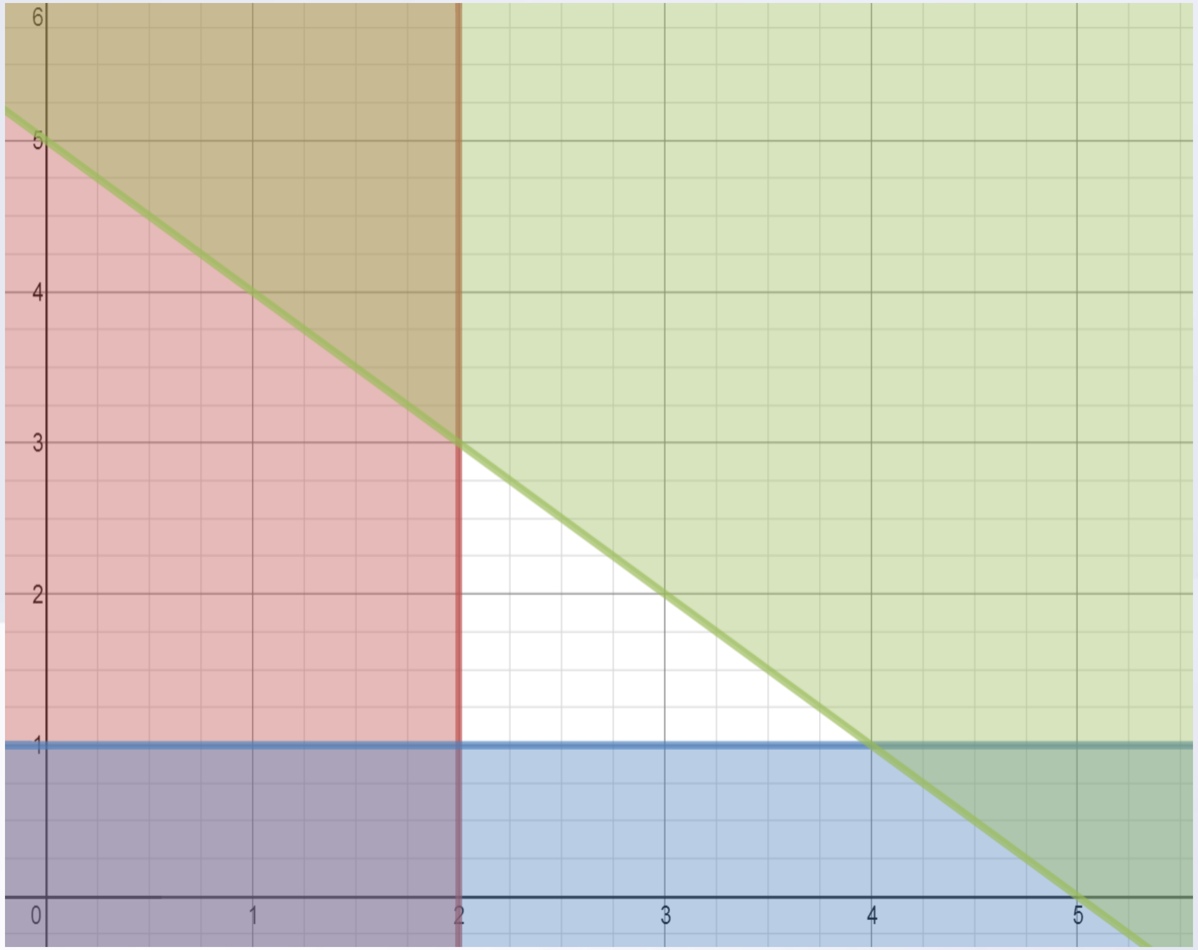
Show graphically the region satisfying the following inequalities

$x \geq 2$

$y \geq 1$

$x+y \leq 5$

Note – required region left unshaded



SMART Board

Quadratic Inequalities Save

desmos

1. $f(x) = (x-a)(x-b)\{x<a\}$

2. $f(x) = (x-a)(x-b)\ \{a<x<b\}$

3. $f(x) = (x-a)(x-b)\{x>b\}$

4. $a = -10$

−10

5. $b = 4$

−10

6. $(a,0),(b,0)$

Label:

7. To solve the inequality f(x)>0 we are asking for what values of x the function is positive. The function is positive where the curve is above the x-axis (blue). We see that f(x)<a and f(x)>b.

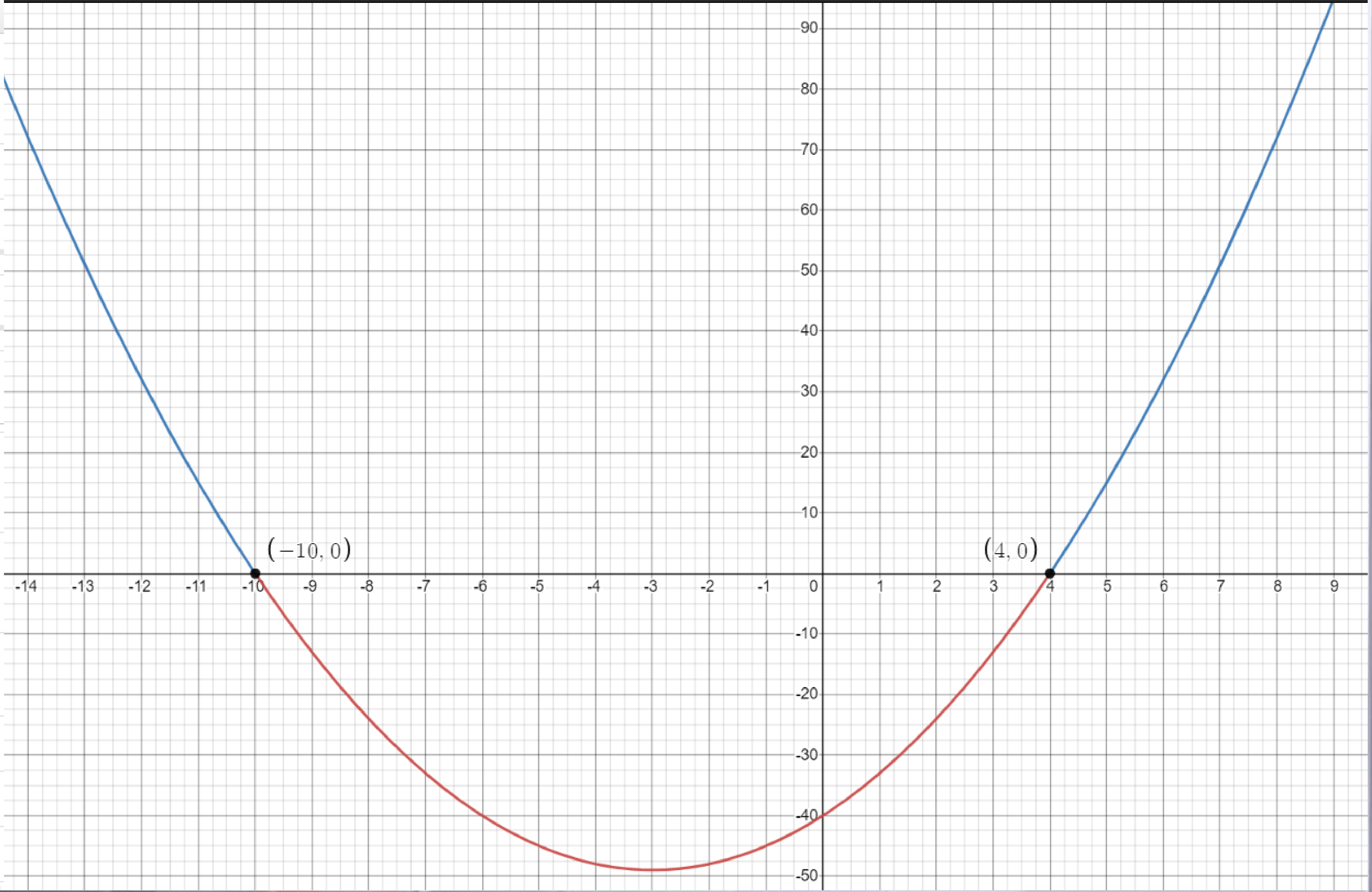
8. To solve the inequality f(x)<0 we are asking for what values of x the function is negative. The function is negative where the curve is below the x-axis (red). We see that f(x)<a and f(x)>b.

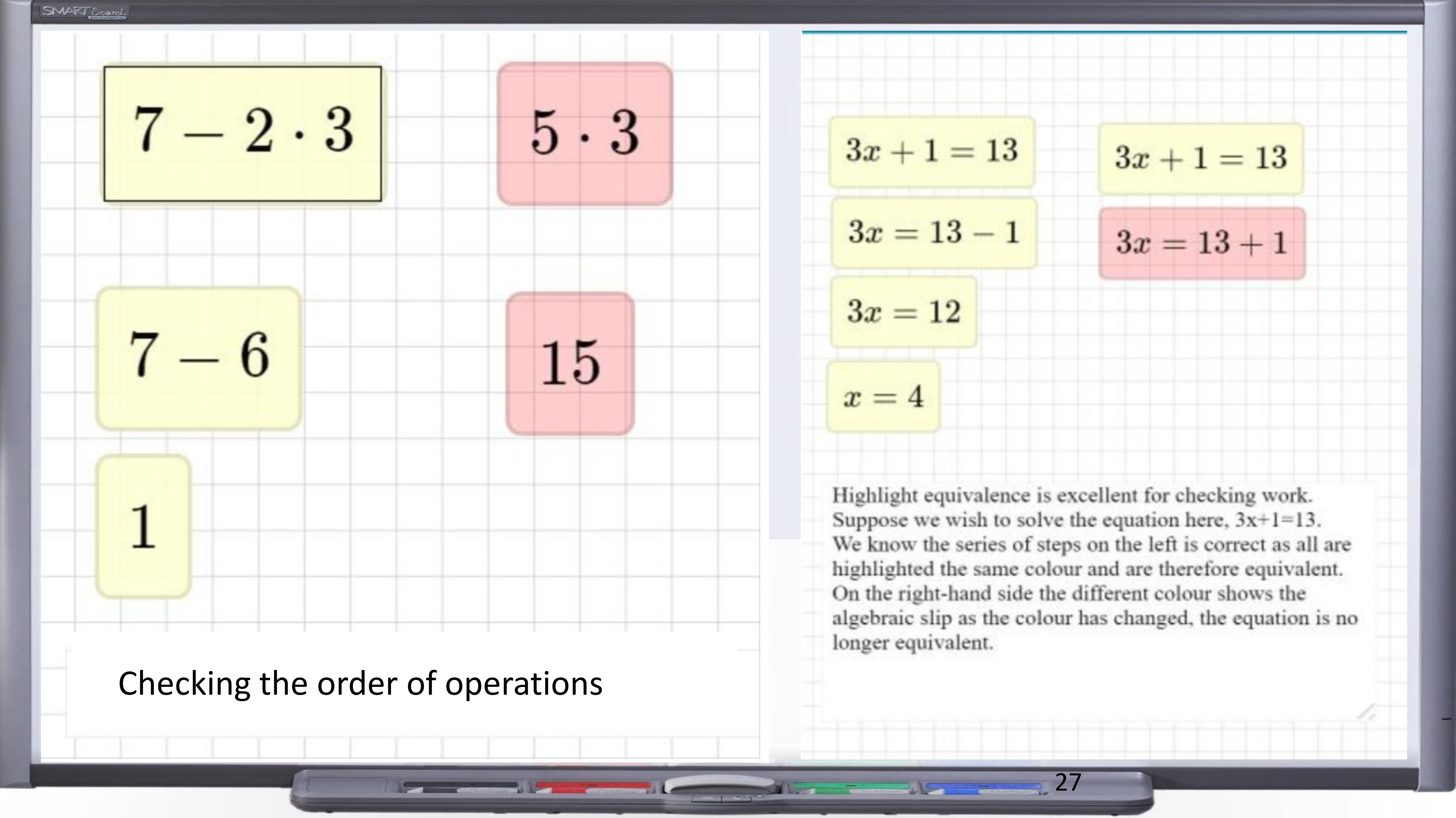
9. Desmos can show the values of x which are the solution to an inequality. Select (touch the circle) g(x)<0 or g(x)>0 to show the values of x satisfying the inequality.

10. $g(x) = (x-a)(x-b)$

11. $g(x) < 0$

12. $g(x) > 0$





$7 - 2 \cdot 3$
$5 \cdot 3$
$7 - 6$
15
1
Checking the order of operations
$3x + 1 = 13$
$3x = 13 - 1$
$3x = 12$
$x = 4$
$3x + 1 = 13$
$3x = 13 + 1$
Highlight equivalence is excellent for checking work.
Suppose we wish to solve the equation here, 3x+1=13.
We know the series of steps on the left is correct as all are
highlighted the same colour and are therefore equivalent.
On the right-hand side the different colour shows the
algebraic slip as the colour has changed, the equation is no
longer equivalent.

