*Improving learning in mathematics

Re-visiting the Standards Unit Box and its successors

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A person of many talents, Malcolm's exceptional skill was in the design of tools that enable typical teachers to make specific research insights into a happy reality in their classrooms.

http://www.mathshell.org/swan.htm

* Aims of the session

To:

* celebrate the life and work of Malcolm Swan;

* experience some of the classroom resources he designed;

* note their applicability to today's classroom.

Reasons for using the resource

* Developed with practitioners in trials and pilots.
* World class
* Interactive.
* Motivating.
* Based on concept development and problem solving.

* What's in the box?

* Activities for learners.
* CPD resources for managers, teachers and trainers.
* Video footage
* Book on effective teaching approaches

*Where do I find it?

* At least 8 copies delivered to every FE college in England. Ask around.
* Sometimes available on eBay!

* National STEM Centre e-library – teaching resources

https://www.stem.org.uk/resources/collection/2938/teaching-activities-and-materials

* National STEM Centre e-library – booklet https://www.stem.org.uk/elibrary/resource/26057
* Mrbartonmaths – teaching resources clearly listed http://mrbartonmaths.com/teachers/rich-tasks/standards-units.html

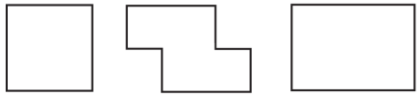
* Principles for effective teaching

* Build on the knowledge learners bring to sessions.
* Expose and discuss misconceptions.
* Develop effective questioning.
* Use cooperative small group work.
* Emphasise methods rather than answers.
* Use rich collaborative tasks.
* Create connections between mathematical topics.
* Use technology in appropriate ways.

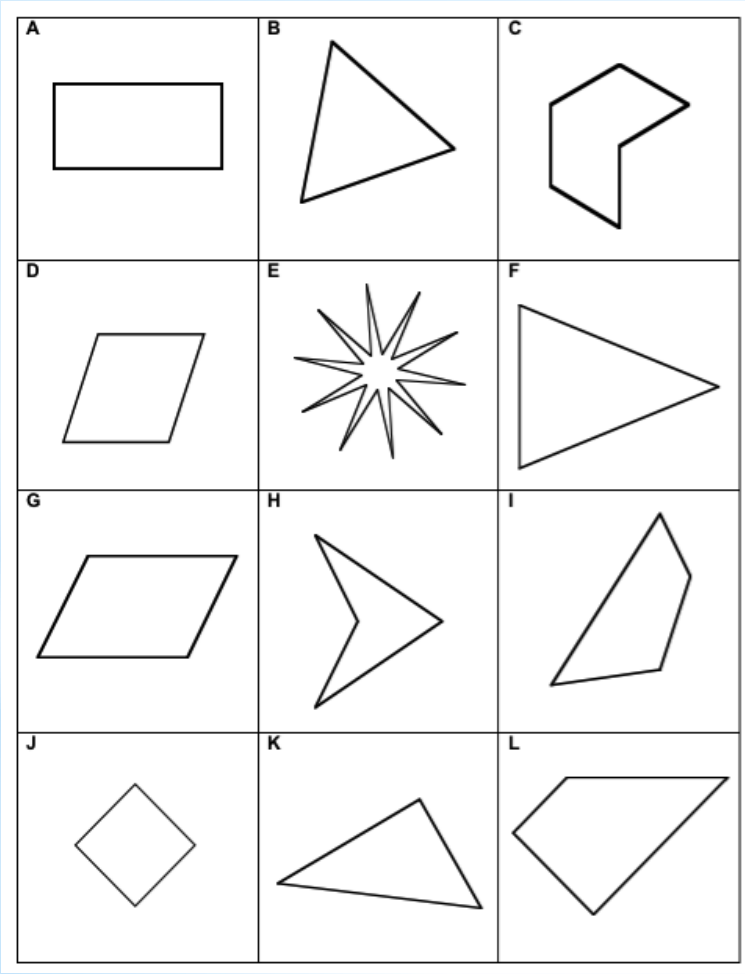
* 1. Classifying mathematical objects

* Learners examine and classify mathematical objects according to their different attributes. They create and use categories to build definitions, learning to discriminate carefully and to recognise the properties of objects. They also develop mathematical language.