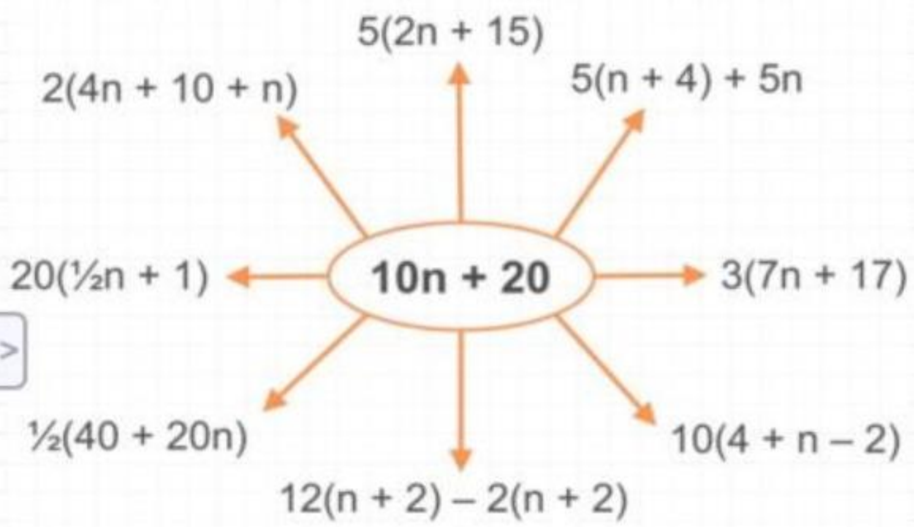
SMART Board

Math Whiteboard

Example Whit

NEW WHITEBOARD Whiteboard Link: https://www.mathwhiteboard.com COPY LINK

Math Write | Math Type | Draw | Text Box | Erase | Insert... | Clear | Undo | Redo



which expressions are **not** the same as 10n + 20 ?

From Don Steward
Don Steward - one incorrect simplification

Note that equivalent expressions are highlighted in the same colour.

$10n + 20$

$5(n+4)+5n$

$5n + 20 + 5n$

$10n + 20$

$10(4+n-2)$

$40 + 10n - 20$

$10n + 20$

$12(n+2)-2(n+2)$

$12n + 24 - 2n - 4$

$10n + 20$

$2(4n + 10 + n)$

$8n + 20 + 2n$

$10n + 20$

$\frac{1}{2}(40 + 20n)$

$20 + 10n$

$20\left(\frac{1}{2}n + 1\right)$

$10n + 20$

$3(7n + 17)$

$21n + 51$

$5(2n + 15)$

$10n + 75$

Math Whiteboard

Example W

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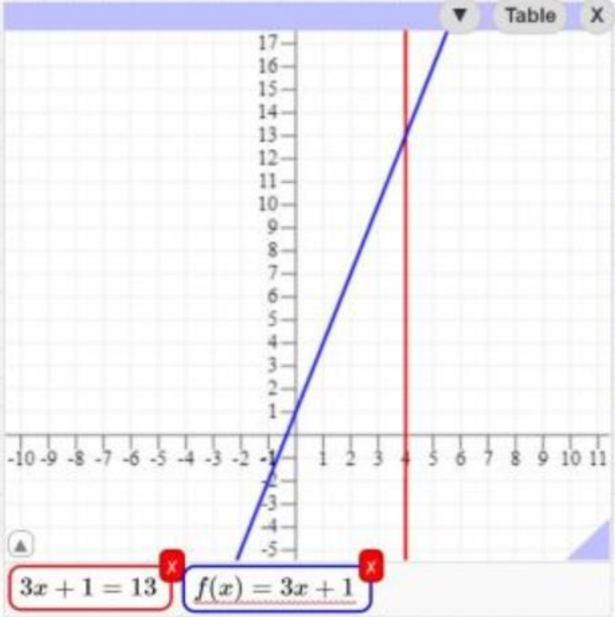
Erase | Insert... | Clear | Undo | Redo

$$3x+1=13$$
$$3x=13-1$$
$$3x=12$$
$$x=4$$

$$3x+1=13$$
$$3x=13+1$$

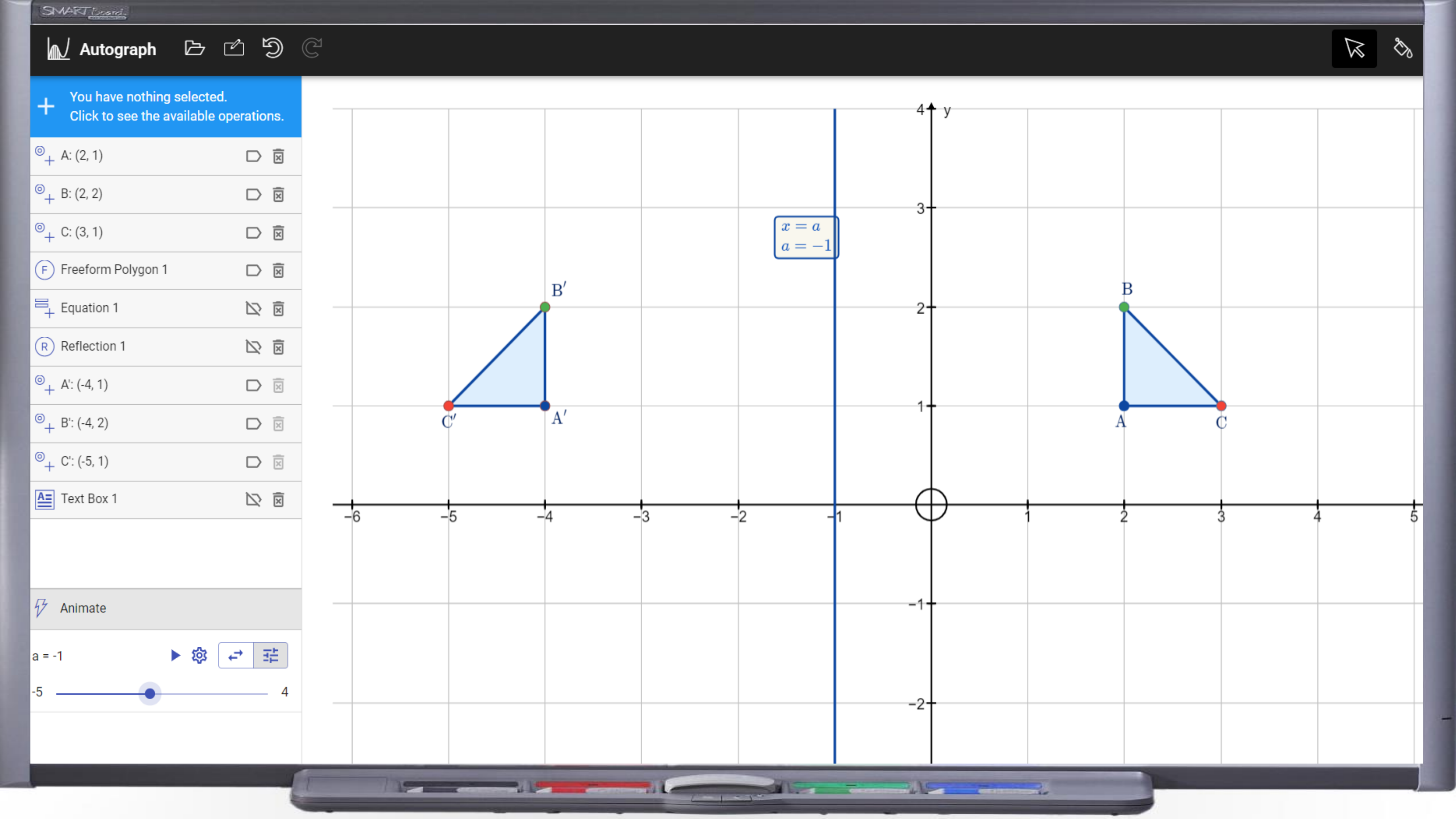
Highlight equivalence is excellent for checking work. Suppose we wish to solve the equation here, 3x+1=13. We know the series of steps on the left is correct as all are highlighted the same colour and are therefore equivalent. On the right-hand side the different colour shows the algebraic slip as the colour has changed, the equation is no longer equivalent.



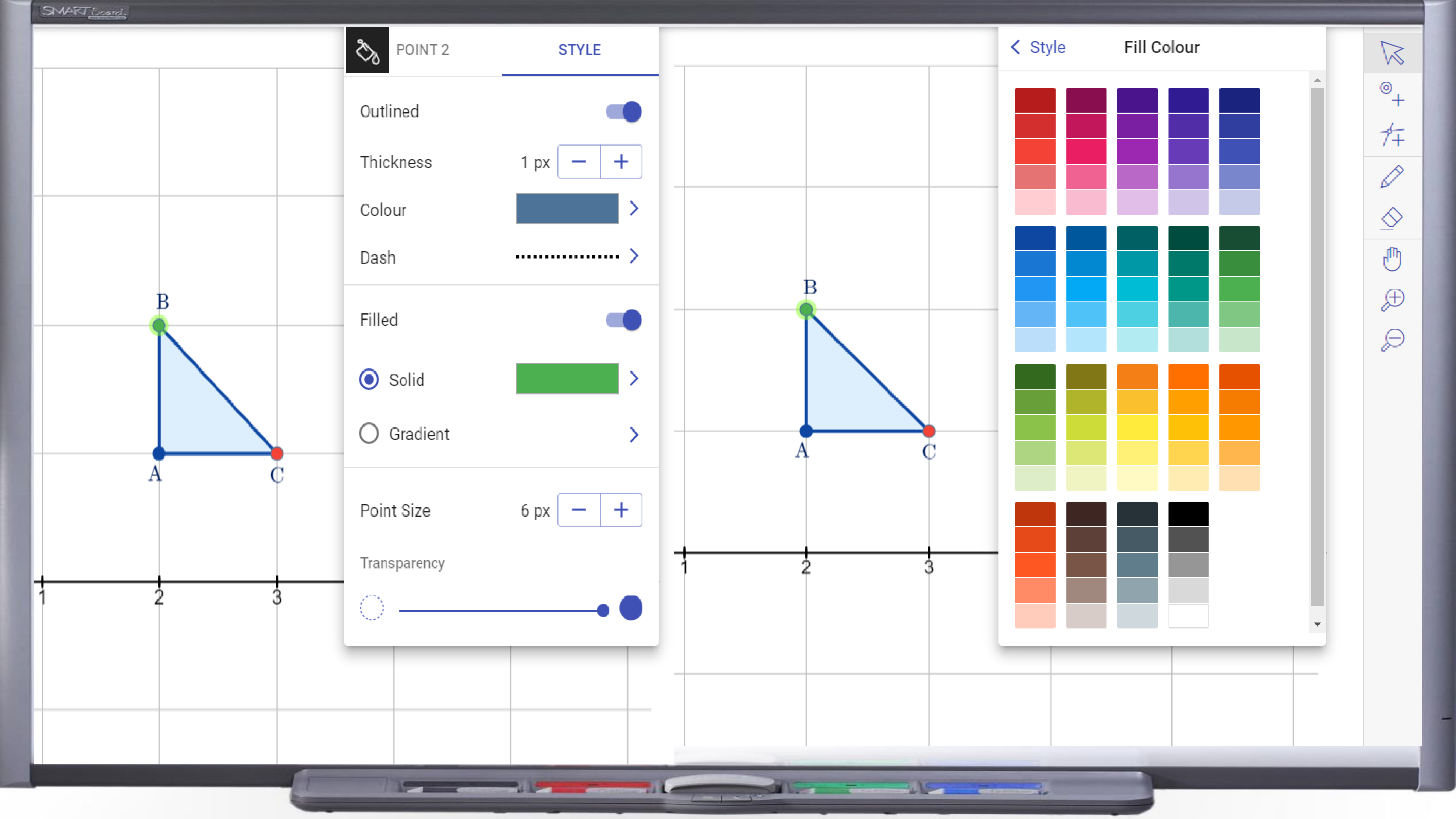
y	$3x+1=13$	$f(x)=3x+1$
-5.000	4.000	-14.000
-4.000	4.000	-11.000
-3.000	4.000	-8.000
-2.000	4.000	-5.000
-1.000	4.000	-2.000
0.000	4.000	1.000
1.000	4.000	4.000
2.000	4.000	7.000
3.000	4.000	10.000
4.000	4.000	13.000