*In each triplet, justify each of a, b, and c as the ‘odd one out’

(a) a fraction (b) a decimal (c) a percentage	(a) sin 60° (b) cos 60° (c) tan 60°
(a) (b) (c) 	(a) $y = x^2 - 6x + 8$ (b) $y = x^2 - 6x + 9$ (c) $y = x^2 - 6x + 10$
(a) (b) (c) 	(a) 20, 14, 8, 2, … (b) 3, 7, 11, 15, … (c) 4, 8, 16, 32, …

* Classifying using 2-way tables



	No rotational symmetry	Rotational symmetry
No lines of symmetry		
One or two lines of symmetry		
More than two lines of symmetry		

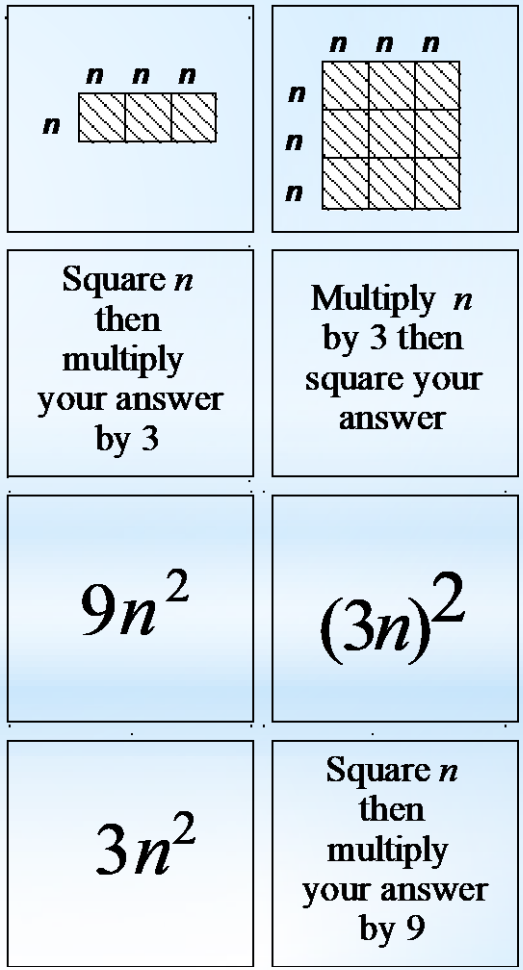
* Classifying using 2-way tables

$y = x^2 + 2x + 4$	$y = x^2 - 5x + 4$
$y = 2x^2 - 5x - 3$	$y = x^2 - 4x + 4$
$y = x^2 + 7x - 3$	$y = 4 + 3x - x^2$
$y = x^2 + 5x - 2$	$y = 6x - x^2 - 9$
$y = x^2 - 3x - 1$	$y = x^2 + 10x + 9$
$y = x^2 + x + 3$	$y = x^2 + 4x + 4$
$y = x^2 - 2\sqrt{3}x + 3$	$y = 3x - x^2 + 7$

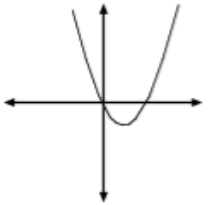
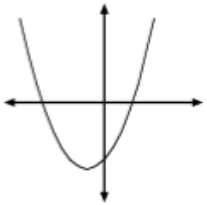
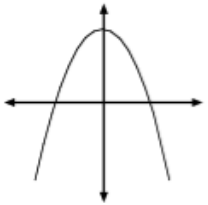
	Factorises with integers	**Does not factorise with integers**
Two *x* intercepts		
No *x* intercepts		
Two equal *x* intercepts		
Has a minimum point		
Has a maximum point		
***y* intercept is 4**		

* 2. Interpreting multiple representations

* Learners match cards showing different representations of the same mathematical idea.
* They draw links between different representations and develop new mental images for concepts.