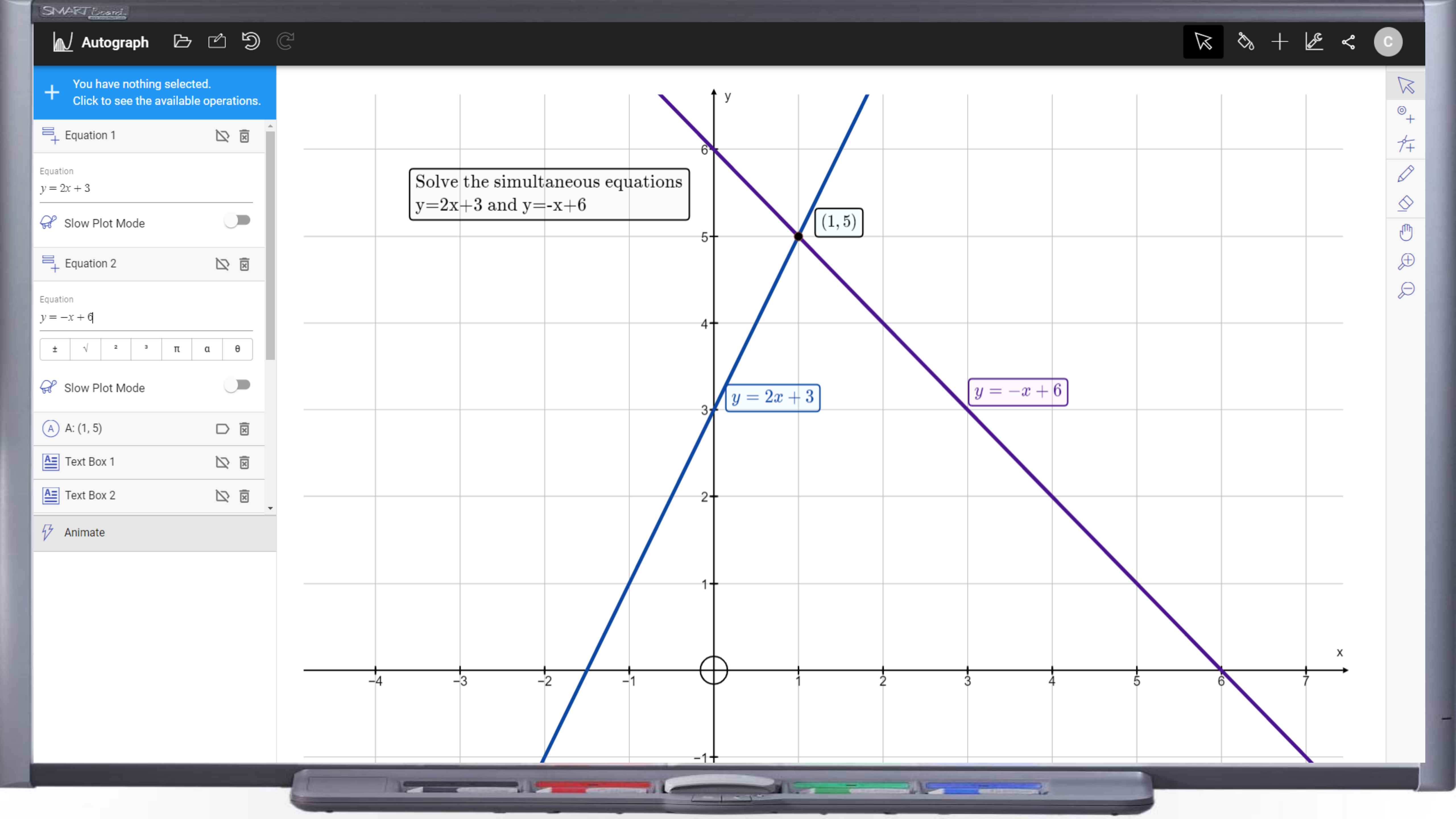
$$f(x)=3x+1$$

To insert a graph use the Insert menu then you can simply drag the Math type expression or expressions to the graph area.
Select Table for a table of values.

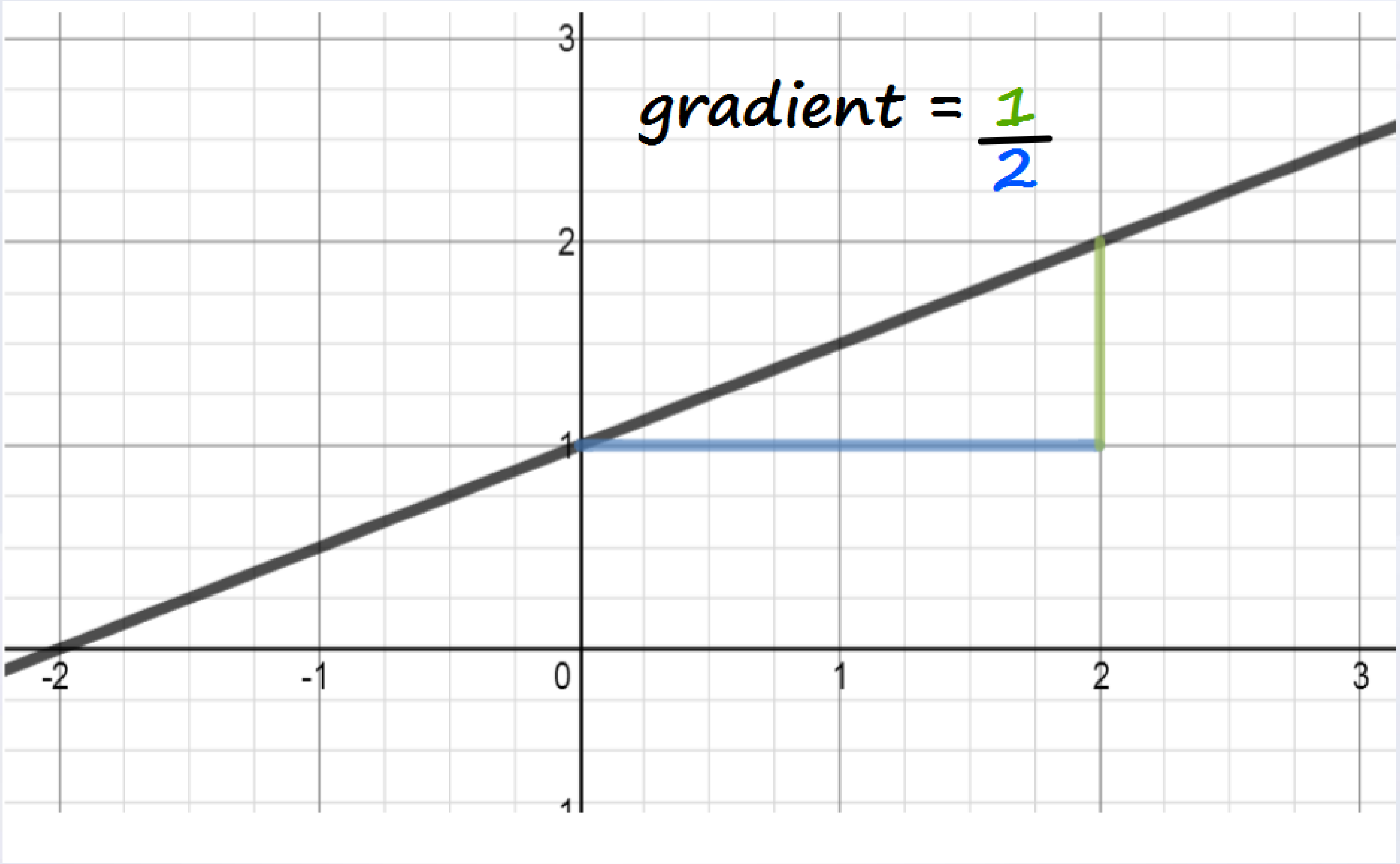


SMART Board
Autograph
You have nothing selected.
Click to see the available operations.
A: (2, 1)
B: (2, 2)
C: (3, 1)
Freeform Polygon 1
Equation 1
Reflection 1
A': (-4, 1)
B': (-4, 2)
C': (-5, 1)
Text Box 1
Animate
a = -1
-5
4
$x = a$
$a = -1$
y
B'
A'
C'
B
A
C


POINT 2
STYLE
Outlined
Thickness
1 px
Colour
Dash
Filled
Solid
Gradient
Point Size
6 px
Transparency
Style
Fill Colour
B
A
C
1
2
3


Autograph
You have nothing selected.
Click to see the available operations.
Equation 1
Equation
y = 2x + 3
Slow Plot Mode
Equation 2
Equation
y = −x + 6
Slow Plot Mode
A: (1, 5)
Text Box 1
Text Box 2
Animate
Solve the simultaneous equations
y=2x+3 and y=-x+6
(1, 5)
y = 2x + 3
y = −x + 6
y
x

Co-ordinate Geometry



Complete the square

$x^2+10x+28$

$= (x+5)^2-25+28$

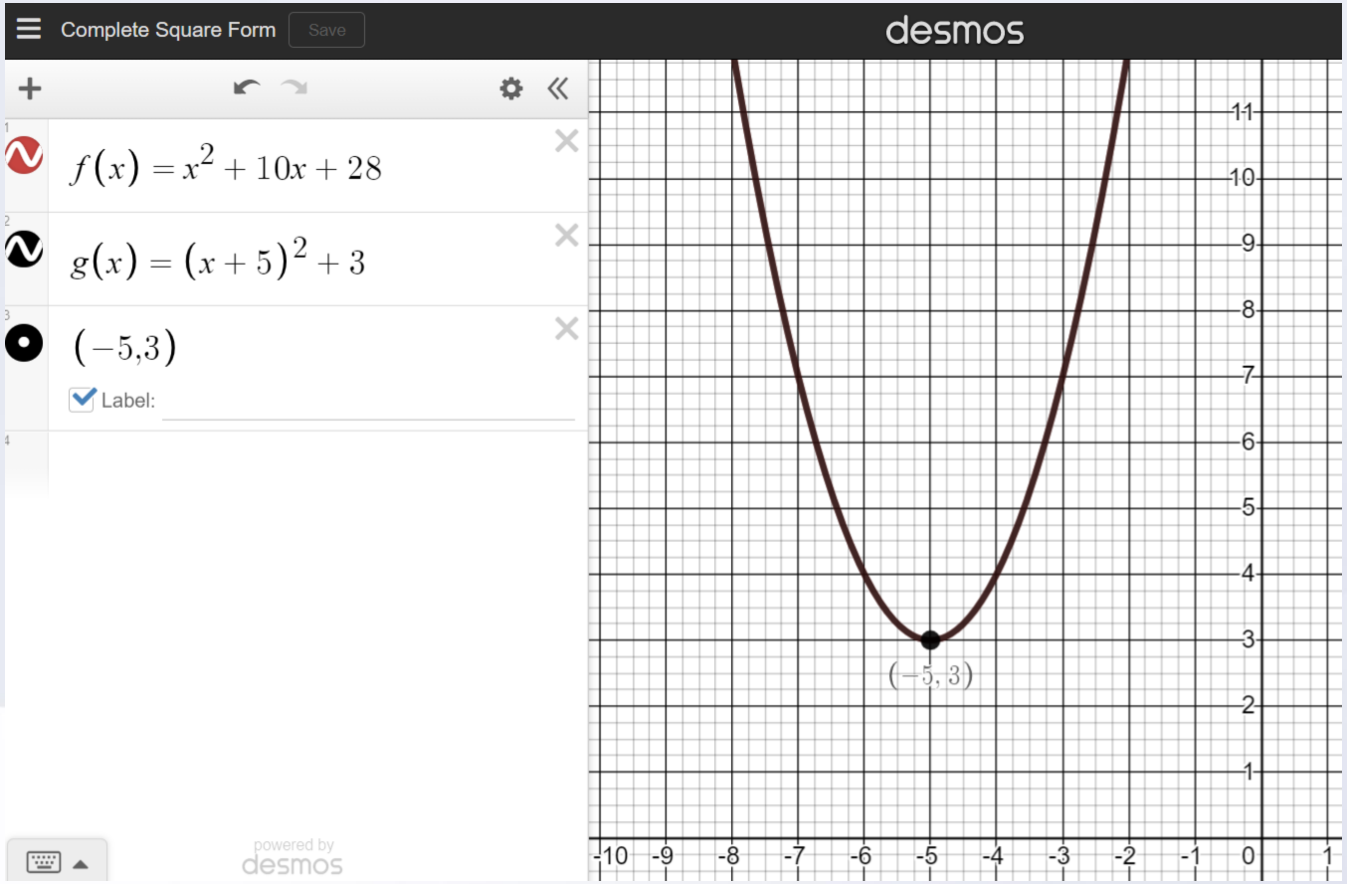
$= (x+5)^2+3$

Complete the square

$$x^2+10x+28$$

$$= (x+5)^2-25+28$$

$$= (x+5)^2+3$$



Circle Geometry

Find the centre and radius:

$x^2-4x+y^2+6y=12$

$(x-2)^2-4+(y+3)^2-9=12$

$(x-2)^2+(y+3)^2=25$

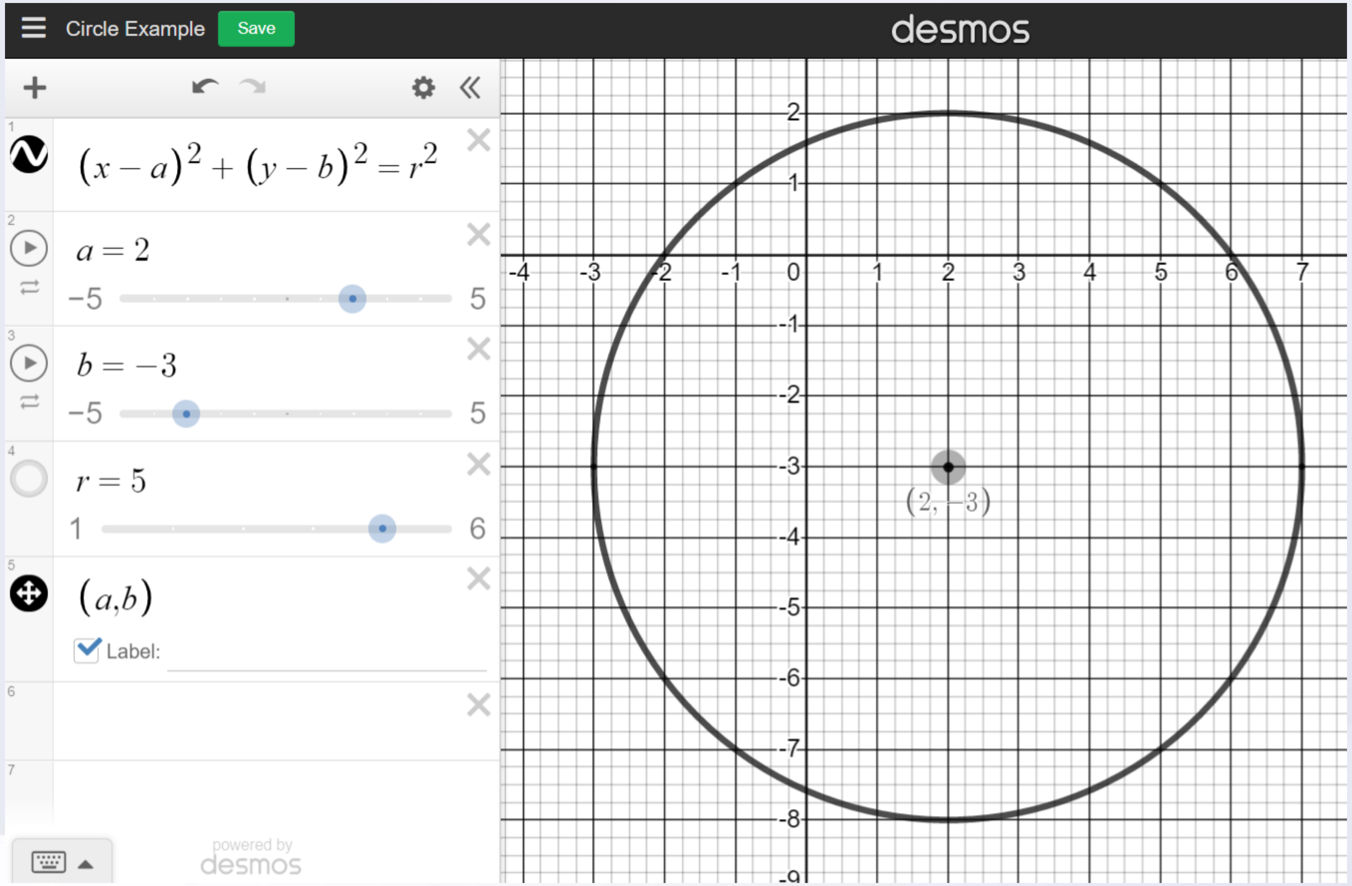
Circle Geometry

$(x-a)^2+(y-b)^2 = r^2$

$(x-2)^2+(y+3)^2 = 25$

$(x-2)^2+(y--3)^2 = 25$

$a=2 \qquad b=-3$



Solve

$$3^{3x} = 9^{x+1}$$

$$3^{3x} = 3^{2(x+1)}$$

$$3^{3x} = 3^{2x+2}$$

$$3^{3x} = 3^{2x+2}$$

$$3x = 2x+2$$

$$x = 2$$

Check

$$3^{3\times 2} = 3^6 = 729$$

$$9^{2+1} = 9^3 = 729 \quad \checkmark$$

Binomial Expansion

WolframAlpha

expand (a+b)^3

Extended Keyboard   Upload

Input interpretation:

expand	$(a+b)^3$

Result:

$a^3 + 3a^2b + 3ab^2 + b^3$

Why $3a^2b$?

(a + b)(a + b)(a + b)	
(a + b)(a + b)(a + b)	a^3
(a + b)(a + b)(a + b)	b^3
(a + b)(a + b)(a + b)	a^2b
(a + b)(a + b)(a + b)	a^2b
(a + b)(a + b)(a + b)	a^2b

$(2+x)^5$

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

$$1\,2^5x^0 + 5\,2^4x^1 + 10\,2^3x^2 + 10\,2^2x^3 + 5\,2^1x^4 + 1\,2^0x^5$$

$$32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$$

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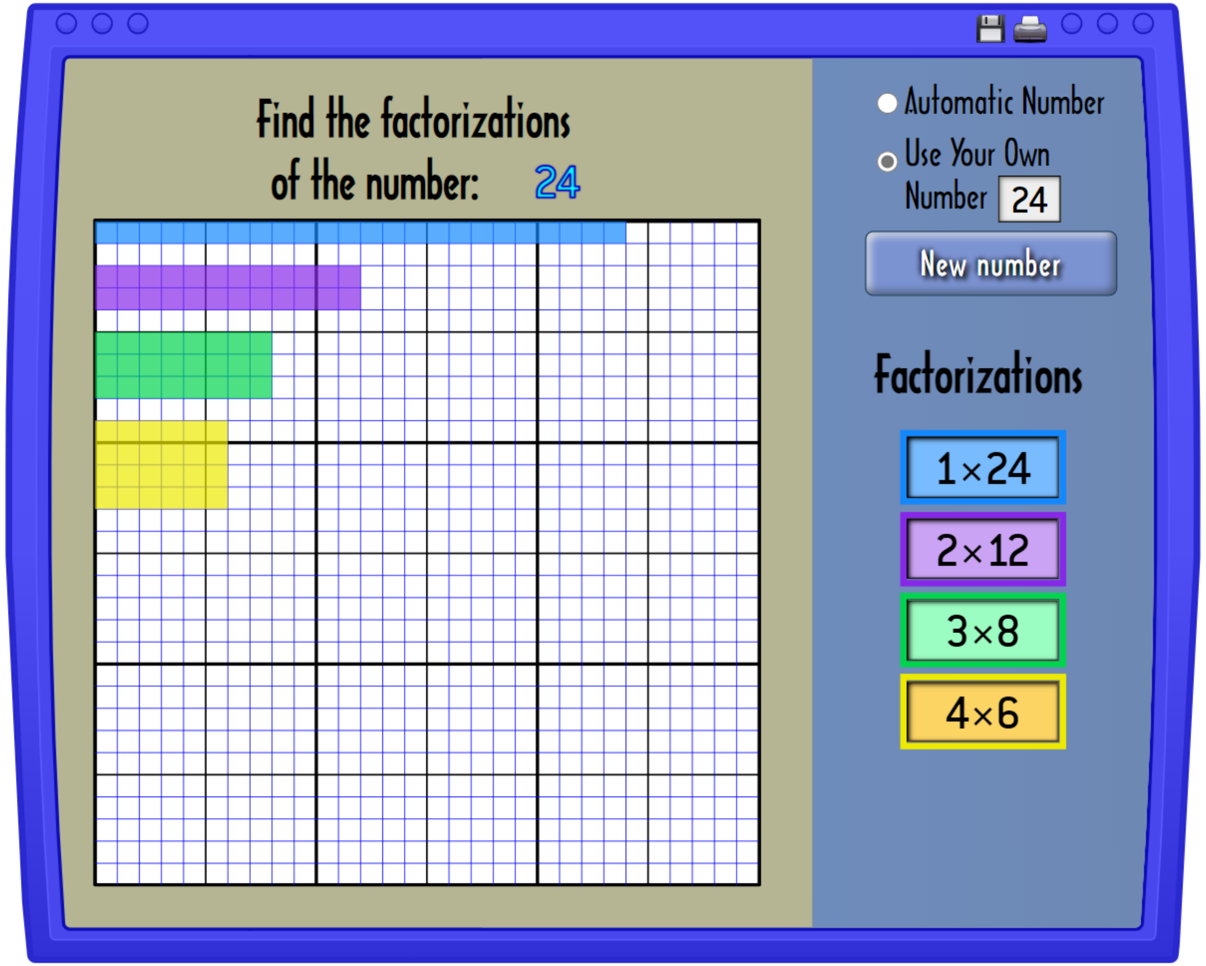
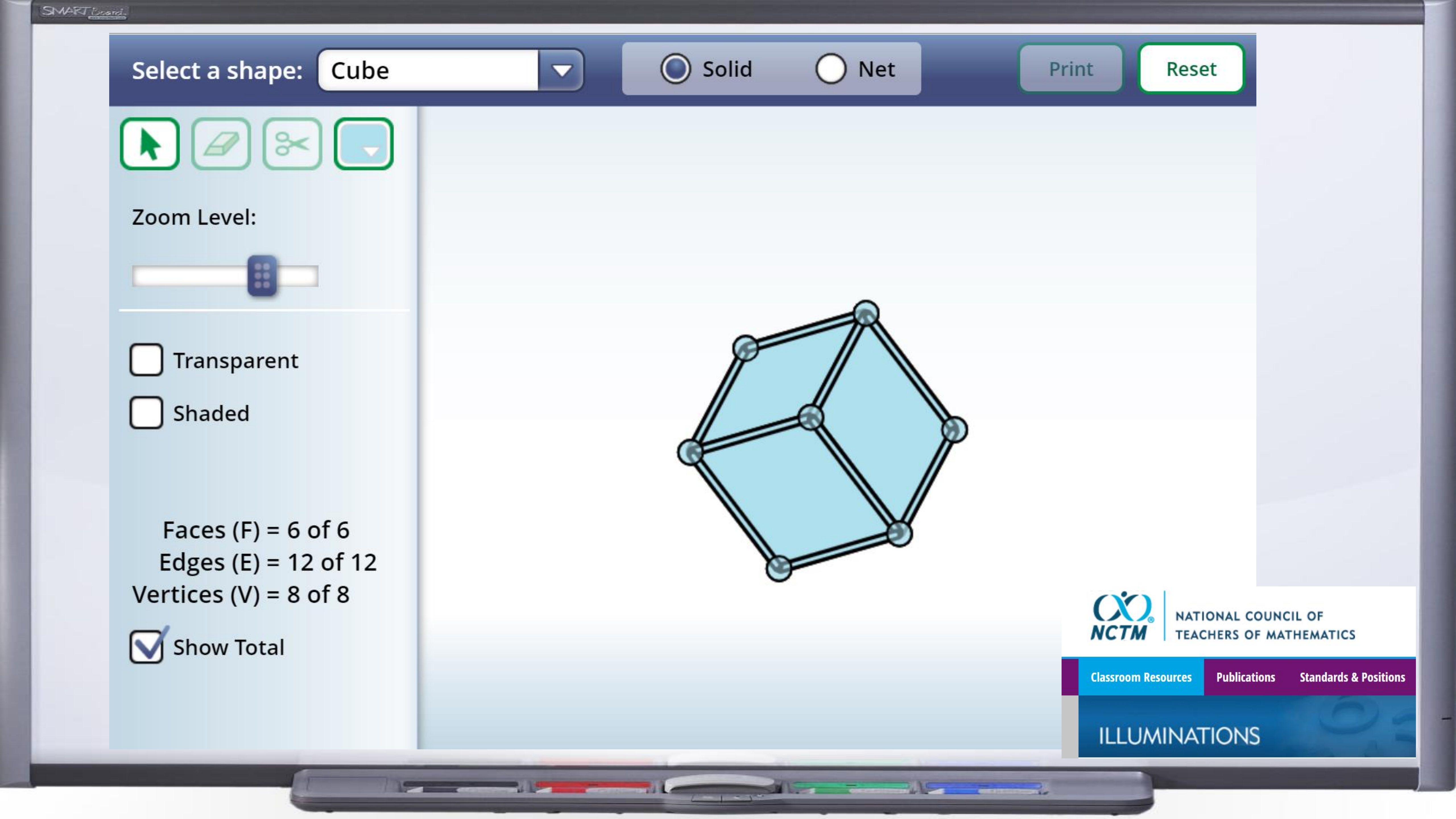
ILLUMINATIONS



Exploration

Follow the instructions to find factorizations for several numbers. As you work, see if you can answer these questions:

- Why do you think the length and width of the rectangles represent the factors of your numbers?
- Which number has the most factorizations? Which has the fewest? Why do you think this is?
- What kinds of numbers have only one factorization? What do the rectangles for these factorizations have in common?
- If you double a number, what happens to the number of factorizations? Do you notice a pattern in the factorizations of your original number and the doubled number?