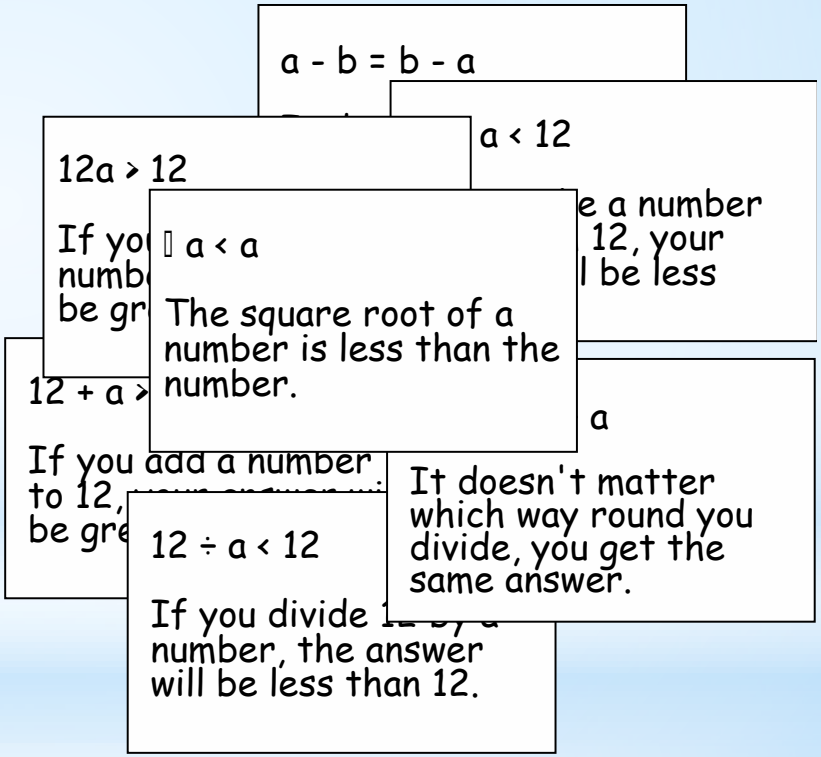


* 2. Interpreting multiple representations

		
$f(x) = 16 - x^2$	$f(0) = 0$	$f(0) > 0$
$f(x) = x^2 + 2x - 3$	$f'(0) = 0$	$f'(0) > 0$
$f(x) = x(x-2)$	$f'(0) < 0$	$f(0) < 0$

* 3. Evaluating mathematical statements

* Learners decide whether given statements are always, sometimes or never true.
* They are encouraged to develop:
  * rigorous mathematical arguments and justifications;
  * examples and counterexamples to defend their reasoning.



* Always, sometimes or never true?

$a \times b = b \times a$ It doesn't matter which way round you multiply, you get the same answer.	$a \div b = b \div a$ It doesn't matter which way round you divide, you get the same answer.
$12 + a > 12$ If you add a number to 12 you get a number greater than 12.	$12 \div a < 12$ If you divide 12 by a number the answer will be less than 12.
$\sqrt{a} < a$ The square root of a number is less than the number.	$a^2 > a$ The square of a number is greater than the number.

*Always, sometimes or never true?

A

When you cut a piece off a shape you reduce its area and perimeter.

*Always, sometimes or never true?

$p + 12 = s + 12$	$3 + 2y = 5y$
$n + 5$ is less than 20	$4p > 9 + p$
$2(x + 3) = 2x + 3$	$2(3 + s) = 6 + 2s$

*True, false or unsure?

When you roll a fair six-sided die, it is harder to roll a six than a four.	Scoring a total of three with two dice is twice as likely as scoring a total of two.
In a lottery, the six numbers 3, 12, 26, 37, 38, 40 are more likely to come up than the numbers 1, 2, 3, 4, 5, 6.	In a 'true or false' quiz with ten questions, you are certain to get five right if you just guess.
If a family has already got four boys, then the next baby is more likely to be a girl than a boy.	The probability of getting exactly three heads in six coin tosses is $\frac{1}{2}$

* 4. Creating and solving problems

* Learners devise their own mathematical problems for other learners to solve.
* Learners are creative and ‘own’ the problems.
* While others attempt to solve them, learners take on the role of teacher and explainer.
* The ‘doing’ and ‘undoing’ processes of mathematics are exemplified.

* Developing an exam question

Van hire

Sanjay wants to hire a van to move some furniture.
He obtains the following information from two hire companies.

Bujit's Van Hire

£30 for the first 50 miles

Every mile after that costs an extra 20p

Hurt's vans

You only pay for the miles you travel.