Select a shape:
Cube
Solid
Net
Print
Reset
Zoom Level:
Transparent
Shaded
Faces (F) = 6 of 6
Edges (E) = 12 of 12
Vertices (V) = 8 of 8
Show Total
NCTM
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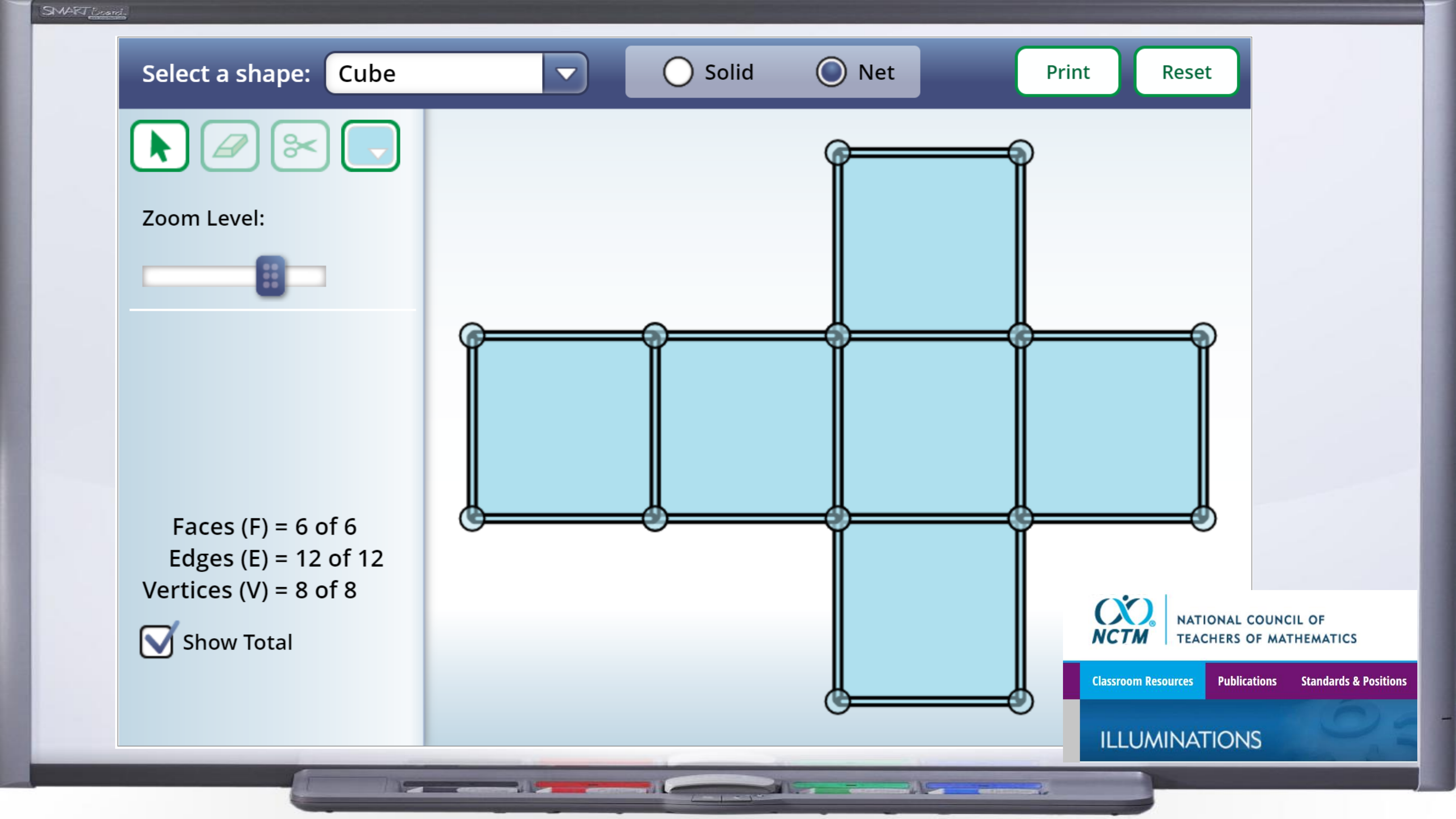
SMART Board
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Zoom Level:
Faces (F) = 6 of 6
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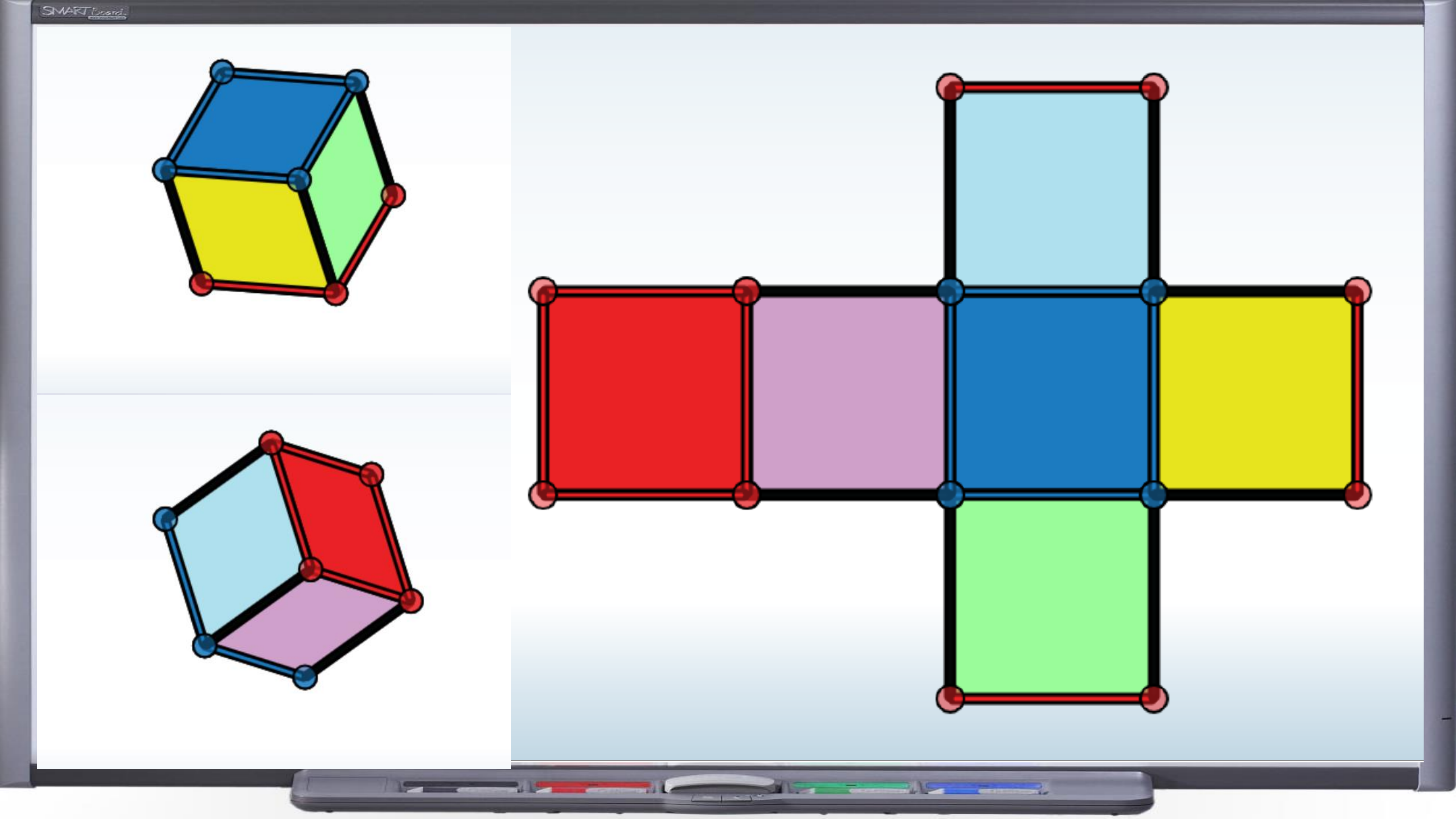
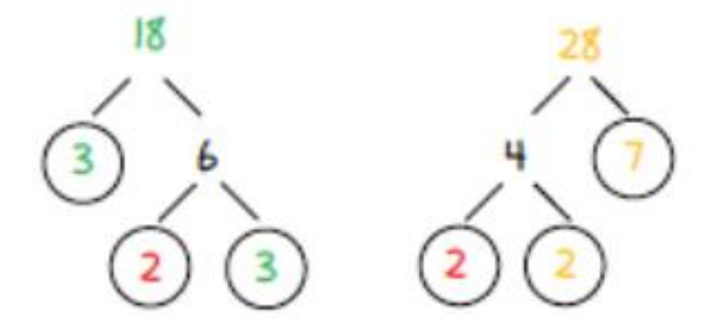
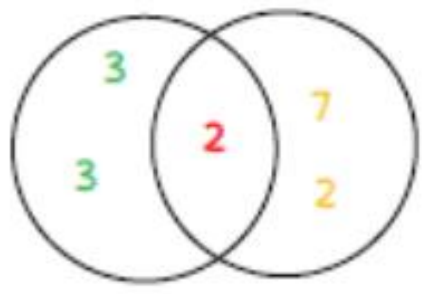
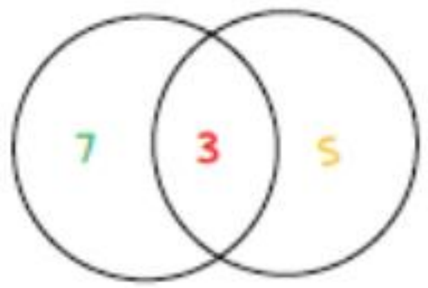
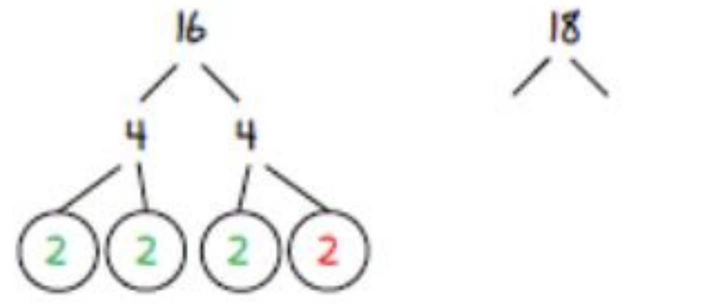
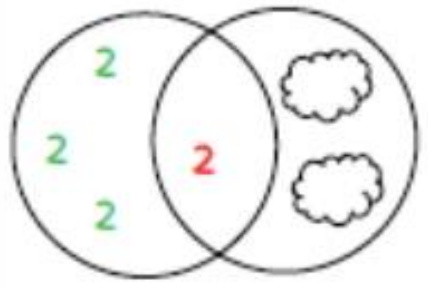


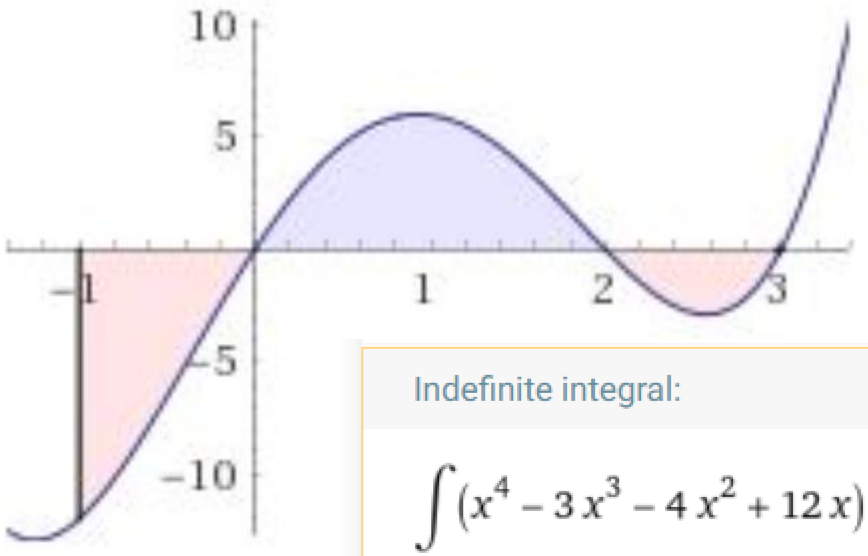
Prime Decomposition	Venn Diagram	LCM	HCF
		3 x 3 x 2 x 2 x 7 = 252	2
			3
		2 x 2 x 2 x 2 x ? x ? =	

Integration

Definite integral:

$$\int_{-1}^{3}(x^4 - 3x^3 - 4x^2 + 12x)\,dx = -\frac{8}{15} \approx -0.53333$$

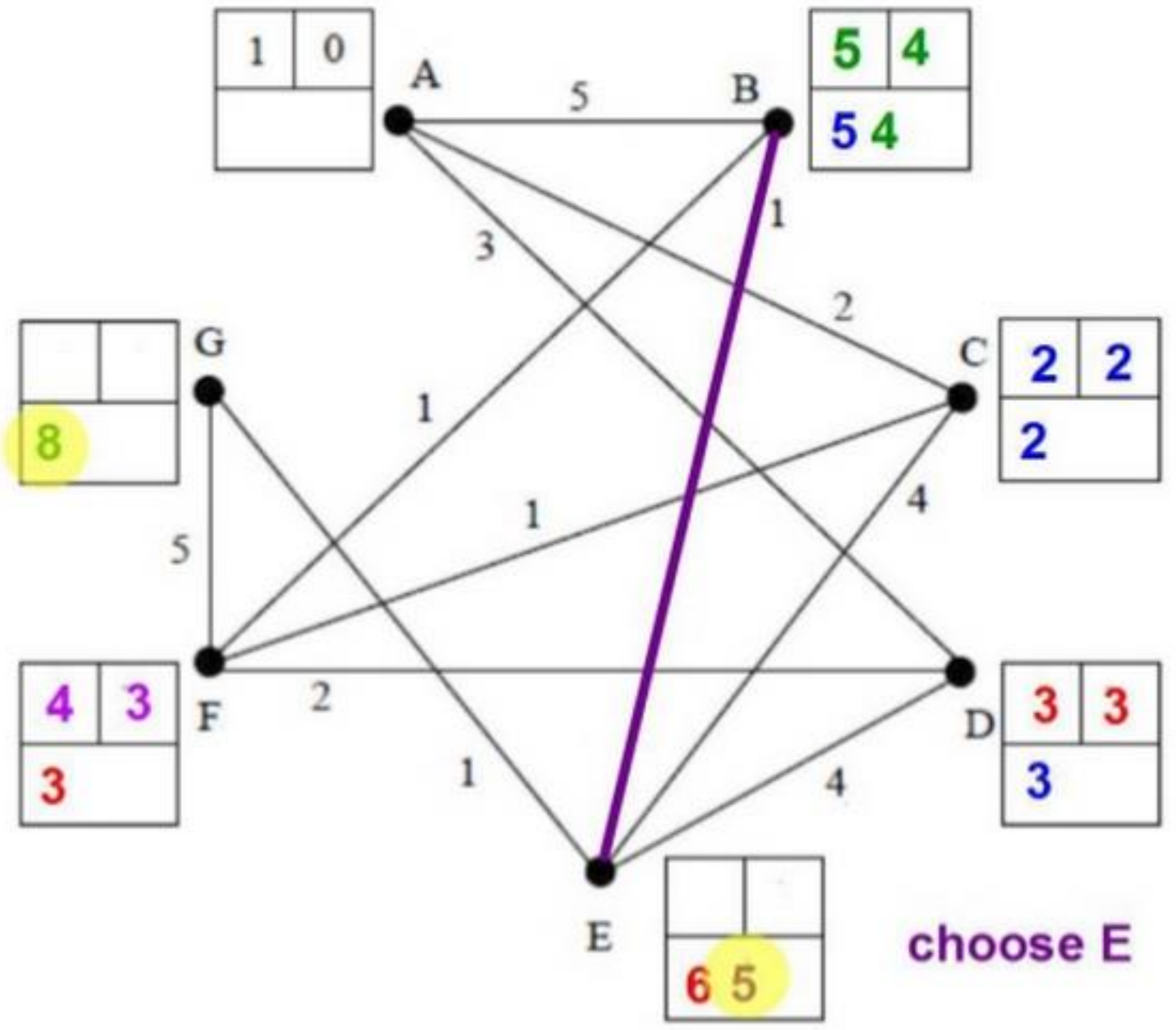
Visual representation of the integral:



Indefinite integral:

$$\int(x^4 - 3x^3 - 4x^2 + 12x)\,dx = \frac{x^5}{5} - \frac{3x^4}{4} - \frac{4x^3}{3} + 6x^2 + \text{constant}$$

Decision Mathematics



Dijkstra's Algorithm

Enter the linear programming problem here:

- ○ Maximize
- ◉ Minimize

$z =$ 0.38x+y subject to the constraints:

- ○ Show only the region defined by the following contraints:

```
3x+2y>=18
2x +   4y   ≥ 16
10x + 50y ≥ 100
```

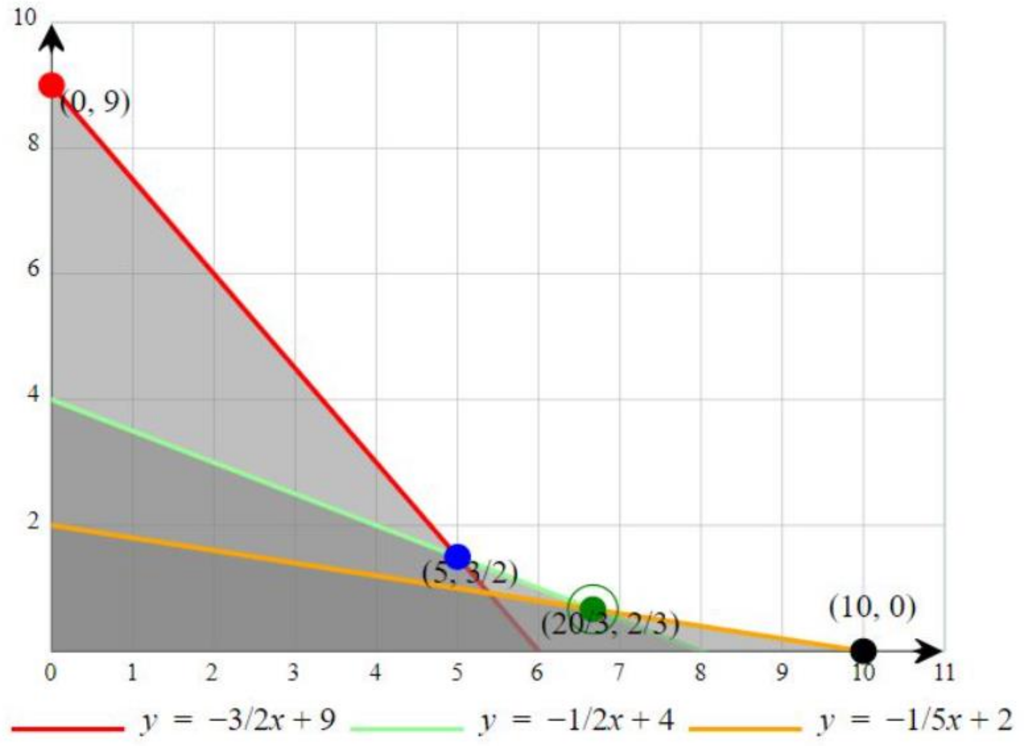
LP Examples | Graphing Examples | Solve

Rounding: 4 decimal places Fraction Mode ☑

Erase Everything

The solution will appear below.

Vertex	Lines through vertex	Value of objective
● (5, 3/2)	$3x + 2y = 18$ $2x + 4y = 16$	17/5
● (0, 9)	$3x + 2y = 18$ $x = 0$	9
● (20/3, 2/3)	$2x + 4y = 16$ $10x + 50y = 100$	16/5 Minimum
● (10, 0)	$10x + 50y = 100$ $y = 0$	19/5



Xmin: 0 Xmax: 11

Ymin: 0 Ymax: 10

Gridlines X: 1 Y: 2

Graph Show vertex coordinates ☑

Worked Solutions

The use of colour can help clarity in online worked solutions for students.

Interestingly, some students prefer a series of still images with no sound to videos as they can really dictate the pace themselves.

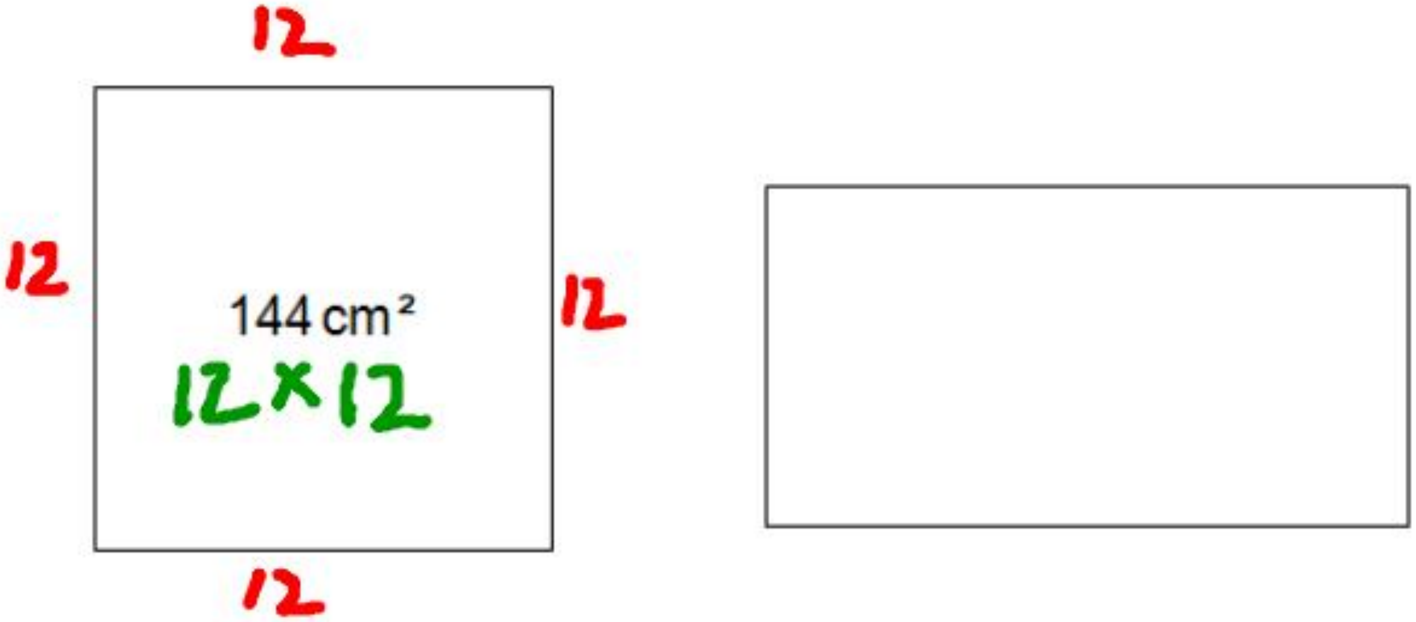
Some examples follow…

Worked Solutions

The perimeter of the square is equal to the perimeter of the rectangle.

The area of the square is 144 cm^2

The length of the rectangle is four times the width of the rectangle.



Not drawn accurately

Work out the width of the rectangle.

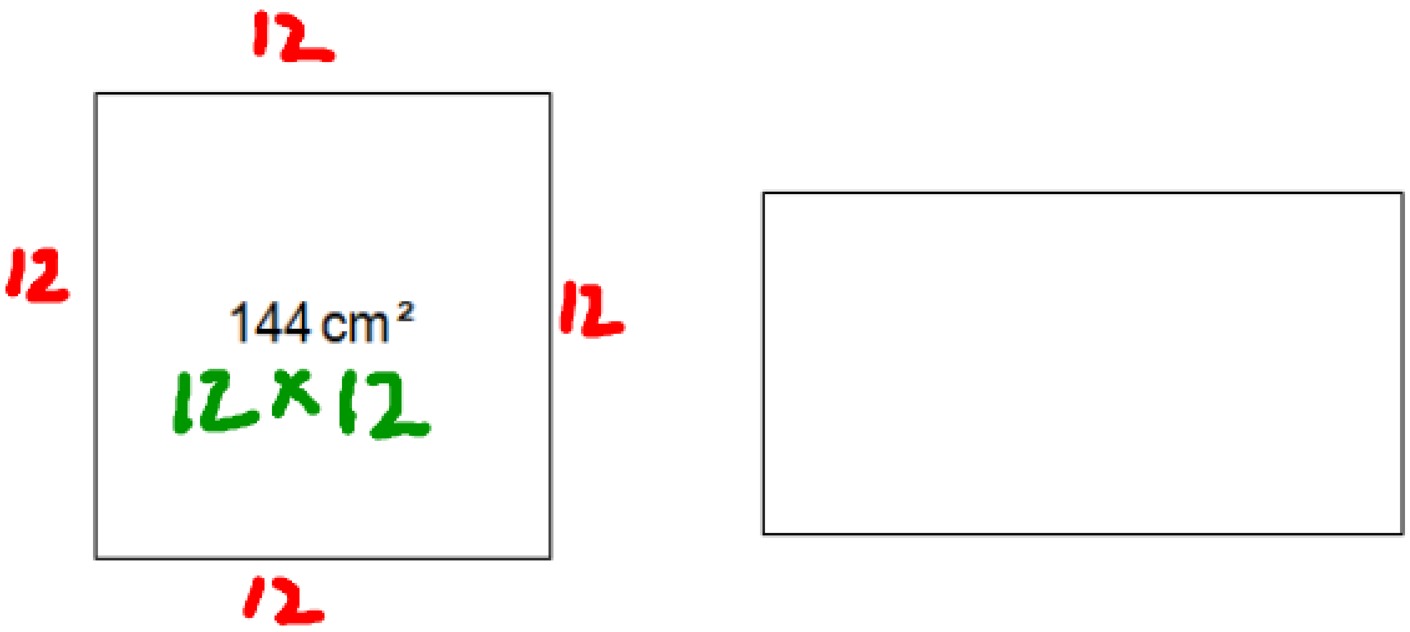
Worked Solutions

48

The perimeter of the square is equal to the perimeter of the rectangle.

The area of the square is 144 cm^2

The length of the rectangle is four times the width of the rectangle.



Not drawn accurately

Work out the width of the rectangle.

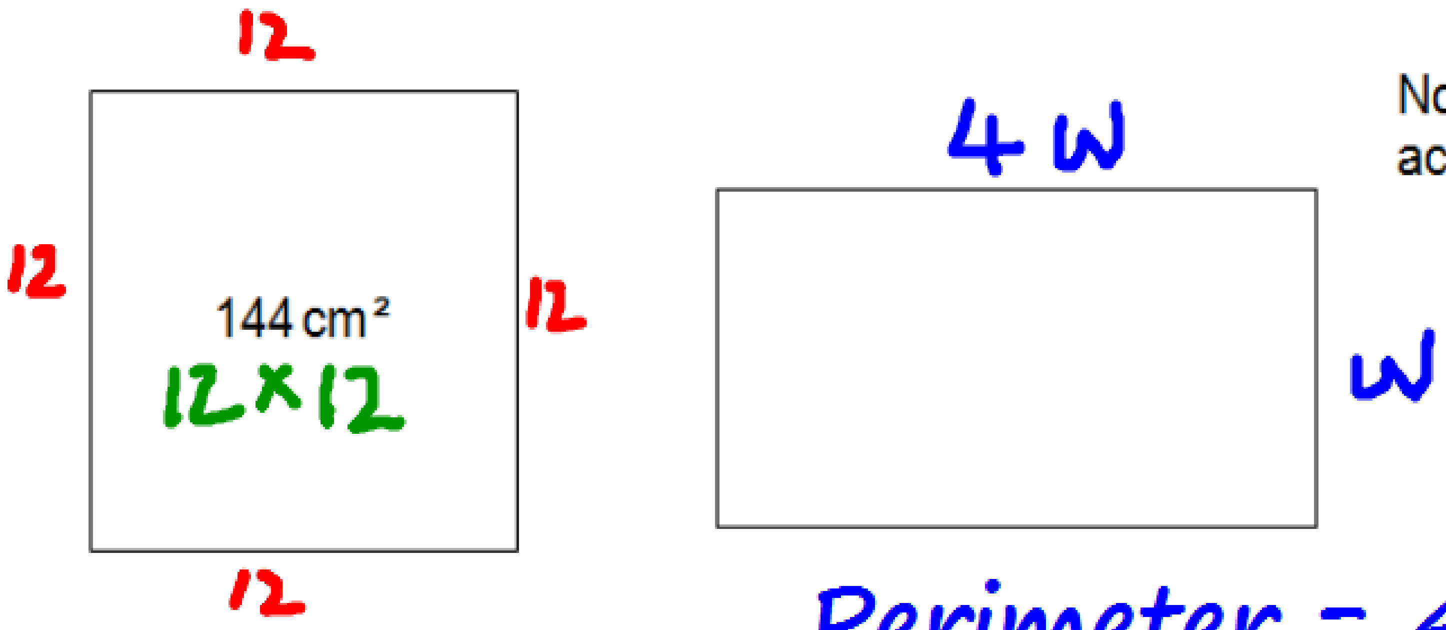
48

The perimeter of the square is equal to the perimeter of the rectangle.