Miles travelled	50	100	150	200
Hire charge	£16	£32	£48	£64

1. How much do Hurt's vans cost per mile?
2. Sanjay expects to travel 175 miles.
   Which company has the lower charge for this distance?
   You must show all your working.

* Developing an exam question

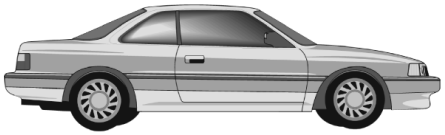
Cath wants to hire a car for a weekend.

She obtains the following information from two hire companies.

..... Car Hire

£ .....for the first .....miles.
Every mile after that costs an extra ..... p.

..... Car Hire



Miles travelled				
Hire charge				

. . . . . .

. . . . . .

. . . . . .

* Doing and undoing processes

Kirsty created an equation, starting with $x = 4$.

$$5\left(\frac{x+2}{3}\right) + 4 = 14$$

She then gave it to another learner to solve.

* Doing and undoing processes

Doing: **The problem poser…**	**Undoing:** **The problem solver…**
generates an equation step-by-step, ‘doing the same to both sides’.	solves the resulting equation.
draws a rectangle and calculates its area and perimeter.	tries to draw a rectangle with the given area and perimeter.
writes down an equation of the form $y=mx+c$ and plots a graph.	tries to find an equation that fits the resulting graph.

* Doing and undoing processes

Doing: **The problem poser…**	**Undoing:** **The problem solver…**
expands an algebraic expression such as (x+3)(x-2).	factorises the resulting expression: x^2+x-6.
writes down a polynomial and differentiates it.	integrates the resulting function.
writes down five numbers and finds their mean, median and range.	tries to find five numbers with the given mean, median and range.

* 5. Analysing reasoning and solutions

Learners compare different methods for doing a problem, organise solutions and/ or diagnose the causes of errors in solutions. They begin to recognise that there are alternative pathways through a problem, and develop their own chains of reasoning.

* Analysing reasoning and solutions

Comparing different solution strategies

Paint prices  1 litre of paint costs £15. What does 0.6 litres cost?	Chris: It is just over a half, so it would be about £8. Sam: I would divide 15 by 0.6. You want a smaller answer. Rani: I would say one fifth of a litre is £3, so 0.6 litres will be three times as much, so £9. Tim: I would multiply 15 by 0.6.

*Analysing reasoning and solutions

Correcting mistakes in reasoning.