The area of the square is 144 cm^2

The length of the rectangle is four times the width of the rectangle. Let width = w

Not drawn accurately



Perimeter = $4w + 4w + w + w = 10w$

Work out the width of the rectangle.

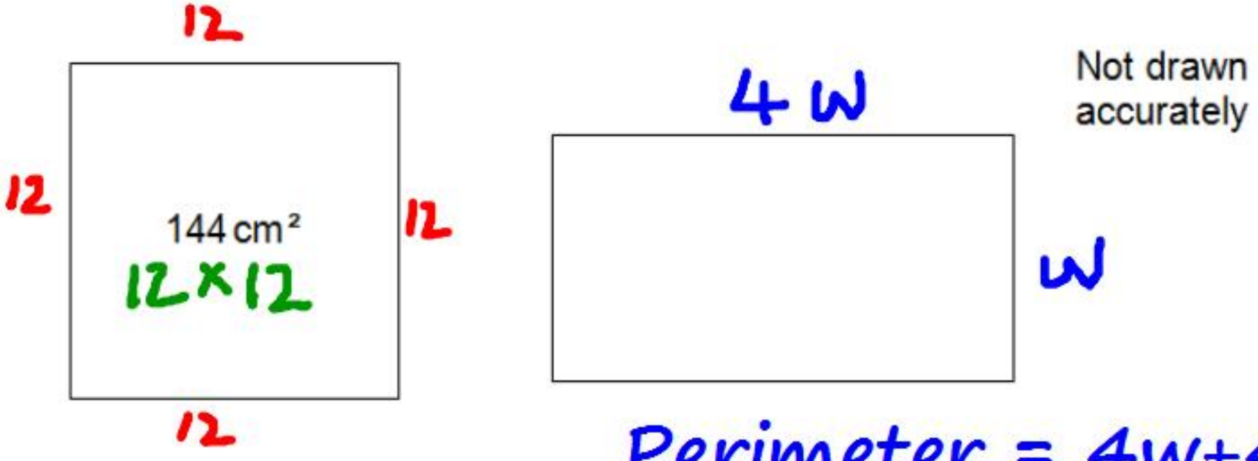
Worked
Solutions

$48 = 10w$

The perimeter of the square is equal to the perimeter of the rectangle.

The area of the square is 144 cm²

The length of the rectangle is four times the width of the rectangle. Let width = w



Perimeter = $4w+4w+w+w=10w$

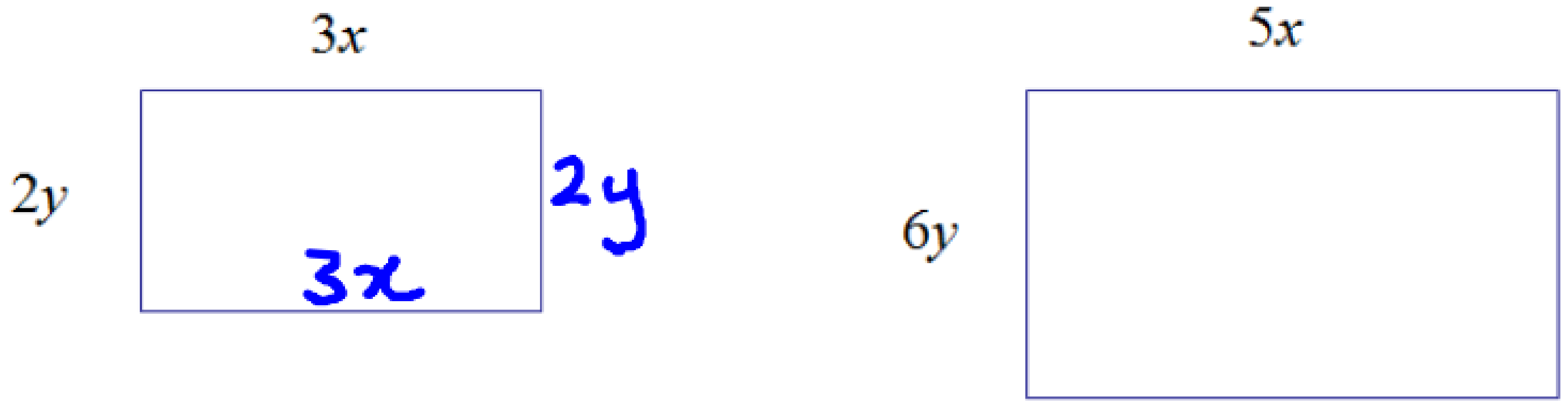
Work out the width of the rectangle.

$48 = 10w$

$w=4.8$

Worked Solutions

Two rectangles have the following dimensions



Perimeter equals 21 cm

$6x + 4y = 21$

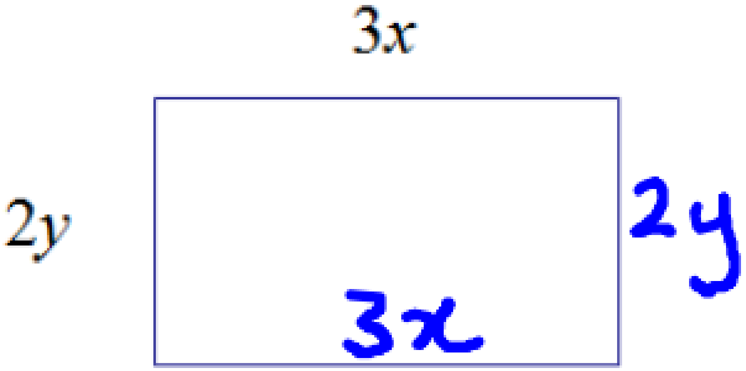
Perimeter equals 43 cm

Work out x and y

$6x + 4y = 21$ (1)

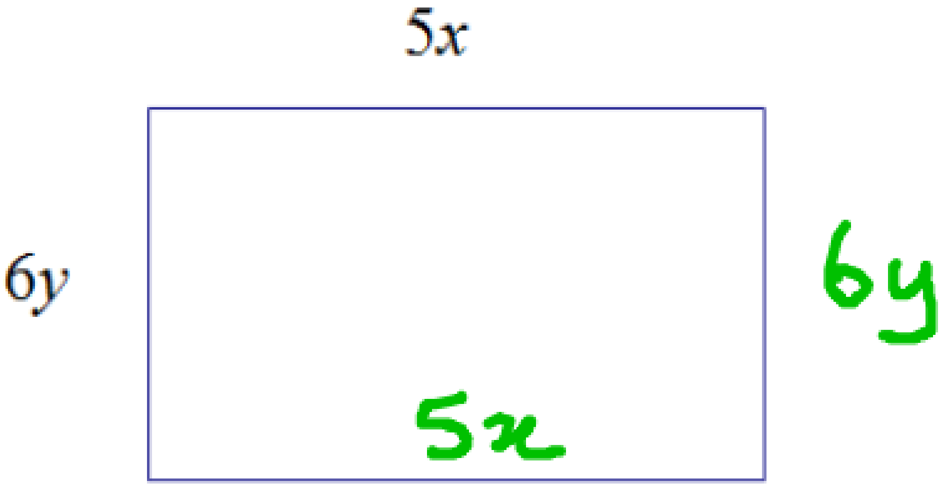
Worked Solutions

Two rectangles have the following dimensions



Perimeter equals 21 cm

$$6x + 4y = 21$$



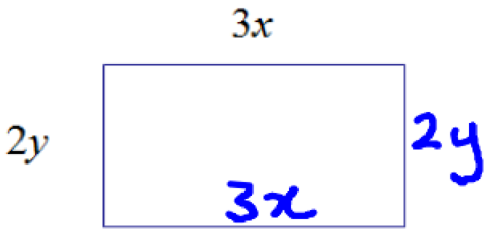
Perimeter equals 43 cm

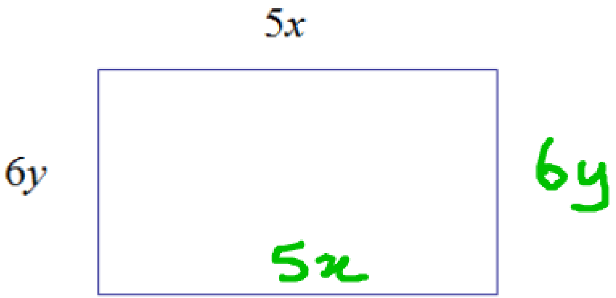
$$10x + 12y = 43$$

Work out x and y

Worked Solutions

Two rectangles have the following dimensions





Perimeter equals 21 cm

$6x + 4y = 21$

Perimeter equals 43 cm

$10x + 12y = 43$

Work out x and y

$6x + 4y = 21 \quad (1)$

$10x + 12y = 43 \quad (2)$

$18x + 12y = 63 \quad (1) \times 3$

$10x + 12y = 43 \quad (2)$

$8x = 20 \quad x = 2.5$

13	63-18x = 43 –10x	M1
	4y =21-15	M1
	x = 2.5 y = 1.5	A1

Worked Solutions

Interpretation following Chi Squared test.

(ii) For each category of runner, comment briefly on how the type of running compares with what would be expected if there were no association. **[6]**

	Category of runner		
	Junior	Senior	Veteran
Track	9	8	2
Road	4	8	12
Both	4	10	6

EXPECTED	Junior	Senior	Veteran
Track	5.13	7.84	6.03
Road	6.48	9.90	7.62
Both	5.40	8.25	6.35

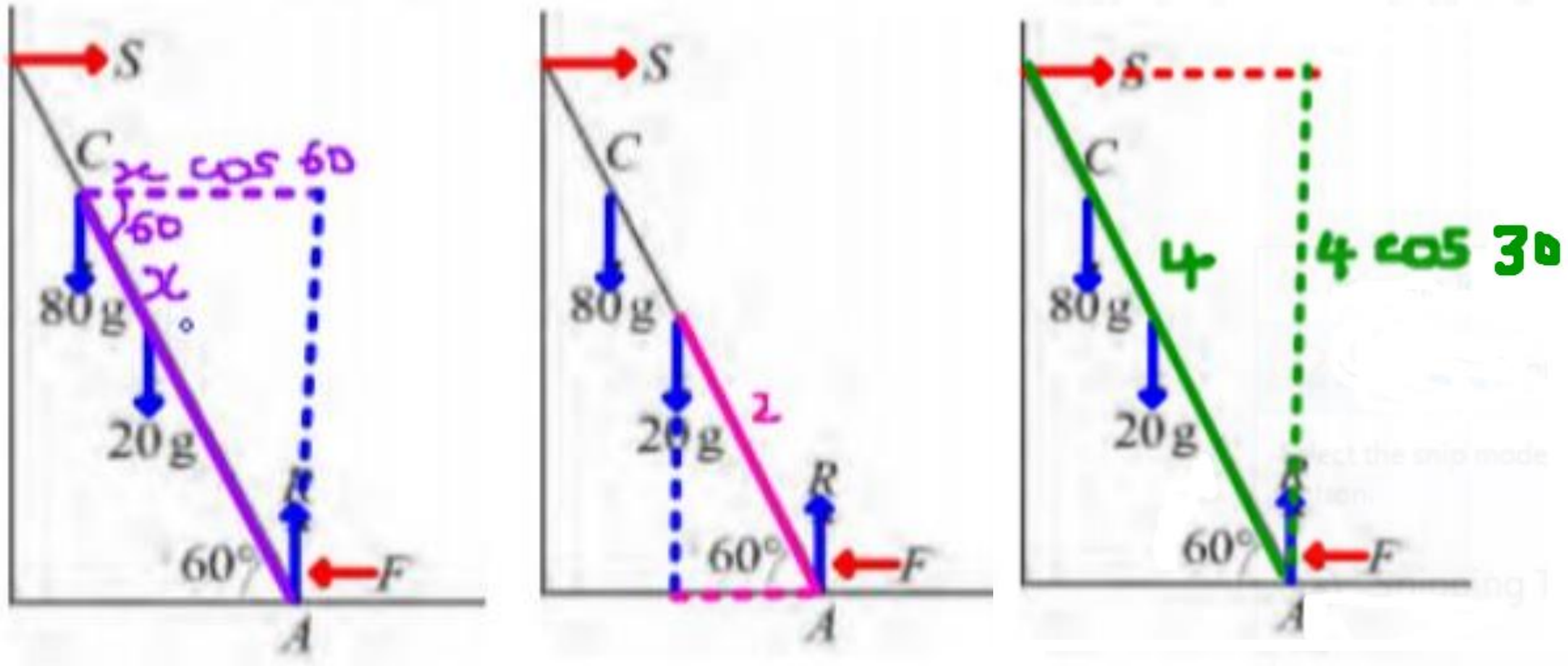
CONTRIBUTN	Junior	Senior	Veteran
Track	2.9257	0.0032	2.6949
Road	0.9468	0.3663	2.5190
Both	0.3615	0.3694	0.0192

• Juniors appear be track runners more often than expected and road less often than expected.	E1 E1	
• Seniors tend to be as expected in all three categories of running.	E1 E1	
• Veterans tend to be road runners more than expected and track runners less than expected.	E1 E1	**6**

Worked Solutions

Mechanics- moments

Uniform ladder, length 4m, mass 20kg



Moments about *A*:

$$80g x \cos 60 + 20g.2\cos 60 = S.4\cos 30$$

$$40gx + 20g = 138.56g$$

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Mathematics support materials

The **quotient rule** states that if u and v are both functions of x and y then

$$\text{if} \quad y = \frac{u}{v}\,, \quad \text{then} \quad \frac{dy}{dx} = \frac{\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)}{v^2}$$

Note the minus sign in the numerator!