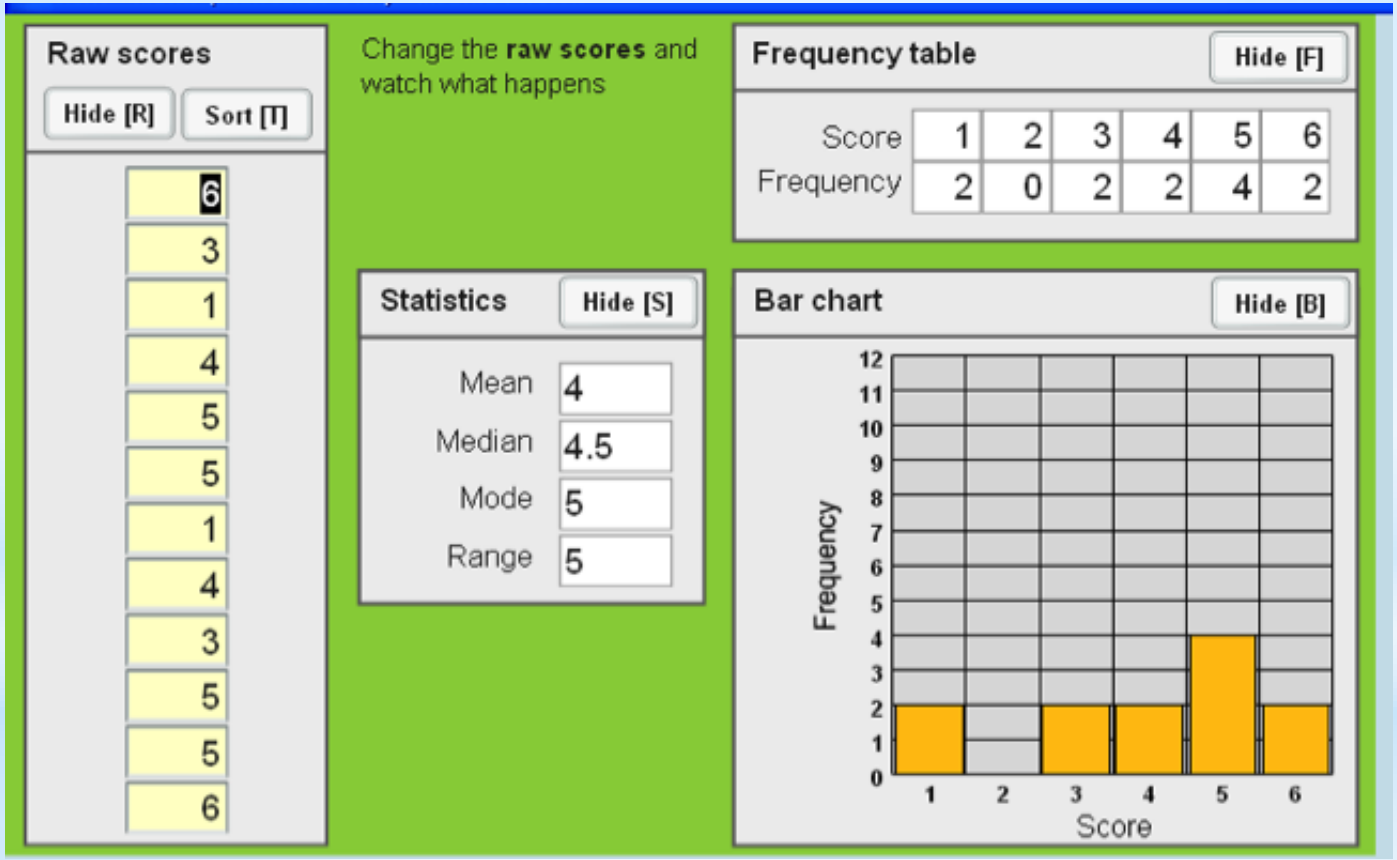
- In January, fares went up by 20%.
- In August, they went down by 20%.
- Sue claims that: “The fares are now back to what they were before the January increase.”
- Do you agree?

* Analysing reasoning

Putting reasoning in order

$y=x^3-4x^2+5x+11$	$y=x^3-x^2-x+5$
$y=x^3-7x^2-5x+9$	$\frac{dy}{dx}=3x^2-2x-1$
$\frac{d^2y}{dx^2}=6x-8$	$\frac{dy}{dx}=3x^2-14x-5$
At $x=-\frac{1}{3}$, $\frac{d^2y}{dx^2}=$	$x=\frac{5}{3}$, $x=1$
$3x^2-8x+5=0$	$\frac{d^2y}{dx^2}=6x-2$
At $x=1$, $\frac{d^2y}{dx^2}=$	$x=-\frac{1}{3}$, $x=5$

$(3x+1)(x-5)=0$	At $x=1$, $\frac{d^2y}{dx^2}=$ .....
$3x^2-14x-5=0$	$3x^2-2x-1=0$
$x=-\frac{1}{3}$, $x=1$	$(3x+1)(x-1)=0$
$x=-\frac{1}{3}$, $\frac{d^2y}{dx^2}=$	$\frac{dy}{dx}=3x^2-8x-5$
At $x=\frac{5}{3}$, $\frac{d^2y}{dx^2}=$	$(3x-5)(x-1)=0$
$\frac{d^2y}{dx^2}=6x-14$	At $x=5$, $\frac{d^2y}{dx^2}=$ .....
Maximum is at	Minimum is at .....
Maximum is at	Minimum is at .....
Maximum is at	Minimum is at .....



http://wirksworthii.nottingham.ac.uk/Improv_Learning_Maths/screens/math_004_008_005/page.html

*“Offspring” of ILIM

Thinking through mathematics (mostly Level 1) https://www.ncetm.org.uk/online-cpd-modules/ttm (free registration required)

Mathematics Assessment Programme (MAP) (designed for USA market) http://map.mathshell.org/ (7,000,000 lessons downloaded to date)