Example 2 Consider $y = 1/\sin(x)$. The derivative may be found by writing $y = u/v$ where:

$$u = 1\,, \quad \Rightarrow \quad \frac{du}{dx} = 0 \qquad \text{and} \qquad v = \sin(x)\,, \quad \Rightarrow \quad \frac{dv}{dx} = \cos(x)$$

Inserting this into the **quotient rule** above yields:

$$\frac{dy}{dx} = \frac{\sin(x) \times 0 - 1 \times \cos(x)}{\sin^2(x)}$$

$$= -\frac{\cos(x)}{\sin^2(x)}$$

The census-taker problem

Example 1.1.3 A census-taker knocks on a door, and asks the woman inside how many children she has and how old they are.

"I have three daughters, their ages are whole numbers, and the product of the ages is 36," says the mother.

"That's not enough information," responds the census-taker.

"I'd tell you the sum of their ages, but you'd still be stumped."

"I wish you'd tell me something more."

"Okay, my oldest daughter Annie likes dogs."

What are the ages of the three daughters?

Zeitz, P., 2007. *The art and craft of problem solving*. 2nd ed. John Wiley & Sons, Inc, p.2

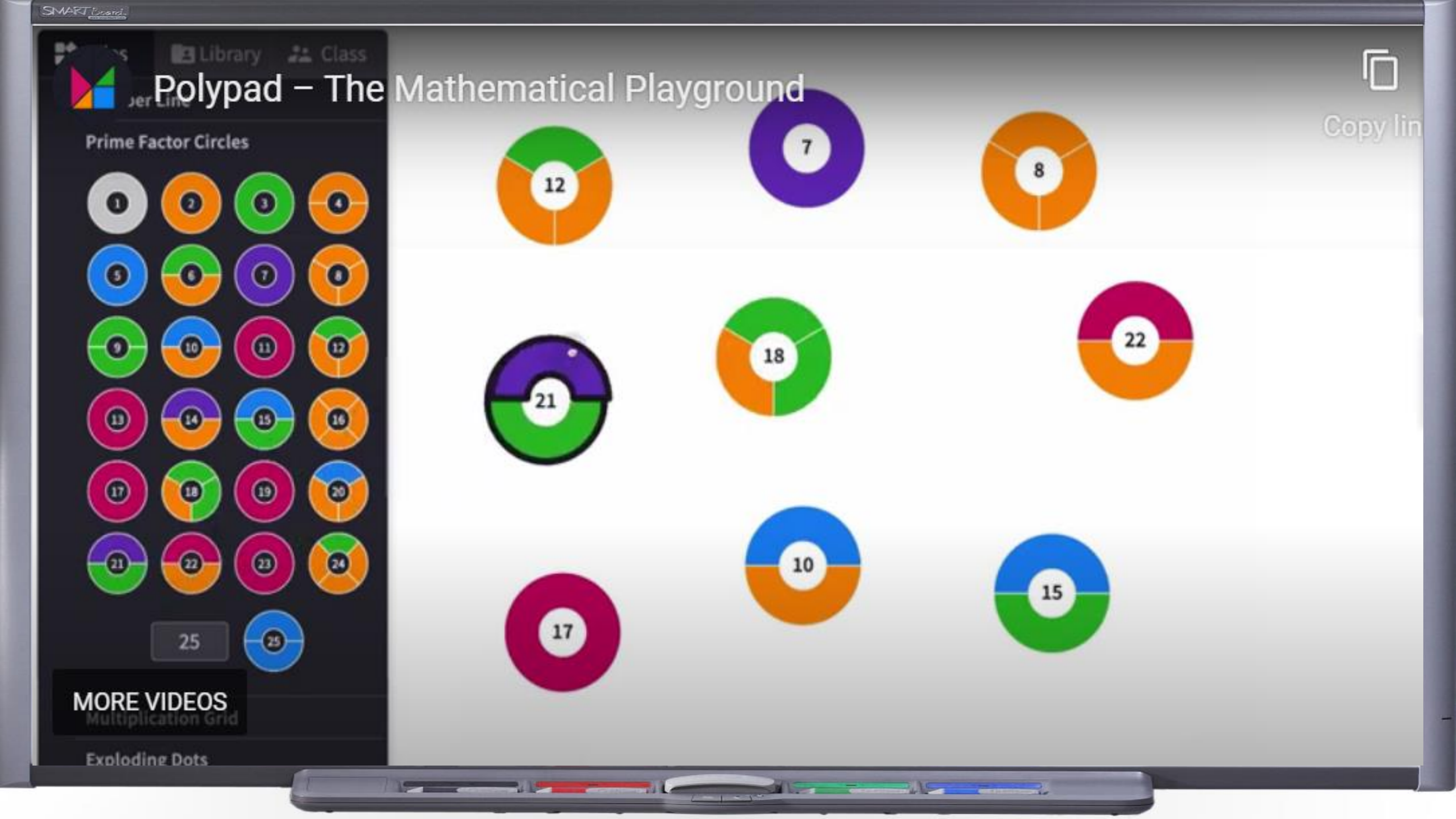
SMART Board

36 2 2 3 3

List the ways you can write 36 as a product of 3 numbers.

Find the sum in each case

1	1	36	38		1	6	6	13
1	2	18	21		2	2	9	13
1	3	12	16		2	3	6	11
1	4	9	14		3	3	4	10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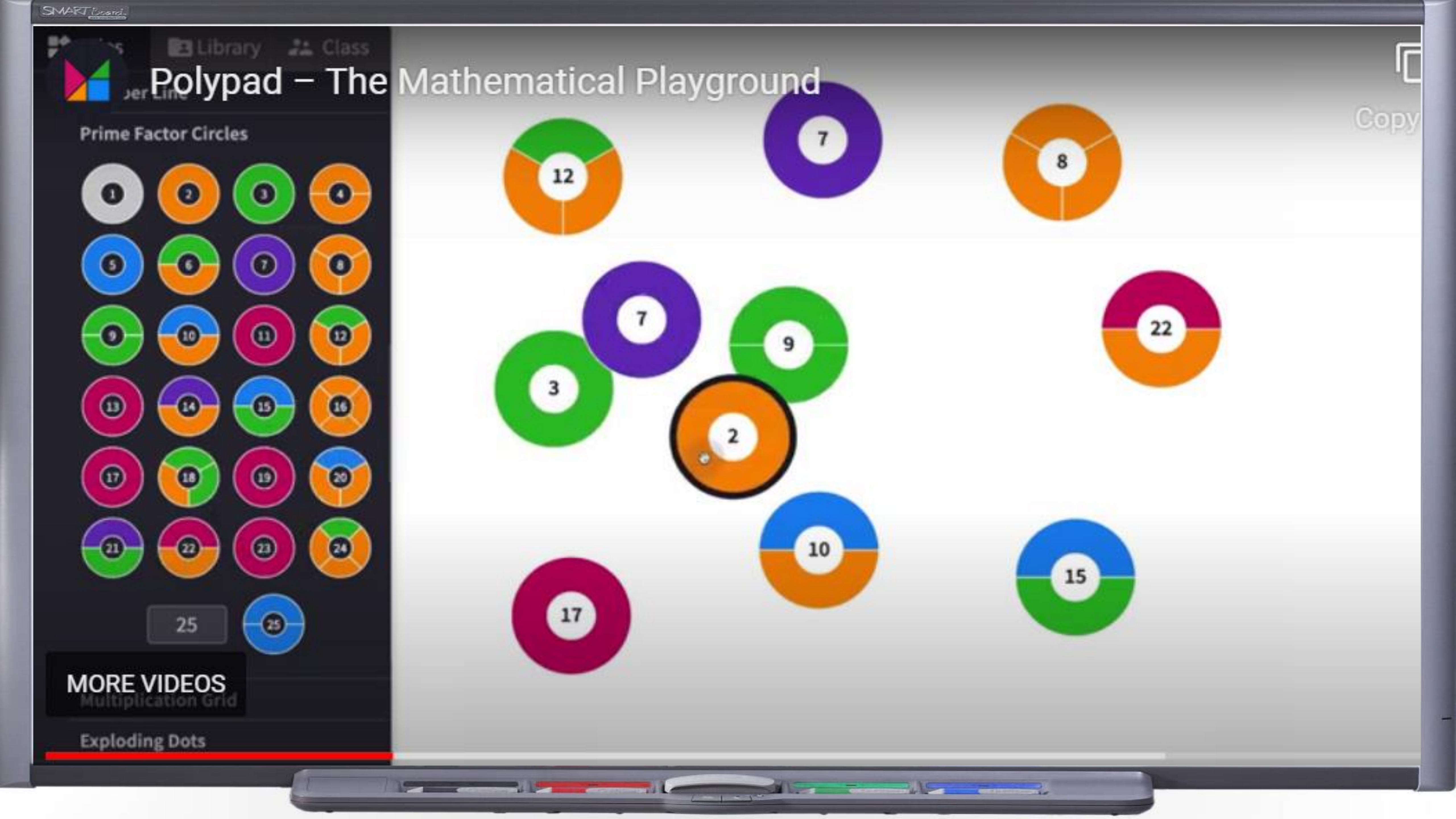
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5 6 7 8
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13 14 15 16
17 18 19 20
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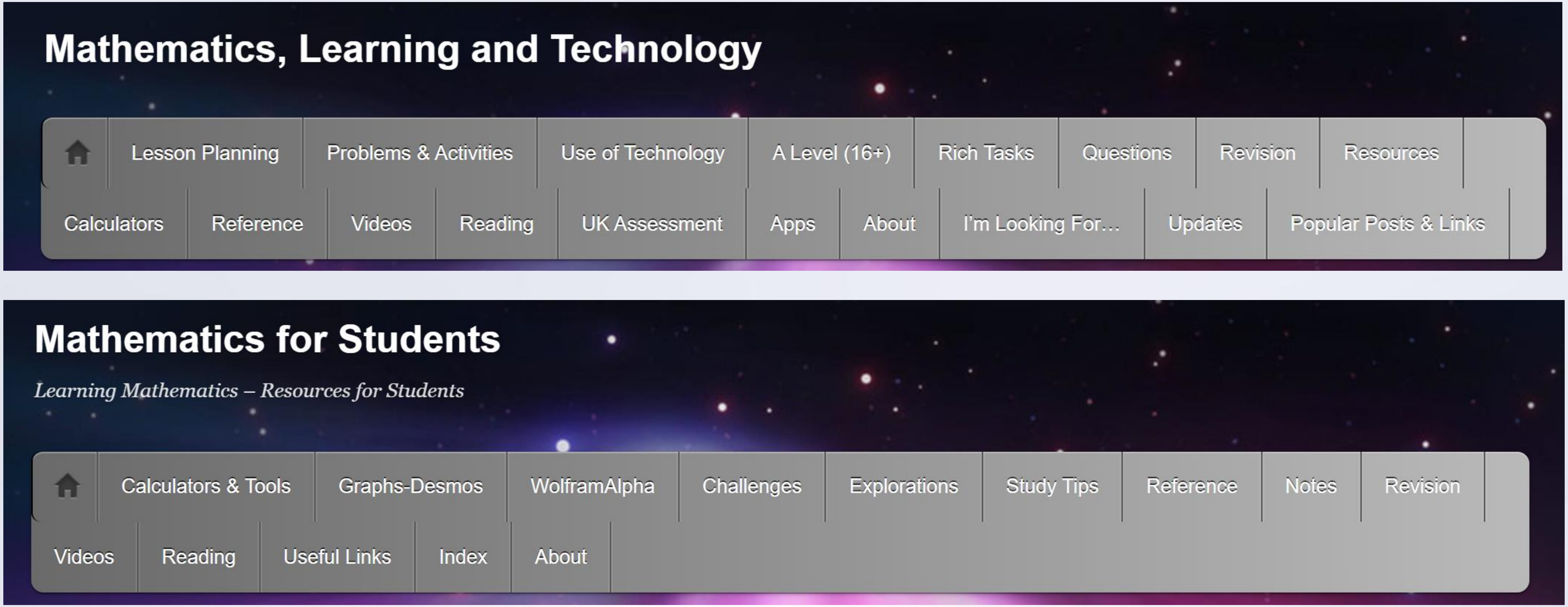

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@ColleenYoung